ივ. ჯავახიშვილის სახელობის

თბილისის სახელმწიფო უნივერსიტეტის

შრომები

PROCEEDINGS

of Javakhishvili

TBILISI STATE UNIVERSITY

გამოყენებითი მათემატიკა • კომპიუტერული მეცნიერებანი

Applied Mathematics • Computer Sciences

ტომი
Volume 342 (20)

თბილისი **Tbilisi**

2000

ივ. ჯავახიშვილის სახელობის
თბილისის სახელმწიფო უნივერსიტეტის
შრომები

PROCEEDINGS
OF JAVAKHISHVILI TBILISI STATE UNIVERSITY

ტომი
Volume **342** (20)

გამოყენებითი მათემატიკა • კომპიუტერული მეცნიერებანი
Applied Mathematics • **Computer Sciences**

თბილისის უნივერსიტეტის გამომცემლობა
Tbilisi University Press
თბილისი **Tbilisi**
2000

PROCEEDINGS

of Javakhishvili

TBILISI STATE UNIVERSITY

ივ. ჯავახიშვილის სახელობის
თბილისის სახელმწიფო უნივერსიტეტის
შრომები

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 5-8

გამოყენებითი მათემატიკა • APPLIED MATHEMATICS

NECESSARY CONDITIONS OF EXTERNALITY OF INITIAL MOMENT FOR ONE CLASS VARIATION PROBLEM WITH DELAY ARGUMENT

L. Alkhazishvili, T. Tadumadze
Control Theory Chair

Abstract. Necessary conditions of extremality are obtained in the form of Euler's equation, the condition of Wierstrass-Erdmann and transversality condition. The condition in the initial moment unike the early known condition, contains a new member.

Let $J=[a,b]$ be a finite interval and $O\subset R^n$ be an open set; the function $f(t,x_1,x_2,x_3,)$ is defined on $J\times O\times O\times R^n$ and satisfies the following conditions: for almost all $t\in J$, the function f is continuously differentiable with respect to (x_1,x_2,x_3) for each fixed $(x_1,x_2,x_3)\in O^2\times R^n$, the functions $f, f_{x_i}, i=1,2,3$ are measurable on J; for arbitrary compacts $K\subset O,\ V\subset R^n$ there exists the summable function $m_{K,V}(t),\ t\in J$, such that

$$\left|f(t,x_1,x_2,x_3)\right|+\sum_{i=1}^{3}\left|f_{x_i}(\cdot)\right|\le m_{K,V}(t),\ \forall(t,x_1,x_2,x_3)\in J\times K^2\times V.$$

Further, let Φ be a set of absolutely continuous functions $x(t)\in O,\ t\in J$, satisfying the condition $|\dot{x}(t)|\le const$. $\dot{\tau}(t)>0,\ t\in J$ is absolutely continuous function satisfying the conditions $\tau(t)\le t,\ \dot{\tau}(t)>0$; $\varphi(t)\in O,\ t\in[\tau(a),b]$ is piecewise continuous function with a finite number of discontinuity points, satisfying the conditon $cl\{\varphi(t):t\in[\tau(a),b]\}\subset O$; $a_0,\ a_1\subset O$ are fixed points.

Let us consider the variational problem

$$I(z)=\int_{t_0}^{t_1}f(t,x(t),x_{t_0}(\tau(t)),\dot{x}(t))dt\to min,\ z=(t_0,t_1,x(\cdot))\in A=J^2\times O.$$

$$x(t_0)=a_0,\ x(t_1)=a_1, \tag{1}$$

where,

$$x_{t_0}(t)=\begin{cases}\varphi(t),\ t\in[\tau(a),t_0),\\ x(t),t\in[t_0,b].\end{cases}$$

<u>DEFINITION1.</u> The element $z\in A$ is said to be admissible, if the condition holds. The set of admissible elements will be denoted by A_0.

<u>DEFINITION2.</u> The element $\widetilde{z}=(\widetilde{t}_0,\widetilde{t}_1,\widetilde{x}(\cdot))\in A_0$ is said to be locally extremal, if there exists a number $\delta>0$ such that for an arbitrary element $z\in A_0$ satisfying

$$|\widetilde{t}_0-t_0|+|\widetilde{t}_1-t_1|+\max_{t\in J}|\widetilde{x}(t)-x(t)|\le\delta$$

the inequality $I(\dot{z})\le I(z)$ holds. Variational problem consists in finding locally extremal element.

<u>THEOREM1</u>. *Let $\widetilde{z}\in A_0$ be a locally extremal element,*

$$\widetilde{t}_0\in(a,b),\ \widetilde{t}_1\in(a,b],\ y_0=y(\widetilde{t}\,)\in[\widetilde{t}_0,\widetilde{t}_1\}$$

And there exists the finite limits: $\dot{\widetilde{x}}(\widetilde{t}_0^{\,-}),\ \dot{\widetilde{x}}(\widetilde{t}_1^{\,-}),\ \dot{y}(\widetilde{t}_0^{\,-}),\ f_{x_3}[\widetilde{t}_1^{\,-}],\ f_{x_3}[\widetilde{t}_0^{\,+}]$;

$$\lim_{\omega\to\omega_0^-}\widetilde{f}(\omega)=f_0^-,\quad \omega\in R^-_{\widetilde{t}_0}\times O^2\quad i=1,2;\quad \omega_0^-=\left(\widetilde{t}_0,a_0,\varphi\left(\tau(\widetilde{t}_0^{\,-})\right)\right),\quad R^-_{\widetilde{t}_0}=\left]-\infty,\widetilde{t}_0\right];$$

$$\lim_{\omega\to\omega_3^-}\widetilde{f}(\omega)=f_2^-,\ \omega\in R^-_{\widetilde{t}_1-}\times O^2,\ \omega_3^-=\left(\widetilde{t}_1,\widetilde{x}(t_1),\widetilde{x}(\tau(t_1))\right).$$

Then the following conditions are fulfilled:

1) for almost all $t\in[\widetilde{t}_0,\widetilde{t}_1]$

$$\widetilde{f}_{x_3}[t]=\widetilde{f}_{x_3}[\widetilde{t}_0^{\,+}]+\int_{\widetilde{t}_0}^{t}\left(\widetilde{f}_{x_1}[s]+\chi(\gamma(s))\widetilde{f}_{x_2}[\gamma(s)]\dot{\gamma}(s)\right)ds,$$

where $\chi(t)$ is characteristic function of interval $[\widetilde{t}_0,\widetilde{t}_1]$, $y(t)$ is the function inverse to $\tau(t)$,

$\widetilde{f}[t]=\widetilde{f}(t,\widetilde{x}(t),\widetilde{x}_{\widetilde{t}_0}(\tau(t)),\dot{\widetilde{x}}(t))$

2) if at poin $t\in(\widetilde{t}_0,\widetilde{t}_1)$ the function $\widetilde{f}_{x_3}[t]$ has the one-side limits, then

$$\widetilde{f}_{x_3}[t^-]=\widetilde{f}_{x_3}[t^+];$$

3)

$$\begin{cases}\widetilde{f}_{x_3}[\widetilde{t}_0^{\,+}]\dot{\widetilde{x}}(t_0^-)\le f_0^-+f_1^-\dot{\gamma}(\widetilde{t}_0^{\,-}),\\ \widetilde{f}_x[\widetilde{t}_1^{\,-}]\dot{\widetilde{x}}(\widetilde{t}_1^{\,-})\ge f_2^-.\end{cases}$$

<u>THEOREM2</u>. *Let $\widetilde{z}\in A_0$ be a locally extremal element, $\widetilde{t}_0\in[a,b),\ \widetilde{t}_1\in(a,b),\ \gamma_0\in[\widetilde{t}_0,\widetilde{t}_1)$; and there exist the finite limits $\dot{\widetilde{x}}(\widetilde{t}_0^{\,+}),\ \dot{\widetilde{x}}(\widetilde{t}_1^{\,+}),\ \dot{y}(\widetilde{t}_0^{\,+}),\ \widetilde{f}_{x_3}[\widetilde{t}_1^{\,-}],\ \widetilde{f}_{x_2}[\widetilde{t}_0^{\,+}]$*

$$\lim_{\omega\to\omega_0}\widetilde{f}(\omega)=f_0^+,\ \omega\in R^+_{\widetilde{t}_0}\times O^2,\ i=1,2;\ \omega_0^+=\left(\widetilde{t}_0,a_0,\varphi(\tau(\widetilde{t}_0))\right);$$

$$\lim_{(\omega_1,\omega_2)\to(\omega_1^0,\omega_2^+)}\left|\widetilde{f}(\omega_1)-\widetilde{f}(\omega_2)\right|=f_1^+,\ \omega_i\in R^+_{\widetilde{t}_0}\times O^2,\ i=1,2;\ \omega_2^+=\left(\gamma_0,\widetilde{x}(\gamma_0),\widetilde{\varphi}(\widetilde{t}_0^{\,+})\right);$$

$$\lim_{\omega\to\omega_3^+}\widetilde{f}(\omega)=f_2^+,\ \omega\in R^+_{\widetilde{t}_1}\times O^2,\ \omega_3^+=\left(\widetilde{t}_1,\widetilde{x}(\widetilde{t}_1),\widetilde{x}(\tau(t_1^+))\right);$$

Then the conditions 1), 2) are fulfilled and, moreover, *4)*

$$\begin{cases}\tilde{f}_{x_3}[\tilde{t}_0^+]\dot{\tilde{x}}(\tilde{t}_0^+) \geq f_0^+ + f_1^+\dot{\gamma}(\tilde{t}_0^+),\\ \tilde{f}_{x_3}[\tilde{t}_1^-]\dot{\tilde{x}}(\tilde{t}_1^+) \leq f_2^+.\end{cases}$$

<u>THEOREM3</u>. *Let* $\tilde{z} \in A_0$ *be a locally extremal element,* $\tilde{t}_0 \in (a,b)$, $\tilde{t}_1 \in (a,b)$, $\gamma_0 \in (\tilde{t}_0, \tilde{t}_1)$ *and the assumptions of theorems 1,2 are fulfilled. Let, besides:*

$$f_0^+ + f_1^+\dot{\gamma}(\tilde{t}_0^+) = f_0^- + f_1^-\dot{\gamma}(\tilde{t}_0^-) = f_1, \quad f_2^+ = f_2^- = f_2,$$
$$\dot{\tilde{x}}(\tilde{t}_0^-) = \dot{\tilde{x}}(\tilde{t}_0^+) = \dot{\tilde{x}}_0, \qquad \dot{\tilde{x}}(\tilde{t}_1^-) = \dot{\tilde{x}}(\tilde{t}_1^+) = \dot{\tilde{x}}_1.$$

Then the conditions 1) ,2) are fulfilled, and

5)

$$\begin{cases}\tilde{f}_{x_3}[\tilde{t}_0^+]\dot{\tilde{x}}_0 = f_1,\\ \tilde{f}_{x_3}[\tilde{t}_1^-]\dot{\tilde{x}}_1 = f_2.\end{cases}$$

<u>REMARK.</u> Assume that the function $\dot{\gamma}(t)$ is continuous at point $\tilde{t}_0$, the function $\varphi(t)$ is continuous on $[\tau(a), b]$; the function $f(t, x_1, x_2, x_3)$ is continuous at points $(\tilde{t}_0, a_0, \varphi(\tau(\tilde{t}_0)))$, $(\gamma_0, \tilde{x}(\gamma_0, a_0))$, $(\gamma_0, \tilde{x}(\gamma_0), \varphi(\tilde{t}_0))$, $(\tilde{t}_1, \tilde{x}(\tilde{t}_1), \tilde{x}(\tau(\tilde{t}_1)))$; $\dot{\tilde{x}}(t)$ is continuous at points $\tilde{t}_0, \tilde{t}_1$. Then it is clear that in Theorem 3

$$\dot{\tilde{x}}_0 = \dot{\tilde{x}}(\tilde{t}_0), \quad \dot{\tilde{x}}_1 = \dot{\tilde{x}}(\tilde{t}_1)$$
$$f_1 = \tilde{f}[\tilde{t}_0] + \big(f(\gamma_0, \tilde{x}(\gamma_0), a_0, \dot{\tilde{x}}(\gamma_0)) - f(\gamma_0, \tilde{x}(\gamma_0)\varphi(\tilde{t}_0), \dot{\tilde{x}}(\gamma_0))\dot{\gamma}(\tilde{t}_0),$$
$$f_2 = \tilde{f}[\tilde{t}_1], \quad \tilde{f}_{x_3}[\tilde{t}_0^+] = \tilde{f}_{x_3}[\tilde{t}_0], \quad \tilde{f}_{x_3}[\tilde{t}_1^-] = \tilde{f}_{x_3}[\tilde{t}_1].$$

These theorems have been proved in standard way [1], and are based on necessary conditions of optimality [2].

REFERENCES

1. L. S. Pontriagin, V. G. Boltyanski, R. V. Gamkrelidze, and E. F. Mishchenko. *The mathematical theory of Optimal Processes*. 1983, Moscow, Nauka, 268p.

2. T. Tadumadze. *On new necessary condition of optimality of initial moment in control problems with delay.* Mem. Differential Equations Math. Phys. 17 (1999) p.p.157-159.

საწყისი მომენტის ექსტრემალურობის აუცილებელი პირობა ერთი კლასის დაგვიანებულარგუმენტიანი ვარიაციული ამოცანისათვის

ლ. ალხაზიშვილი, თ. თადუმაძე
მართვის თეორიის კათედრა

მიღებულია ექსტრემალურობის აუცილებელი პირობები ეილერის განტოლების, ვეიერშტრას–ერდმანისა და ტრანსვერსალობის პირობების ფორმით. პირობა საწყის მომენტში ადრე ცნობილი პირობისაგან განსხვავებით შეიცავს ახალ წევრს.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 9-14

DECOMPOSITION SEQUENTIAL-PARALLEL SCHEME OF HIGH DEGREE PRECISION FOR NON-HOMOGENOUS EVOLUTION EQUATION

Z. Gegechkori, J. Rogava, M. Tsiklauri
Computer Software and Informational Technologies chair

Abstract. The Cauchy abstract problem for non-homogenous evolution equations is considered in Banach space in case of limited operator. Is constructed sequential-parallel decomposition scheme with third degree precision. For the approximation of solution explicit prior estimations are obtained.

As is well known, decomposition method is sufficiently general for obtaining the economical schemes for the solution of the multidimensional problems of mathematical physics. They can be divided in two groups: the schemes of sequential account (N. N. Ianenko [1], A. A. Samarskii [2], E. G. Diakonov [3], Marchuk G. I. [4], D.G. Gordeziani [5], Temam R. [6], Gegechkori Z. G. and Demidov G. V. [7]) and the schemes of parallel account (D. G. Gordeziani and H. V. Meladze [8], [9], D. G. Gordeziani and A. A. Samarskii [10]).

In the above-stated works the considered schemes are of the first or second precision order. As far as we know, the high degree precision decomposition formulas in case of two addands $\left(A = A_1 + A_2\right)$ for the first time were obtained in the work [11].

In the present work, there a symmetrized sequential-parallel method of the third degree precision is offered for the solution of Cauchy abstract problem in case of bounded operator.

The present scheme may be generalized for any finite number of addends $\left(A = A_1 + A_2 + \ldots + A_m,\quad m \geq 2\right)$.

Let us consider Cauchy abstract problem in Banach space X:

$$\frac{du(t)}{dt} + Au(t) = f(t), \quad t > 0,\ u(0) = \varphi. \tag{1}$$

Here A is bounded linear operator, φ is a given element from X, $f(t) \in C([0,\infty); X)$.

The solution of the problem (1) is given by the following formula:

$$u(t) = U(t, A)\varphi + \int_0^t U(t-s, A) f(s) ds, \tag{2}$$

where

$$\mathrm{U(t, A)} = \exp(-\mathrm{tA}) = \sum_{\mathrm{k}=0}^{\infty} (-1)^{\mathrm{k}} \frac{t^k}{k!} A^k.$$

Let $A = A_1 + A_2$, where A_i, (i=1,2) are bounded linear operators in X.

Let us introduce difference net domain:

$$\varpi_\tau = \{ t_k = k\tau : \tau > 0, k = 1,2,..\}.$$

Along with problem (1) on each $[t_{k-1}, t_k]$ interval we consider two sequences of the following problems:

$$\begin{cases} \dfrac{dv_k^1(t)}{dt} + \alpha A_1 v_k^1(t) = \dfrac{\alpha}{2} f(t_k) - 2\sigma_0 (t_k - t) f'(t_k), \\ v_k^1(t_{k-1}) = u_{k-1}(t_{k-1}), \\ \dfrac{dv_k^2(t)}{dt} + A_2 v_k^2(t) = \dfrac{1}{2} f(t_k) - 2\sigma_1 (t_k - t) f'(t_k), \\ v_k^2(t_{k-1}) = v_k^1(t_k), \\ \dfrac{dv_k^3(t)}{dt} + \bar{\alpha} A_1 v_k^3(t) = \dfrac{\bar{\alpha}}{2} f(t_k) - 2\sigma_2 (t_k - t) f'(t_k) + \dfrac{(t_k - t)^2}{2} f''(t_k), \\ v_k^3(t_{k-1}) = v_k^2(t_k); \end{cases}$$

$$\begin{cases} \dfrac{dw_k^1(t)}{dt} + \alpha A_2 w_k^1(t) = \dfrac{\alpha}{2} f(t_k) - 2\sigma_0 (t_k - t) f'(t_k), \\ w_k^1(t_{k-1}) = u_{k-1}(t_{k-1}), \\ \dfrac{dw_k^2(t)}{dt} + A_1 w_k^2(t) = \dfrac{1}{2} f(t_k) - 2\sigma_1 (t_k - t) f'(t_k), \\ w_k^2(t_{k-1}) = w_k^1(t_k), \\ \dfrac{dw_k^3(t)}{dt} + \bar{\alpha} A_2 w_k^3(t) = \dfrac{\bar{\alpha}}{2} f(t_k) - 2\sigma_2 (t_k - t) f'(t_k) + \dfrac{(t_k - t)^2}{2} f''(t_k), \\ w_k^3(t_{k-1}) = w_k^2(t_k). \end{cases}$$

Here $\alpha, \sigma_0, \sigma_1, \sigma_2$ are numerical complex parameters, which will be defined later, $u_0(0) = \varphi$.

The function $u_k(t)$ on each $[t_{k-1}, t_k]$, $(k = 1,2,...)$ interval is defined as follows:

$$u_k(t) = \frac{1}{2}\left(v_k^3(t) + w_k^3(t)\right).$$

We declare the function $u_k(t)$ as the approached solution of the problem (1).

Above-stated scheme in case of homogenous equation is considered in [12].

<u>THEOREM.</u> *If* $\alpha = \dfrac{1}{2} \pm i \dfrac{1}{2\sqrt{3}}$ $\left(i = \sqrt{-1}\right)$, $f(t) \in C^3([0,\infty); X)$, *and the parameters* $\sigma_0, \sigma_1, \sigma_2$ *satisfy the following relations:*

$$\sigma_0 = \frac{2-\overline{\alpha}}{4+\alpha} - \frac{2+\overline{\alpha}}{4+\alpha}\sigma_1, \ \ \sigma_2 = \frac{1+\overline{\alpha}}{2(4+\alpha)} - \frac{3-2\overline{\alpha}}{4+\alpha}\sigma_1$$

where σ_1 is any complex number, then

$$\|u(t_k) - u_k(t_k)\| \le ce^{\omega t_k} t_k \tau^3 \left(\|\varphi\| + t_k \sup_{t\in[0,t_k]} \|f(t)\| + \right. \tag{3}$$
$$\left. + \sup_{t\in[0,t_k]} \|f'(t)\| + \sup_{t\in[0,t_k]} \|f''(t)\| + \sup_{t\in[0,t_k]} \|f'''(t)\| \right)$$

where c, ω are positive constants.

SCHEME OF THE PROOF:

According to the property of semigroup the formula (2) we can transform as follows:

$$u(t_k) = U^k(\tau, A)\varphi + \sum_{i=1}^{k} U^{k-i}(\tau, A) F_i^{(1)}, \tag{4}$$

where

$$F_i^{(1)} = \int_{t_{i-1}}^{t_i} U(t_i - s, A) f(s) ds .$$

$u_k(t_k)$ can be written in the following expression:

$$u_k(t_k) = V^k(\tau)\varphi + \sum_{i=1}^{k} V^{k-i}(\tau) F_i^{(2)}, \tag{5}$$

where

$$V(\tau) = \frac{1}{2}\left[U(\tau, \overline{\alpha}A_1)U(\tau, A_2)U(\tau, \alpha A_1) + U(\tau, \overline{\alpha}A_2)U(\tau, A_1)U(\tau, \alpha A_2)\right]$$

$$F_i^{(2)} = \int_{t_{i-1}}^{t_i} V_0(\tau, t_i - s)\left(\frac{\alpha}{2} f(t_i) - 2\sigma_0(t_i - t) f'(t_i)\right) ds +$$
$$+ \int_{t_{i-1}}^{t_i} V_1(\tau, t_i - s)\left[\frac{1}{2} f(t_i) - 2\sigma_1(t_i - t) f'(t_i)\right] ds +$$
$$+ \int_{t_{i-1}}^{t_i} V_2(t_i - s)\left[\frac{\overline{\alpha}}{2} f(t_i) - 2\sigma_2(t_i - t) f'(t_i) + \frac{(t_i - t)^2}{2} f''(t_i)\right] ds$$

and

$$V_o(\tau,t)=\frac{1}{2}\left[U(\tau,\overline{\alpha}A_1)U(\tau,A_2)U(t,\alpha A_1)+U(\tau,\overline{\alpha}A_2)U(\tau,A_1)U(t,\alpha A_2)\right]$$

$$V_1(\tau,t)=\frac{1}{2}\left[U(\tau,\overline{\alpha}A_1)U(t,A_2)+U(\tau,\overline{\alpha}A_2)U(t,A_1)\right]$$

$$V_0(t)=\frac{1}{2}\left[U(t,\overline{\alpha}A_1)+U(t,\overline{\alpha}A_2)\right]$$

From the equalities (4) and (5) we have:

$$\begin{aligned} u(t_k)-u_k(t_k)&=\left[U^k(\tau,A)-V^k(\tau)\right]\varphi+\\ &+\sum_{i=1}^{k}\left[\left(U^{k-i}(\tau,A)-V^{k-i}(\tau)\right)F_i^{(1)}+V^{k-i}(\tau)\left(F_i^{(1)}-F_i^{(2)}\right)\right]. \end{aligned} \tag{6}$$

It is proved that, (see[12]):

$$\left\|U^k(\tau,A)-V^k(\tau)\right\|\le ce^{\omega t_k}t_k\tau^3.$$

Also the following estimation takes place:

$$\left\|F_k^{(1)}-F_k^{(2)}\right\|\le ce^{\omega t_k}t_k\tau^4\left(\left\|f(t_k)\right\|+\left\|f'(t_k)\right\|+\left\|f''(t_k)\right\|+\sup_{t\in[0,t_k]}\left\|f'''(t)\right\|\right).$$

According to this estimations and formula (6) we obtain estimation (3).

Scheme of the proof is finished.

REFERENCES

1. Н.Н. Яненко. *Об одном разностном методе счета многомерного уравнения теплопроводности.* ДАН СССР, 1959, Т. 125,№6.

2. А.А. Самарский. *О сходимости метода дробных шагов для уравнения теплопроводности.* ЖВМ МФ, 1962, Т. 2, №6.

5. E.G. Diakonov. *Difference schemes with decomposition operator for multidimensional problems.* J.V.M. and M.F., 1962, Vol. 2, N 4, pp. 311-319.

4. Г.И. Марчук. *Численный алгоритм решения уравнений прогноза погоды.* ДАН СССР, 1964, Т. 156,№2.

5. D.G. Gordeziani. *On application of local one dimensional method for solving parabolic type multidimensional problems of 2m-degree*. Bulletin of GSSR Science Academy, 1965, Vol. 3, N 39, pp 535-542.

6. R. Temam. *Quelques methods de decomposition en analyse merique*. Actes, Congress, intern. Math., 1970, N3, pp. 311-319.

7. Z.G. Gegechkori, G.V. Demidov. *On the convergence of weak the approximation method*. Soviet Math. Dokl., Vol. 14() 1973A, No.6.

8. D.G. Gordeziani, H.V. Meladze. *On modeling multidimensional quasi-linear equation* of *parabolic type by one-dimensional ones*. Billetin of the Academy of Sciences of the Georgian SSR, 1970,v.60,3. pp. 537-540.

9. D.G. Gordeziani, H.V. Meladze. *On modeling of third boundary value problem for the multydimensional parabolic equations of arbitrary area by the one-dimensional equations*. J.V.M. and M.F., 1974, Vol. 14, N 1, pp. 246-250.

10. D.G. Gordeziani, A.A. Samarskii. *Some problems of plates and shells thermo elasticity and method of summary approximation*. Complex analysis and it's applications, M.: Nauka, 1978, pp. 173-186.

11. B. O. Dia, M. Schatzman. *Comutatuers semi-groupes holomorphes et applications aux directions alternees*. M2AN, Vol. 30, n^0, 3, 1996.

12. Z. G. Gegechkori, J. A. Rogava, M. A. Tsiklauri. *Sequential-Parallel method of high degree precision for Cauchy abstract problem solution*. Tbilisi, Reports of Enlarged Session of the Seminar of I.Vekua Institute of Applied Mathematics. Vol. 14, N 3, 1999.

მაღალი რიგის სიზუსტის დეკომპოზიციის მიმდევრობით–პარალელური სქემა არაერთგვაროვანი ევოლუციური განტოლებისათვის

ზ. გეგეჭკორი, ჯ. როგავა, მ. წიკლაური
კომპიუტერების მათემატიკური უზრუნველყოფისა და ინფორმაციული ტექნოლოგიების კათედრა

ბანახის სივრცეში განხილულია კოშის აბსტრაქტული ამოცანა არაერთგვაროვანი ევოლუციური განტოლებისთვის შემოსაზღვრული ოპერატორის შემთხვევაში. აგებულია მესამე რიგის სიზუსტის მიმდევრობით–პარალელური დეკომპოზიციის სქემა. ამონახსნის ცდომილებისათვის მიღებულია ცხადი აპრიორული შეფასება.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 15-18

ON ONE COMMON GENERALIZATION OF SOME WELL-KNOWN ANALYTICAL CONSTRUCTIONS

K. Gelashvili
Control Theory Chair

Abstract. The function, the private cases of which are Riemannian integral, the functions of type of multiplicative integral, the directional derivative, the total variation of function and some others, are defined.

1. The Basic Definitions

The present paper describes one situation of such a type, when some mathematical constructions are the private cases of other, more complicate construction, owing to selection of values of parameters. So, it can be useful for young mathematicians. Moreover, independently important is observation of relations of different mathematical constructions.

Let us define function $K(a,b,f(t,dt),*,Step,Ord)$ and its arguments. In what follows, we shall assume that: $a,b \in R$; $\{(t,s) \mapsto f(t,s)\} : V \to M$,where $V \subset R^2$, and the limiting structure in M is determined by means of directednesses, as in [1]. M is an abstract monoid whose algebraic structure is defined by a binary associative and continuous operation $*$ and by the unity e. The admissible values of parameter $Step$ are: $ZeroStep = 0$, $FullStep = 1$ and $RndStep$-the rundom number from [0,1] (note that $RndStep + RndStep$ can be equal to $\frac{1}{2} + \frac{1}{4}$). Σ denotes the set of all partitions of the form $\sigma = \{0 = s_0 < \cdots < s_n = 1\}$ and $\Delta s_i = s_i - s_{i-1}$, $|\sigma| = \max\{\Delta s_i \mid i = 1,\dots,n\}$.

Now, $Ord \in \{NormOrd, InscrOrd\}$ and is the relation on the Σ:

$$\sigma_1 (NormOrd) \sigma_2 \Leftrightarrow |\sigma_1| \le |\sigma_2| ;$$

$$\sigma_1 (InscrOrd) \sigma_2 \Leftrightarrow (\sigma_1 \text{ is inscribed in } \sigma_2).$$

Preliminarily define $K(a,b,f(t,dt),*,Step,\sigma)$, where $\sigma = \{0 = s_0 < \cdots < s_n = 1\}$. Take $g = e$. If the following loop

$$for \quad i := 1 \ to \quad n \quad do$$

$$g = g * f(a + (s_{i-1} + Step \cdot \Delta s_i)(b-a), \Delta s_i (b-a)),$$

will be performed correctly, then we shall denote

$$K(a,b,f(t,dt),*,Step,\sigma) = g.$$

<u>Definition 1</u>. Let there exist $\sigma_0 \in \Sigma$ and $\tilde{g} \in M$ such that the directedness

$$\{K(a,b,f(t,dt),*,Step,\sigma)\}_{\sigma \in (\Sigma, Ord), \sigma (Ord) \sigma_0},$$

converges to $\tilde{g}$ .Then we denote

$$K(a,b,f(t,dt),*,Step,Ord)=\tilde{g}.$$

(Σ, Ord) is the directed set, and for every σ_1,σ_2 there exists their majorant. Therefore in Definition 1 the values of $K(a,b,f(t,dt),*,Step,Ord)$ do not depend on the choice of σ_0. The order of co-factors in the right-hand side of (1) is important in the non-commutative case.

2. Some Examples

2.1. Reimannian integral in monoid. Let $t_1,t_2\in R$, $\{(t,s)\mapsto f(t,s)\}:V\to M$, where $V\subset R^2$ and $(M,*)$ is monoid, endowed by a limiting structure. Then ${}^{(*)}\int_{t_1}^{t_2} f(t,dt)$, determined in [2], is the same as $K(t_1,t_2,f(t,dt),*,RndStep,NormOrd)$, determined by Definition 1. Thus, private cases of ${}^{(*)}\int_{t_1}^{t_2} f(t,dt)$ are also private cases of Definition 1 (see examples 2.2, 2.3, 2.4).

2.2. Riemannian integral. Let $f:[a,b]\to X$, where $[a,b]\in R$ and X is the Banach space and $t_1, t_2\in R$. Obviously, $\{(t,s)\mapsto s\cdot f(t)\}:[a,b]\times R\to X$. It is easily to seen, that $K(a,b,f(t)dt,+,RndStep,NormOrd)$, determined by Definition 1, is the same as Riemannian integral $\int_{t_1}^{t_2} f(t)dt$, i. e. $K(t_1,t_2,f(t)dt,+,RndStep,NormOrd)$ exists then and only then, when exists $\int_{t_1}^{t_2} f(t)dt$ and $\int_{t_1}^{t_2} f(t)dt=K(t_1,t_2,f(t)dt,+,RndStep,NormOrd)$.

This fact and some others bellow (examples 2.3, 2.4) can be proved in the standard way (see [1] and [2]).

2.3. T-exponent. Let $A(t)$, $t\in[a,b]$, be a piecewise-continuous mapping in a noncommutative Banakh algebra. Then $K(a,b,\exp(A(t)dt,\cdot,UpStep,NormOrd)$ is the same, as T-exponent $Exp\int_a^b A(t)dt$; they exist simultaneously and are equal.

2.4. The multiplicative integral. Let $A(\cdot)$ be a continuous mapping from $[a,b]$ to $B(X)$ ($B(X)$-the set of bounded linear operators in the Banach space X). Then $K(a,b,\exp(A(t)dt, \circ, UpStep, NormOrd)$ is the same as the multiplicative integral.

2.5. The total variation of function. Let $f:[a,b]\to X$, where X is a Banach space. Taking into account the simple equality

$$\sup_{\sigma\in\Sigma}\sum_{i=0}^{n-1}\|f(s_{i+1})-f(s_i)\| = \lim_{\sigma\in(\Sigma(a,b),InscOrd)}\sum_{i=0}^{n-1}\|f(s_{i+1})-f(s_i)\|$$

$$\forall\sigma=\{0=s_0<\dots<s_n=b\}\in\Sigma(a,b),$$

we see that

$$K(a,b,\|f(t+dt)-f(t)\|,+,DownStep,InscOrd)$$

is the same as total variation of f on $[a,b]$, i.e. $V_a^b[f]$.

2.6. The directional derivative. Let X and Y be Banakh spaces, O is open subset in X, $f:O\to Y$, $x\in O$, $h\in X$ and there exist $f'(x;h)$ ($f'(x;h)$ denotes the derivative of f in x with direction h). Then there exists

$$K(0,1,f(x+dt\cdot h)-f(x),+,Step,NormOrd)$$

and

$$K(0,1,f(x+dt\cdot h)-f(x),+,Step,NormOrd)=f'(x;h),$$

$\forall Step\in\{DownStep,UpStep,RndStep\}$. The inverse result is also valid in certain assumptions (see [2]).

2.7. Representation of c_0-semigroups of operators. In [1] is proved an interesting result, which in terms of Definition 1 takes the following face:

<u>THEOREM 1.</u> *Let a linear operator A in the Banach space X generate the strongly continuous semigroup $\{U(s)\}_{s\geq 0}$. Then for sufficiently small s is determined*

$$(I_X-sA)^{-1}\in B(X)$$

and for each $s\geq 0$ takes place:

$$U(s)=K(0,s,(I_X-dt\cdot A)^{-1},\circ,Step,NormOrd),$$

$\forall Step\in\{DownStep,UpStep,RndStep\}$, where $B(X)$ is considered to have the unity I_X, operation of composition and strongly convergence.

2.8. The integral representation of Cauchy's problem solution (see [4]). Consider the Cauchy's problem:

$$\dot{x}=f_t(x),\quad x(t_0)=x_0$$

and suppose that the field $f_t(x)$ has the following properties: $(t,x) \mapsto f_t(x)$ maps $[a,b] \times R^r$ into R^r, $-\infty < a < b < +\infty$, $f_t(x)$ is continuous with t for each $x \in R^r$ and there exists $k \geq 0$ such that

$$|f_t(x_1) - f_t(x_2)| \leq k|x_1 - x_2|, \quad \forall t \in [a,b], \quad \forall x_1, x_2 \in R^r.$$

$C_{Lip}(R^r)$ denotes the set of Lipschitz's mappings $g : R^r \to R^r$. The identical mapping $I : R^r \to R^r$ and the operation of composition $\circ$ create a structure of monoid on $C_{Lip}(R^r)$. $C_{Lip}(R^r)$ is endowed by a limiting structure too, and the composition is continuous.

Under these constraints,

$$\{(t,s) \mapsto (I + sf_t)\} : [a,b] \times R^r \to C_{Lip}(R^r)$$

and in terms of Definition 1 the proved in [4] result takes the form:

<u>THEOREM 2.</u> *Let* $(t_0, t, x) \in (a,b)^2 \times R^r$ *be given arbitrarily. Then there exists*

$$K(t_0, t, (I + dtf_t), \circ, RndStep, NormOrd)$$

and

$$\varphi(t) = K(t_0, t, (I + dtf_t), \circ, RndStep, NormOrd)(x_0)$$

is the solution of $\dot{x} = f_t(x)$ *with initial conditions* $\varphi(t_0) = x_0$.

REFERENCES

1. I.Gogodze, K.Gelashvili. *Representation of* c_0*-semigroups of operators by a chronological integral.* Mem. Diff. Equat. Math Physics, 11(1997), 47-66.
2. K. Gelashvili. *Ordinary differential equations in a monoid of special type*. Proceedings of A. Razmadze Mathematical institute, 121(1999) (In print).
3. M. V. Karasev, M.V. Mosolova. *Infinite products and T-products of exponents.* Teoret. at. Fiz. 28(1976), No.2, 189-200. (Russian).
4. K. Gelashvili. *The Cauchy's problem solution as the function of the field.* Reports of enlarged session of the seminar of I. Vekua institute of applied mathematics (In print).

რამდენიმე ცნობილი ანალიზური კონსტრუქციის საერთო განზოგადების შესახებ

ქ. გელაშვილი
მართვის თეორიის კათედრა

განმარტებულია ფუნქცია, რომლის კერძო შემთხვევები არის რიმანის ინტეგრალი, მულტიპლიკაციური ინტეგრალის ტიპის ფუნქციები, ფუნქციის სრული ვარიაცია და ზოგიერთი სხვა ცნობილი კონსტრუქცია.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 19-22

NECESSARY CONDITIONS OF OPTIMALITY FOR ONE CLASS NEUTRAL TYPE PROBLEMS OF OPTIMAL CONTROL

N. Gorgodze
Control Theory Chair

Abstract. For the quasi-linear neutral type problem of optimal control, necessary conditions of optimality in the form of an integral maximum principle and the transversality conditions are obtained.

Let $J=[a,b]$ be a finite interval; $O\subset R^n$ be a open set; $M\subset O$ be a convex bounded set; $U\subset R^r$ be a compact set; $V\subset R^p$ be a convex bounded set; $\tau:R^1\to R^1$, $\eta:R^1\to R^1$ are an absolutely continuous and continuously differentiable functions, respectively, and satisfying the conditions:

$$\tau(t)\le t, \dot\tau(t)>0, \quad \eta(t)<t, \dot\eta(t)>0;$$

$\gamma(t)=\tau^{-1}(t)$, $\quad\sigma(t)=\eta^{-1}(t)$; $\quad q^i:J^2\times O^2\to R^1, i=0,\dots,l$, are continuously differentiable functions; $\Delta=\Delta(J_1,M)$ is a set of continuously differentiable functions $\varphi:J_1\to M$, $J_1=[\rho(a),b]$, $\rho(t)=\min\{\eta(t),\tau(t)\}$, $t\in J$, $\|\varphi\|=\sup\{|\varphi(a)|+|\dot\varphi(t)|:t\in J_1\}$; Ω_1 is a set of measurable functions $u:J\to U$; Ω_2 is a set of measurable functions $v:J\to V$; $A(t,v)$ is a $n\times n$ dimensional matrix function, continuous on $J\times V$ and continuously differentiable with respect to $v\in V$;

Next, the function $f:J\times O^2\times U\to R^n$ satisfies the following conditions:

1) for a fixed $t\in J$ the function $f(t,x_1,x_2,u)$ is continuous with respect to $(x_1,x_2,u)\in O^2\times U$ and continuously differentiable with respect to $(x_1,x_2)\in O^2$;

2) for a fixed $(x_1,x_2,u)\in O^2\times U$ the functions f, $f_{x_i}, i=1,2$, are measurable with respect to *t*; for an arbitrary compact $K\subset O$ there exist $m_K=const>0$, $L_K(t)\in L_1(J,R_0^+)$, $R_0^+=[0,\infty)$ such that $|f(t,x_1,x_2,u)|\le m_K$, $\quad \sum_{i=1}^{2}|f_{x_i}(t,x_1,x_2,u)|\le L_K(t)$, $\forall(t,x_1,x_2,u)\in J\times K^2\times U.$

To every element $\mu=(t_0,t_1,x_0,\varphi,u,v)\in B=J^2\times O\times\Delta\times\Omega_1\times\Omega_2$, $t_0<t_1$, corresponds the differential equation

$$\dot x(t)=A(t,v(t))\dot x(\eta(t))+f(t,x(t),x(\tau(t),u(t))),\quad t\in[t_0,t_1] \tag{1}$$

with initial condition

$$x(t)=\varphi(t),\quad t\in[\rho(t_0),t_0),\quad x(t_0)=x_0. \tag{2}$$

DEFINITION 1. The function $x(t)=x(t,\mu)\in O, t\in[\rho(t_0),t_1], t_0\in[a,t_1)$, is said to be solution corresponding to the element $\mu\in B$, if on $[\rho(t_0),t_0]$ it satisfies the condition (2), while on the interval $[t_0,t_1]$ is absolutely continuous and satisfies the equation (1) almost everywhere.

DEFINITION 2. The element $\mu\in B$ is said to be admissible, if the corresponding solution $x(t)$ satisfies the conditions $q^i(t_0,t_1,x_0,x(t_1))=0,\ i=1...l$.

The set of admissible elements will denoted by B_0.

DEFINITION 3. The element $\tilde{\mu}=(\tilde{t}_0,\tilde{t}_1,\tilde{x}_0,\tilde{u},\tilde{v})\in B_0$ is said to be locally optimal, if there exist a number $\delta>0$ and compact set $X\subset O$ such that for an arbitrary element $\mu\in B_0$ satisfying $|\tilde{t}_0-t_0|+|\tilde{t}_1-t_1|+|\tilde{x}_0-x_0|+\|\tilde{\varphi}-\varphi\|+\|\tilde{f}-f\|_X+\sup_{t\in J}|\tilde{v}(t)-v(t)|\le\delta$, the inequality $q^0(\tilde{t}_0,\tilde{t}_1,\tilde{x}_0,)\le q^0(t_0,t_1,x_0,x(t))$ is holds.

Here

$$\|\tilde{f}-f\|_X=\int_J H(t;f,X)dt,$$

$$H(t;f,X)=\sup\{|\tilde{f}(t,x_1,x_2)-f(t,x_1,x_2)|+\sum_{i=1}^{2}|\tilde{f}_{x_i}(\cdot)-f_{x_i}(\cdot)|:(x_1,x_2)\in X^2\};$$

$\tilde{f}(t,x_1,x_2)=f(t,x_1,x_2,\tilde{u}(t)),\quad f(t,x_1,x_2)=f(t,x_1,x_2,u(t)),\quad \tilde{x}(t)=x(t,\tilde{\mu})$.

The problem of optimal control consists in finding a locally optimal element.

THEOREM 1. *Let $\tilde{\mu}\in B_0$, $\tilde{t}_i\in(a,b)$, $i=0,1$, be a locally optimal element; $\gamma_0=\gamma(\tilde{t}_0)\in(\tilde{t}_0,\tilde{t}_1)$, $\sigma_0=\sigma(\tilde{t}_0)\in(\tilde{t}_0,\tilde{t}_1)$, there exist integer numbers $m_i\ge 0, i=1,2$, such that $\gamma_0\in(\eta^{m_1+1}(\tilde{t}_1),\eta^{m_1}(\tilde{t}_1))$, $\sigma_0\in(\eta^{m_2+1}(\tilde{t}_1),\eta^{m_2}(\tilde{t}_1))$; the function $\dot{\tau}(t)$ is continuous at point $\tilde{t}_0$; the function $\tilde{f}(\omega)=\tilde{f}(t,x_1,x_2)$ is continuous at points $\omega_0=(\tilde{t}_0,\tilde{x}_0,\tilde{\varphi}(\tau(\tilde{t}_0)))$, $\omega_1=(\gamma_0,\tilde{x}(\gamma_0),\tilde{x}_0)$, $\omega_2=(\gamma_0,\tilde{x}(\gamma_0),\tilde{\varphi}(\tilde{t}_0))$, $\omega_3=(\tilde{t}_1,\tilde{x}(\tilde{t}_1),\tilde{x}(\tau(\tilde{t}_1)))$; the function $\tilde{A}(t)=A(t,\tilde{v}(t))$ is continuous at points $\tilde{t}_0,\tilde{t}_1,\sigma^i(\gamma_0)$, $i=1,..,m_1$, $\sigma^i(\sigma_0)$, $i=0,..,m_2$; the function $\dot{\tilde{x}}(\eta(t))$ is continuous at point $\tilde{t}_1$. Then there exists a non-zero vector $\pi=(\pi_0,...,\pi_l)$, $\pi_0\le 0$ and solutions $\psi(t)$, $\chi(t)$ of the system*

$$\begin{cases}\dot{\chi}(t)=-\psi(t)\widetilde{f}_{x_1}[t]-\psi(\gamma(t))\widetilde{f}_{x_2}[\gamma(t)]\dot{\gamma}(t),\\ \dot{\psi}(t)=\chi(t)+\psi(\sigma(t))\widetilde{A}(\sigma(t))\dot{\sigma}(t),\quad t\in[\widetilde{t}_0,\widetilde{t}_1],\quad \psi(t)=0,\quad t>\widetilde{t}_1,\end{cases}$$

such that following conditions are fulfilled:

$$\int_{\tau(\widetilde{t}_0)}^{\widetilde{t}_0}\psi(\gamma(t))\widetilde{f}_{x_2}[\gamma(t)]\dot{\gamma}(t)\widetilde{\varphi}(t)dt+\int_{\eta(\widetilde{t}_0)}^{\widetilde{t}_0}\psi(\sigma(t))\widetilde{A}(\sigma(t))\dot{\sigma}(t)\,\dot{\widetilde{\varphi}}(t)dt\geq$$

$$\geq\int_{\tau(\widetilde{t}_0)}^{\widetilde{t}_0}\psi(\gamma(t))\widetilde{f}_{x_2}[\gamma(t)]\dot{\gamma}(t)\varphi(t)dt+\int_{\eta(\widetilde{t}_0)}^{\widetilde{t}_0}\psi(\sigma(t))\widetilde{A}(\sigma(t))\dot{\sigma}(t)\,\dot{\varphi}(t)dt,\quad\forall\varphi\in\Delta;$$

$$\int_{\widetilde{t}_0}^{\widetilde{t}_1}\psi(t)f(t,\widetilde{x}(t),\widetilde{x}(\tau(t)),\widetilde{u}(t))dt\geq\int_{\widetilde{t}_0}^{\widetilde{t}_1}\psi(t)f(t,\widetilde{x}(t),\widetilde{x}(\tau(t)),u(t))dt,\quad\forall u\in\Omega_1;$$

$$\int_{\widetilde{t}_0}^{\widetilde{t}_1}\psi(t)\widetilde{A}_v(t)\dot{\widetilde{x}}(\eta(t))\,\widetilde{v}(t)dt\geq\int_{\widetilde{t}_0}^{\widetilde{t}_1}\psi(t)\widetilde{A}_v(t)\dot{\widetilde{x}}(\eta(t))\,v(t)dt,\qquad\forall v\in\Omega_2;$$

$$\pi\widetilde{Q}_{t_0}=\chi(\widetilde{t}_0)\,[\widetilde{A}(\widetilde{t}_0)\dot{\widetilde{\varphi}}(\eta(\widetilde{t}_0))+\widetilde{f}(\omega_0)]+$$

$$+\psi(\sigma_0)\widetilde{A}(\sigma_0)\,[\widetilde{A}(\widetilde{t}_0)\dot{\widetilde{\varphi}}(\eta(\widetilde{t}_0))+\widetilde{f}(\omega_0)-\dot{\widetilde{\varphi}}(\widetilde{t}_0)]\dot{\sigma}(\widetilde{t}_0)\,\psi(\gamma_0)[\widetilde{f}(\omega_1)-\widetilde{f}(\omega_2)]\dot{\gamma}(\widetilde{t}_0),$$

$$\pi\widetilde{Q}_{t_1}=-\psi(\widetilde{t}_1)[\widetilde{A}(\widetilde{t}_1)\dot{\widetilde{x}}(\eta(\widetilde{t}_1))+\widetilde{f}(\omega_3)].,$$

$$\pi\widetilde{Q}_{x_0}=-\chi(\widetilde{t}_0),\qquad\pi\widetilde{Q}_{x_1}=\chi(\widetilde{t}_1).$$

Here $Q=(q^0,\cdots,q^l)^T$, $\widetilde{Q}$ means that the corresponding gradient is calculated at the point $(\widetilde{t}_0,\widetilde{t}_1,\widetilde{x}_0,\widetilde{x}(\widetilde{t}_1))$; $\widetilde{f}_{x_i}[t]=\widetilde{f}_{x_i}(t,\widetilde{x}(t),\widetilde{x}(\tau(t)))$.

Finally we note that the theorem formulated above are an analogue of theorem given [1]. This theorem is proved, using formula of the differential of solutions with respect to the initial data and the right-hand side [2], by the scheme described in [3]. The case, when $\widetilde{v}(t)$ is the piecewise continuous function, is considered in [4].

REFERENCES

1. T. Tadumadze. *On new necessary condition of optimality of initial moment in control problems with delay*. Mem. Differential Equations Math. Phys. 17(1999), 157-159.

2. T. Tadumadze and N. Gorgodz. *Differentiability of solution of differential equations with deviating argument with respect to the initial data and the right-hand side*. Seminar of I. Vekua Institute of Applied Mathematics. Reports. 23(1997), 108-119.

3. R. V. Gamkrelidze, G. L. Kharatishvili. *Extremal problems in linear topological spaces*. (Russian), Izv. AN SSSR, Sem. Math. 1969(33), N4, 781-839.

4. N. Gorgodze. *Necessary conditions of optimality in neutral type optimal problems with non-fixed initial moment*. Mem. Differential Equations Math. Phys. 19(2000). (In print)

ოპტიმალურობის აუცილებელი პირობები ერთი კლასის ნეიტრალური ტიპის ოპტიმალური მართვის ამოცანისათვის

ნ. გორგოძე
მართვის თეორიის კათედრა

კვაზიწრფივი ნეიტრალური ტიპის ოპტიმალური მართვის ამოცანისათვის მიღებულია ოპტიმალურობის აუცილებელი პირობები ინტეგრალური მაქსიმუმის პრინციპისა და ტრანსვერსალობის პირობების სახით.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 23-26

CONTRARY PAIRS IN PSEUDO-BOOLEAN ALGEBRA

G. Kashmadze
Chair of Cybernetics

Abstract. The lattice operations for split subsets are considered when the values of membership function belong to the pseudo-boolean algebra.

Let us consider a set of pairs of kind $(\mu I_A,(1-\mu)I_A)$, where $A\subseteq\Omega$, Ω is universal domain, A - his fixed subset, μ - any mapping of Ω into [0,1] and I_A - characteristic function or indicator of A /1/.

Introduce $\wedge$ (greatest lower bound) and $\vee$ (least upper bound) operations traditionally componentwise:

$$(X,Y)\vee(Z,T)=(X\vee Z,Y\vee T)$$
$$(X,Y)\wedge(Z,T)=(X\wedge Z,Y\wedge T)$$

Take arbitrarily two pairs $(\mu I_A,(1-\mu)I_A)$, $(\nu I_A,(1-\nu)I_A)$, $\mu,\nu\in[0,1]^\Omega$ and consider their l.u.b. and g.l.b:

$$((\mu I_A\vee\nu I_A)(x),((1-\mu)I_A\vee(1-\nu)I_A)(x))=$$
$$=\begin{cases}(0,0) & \text{if } x\notin A\\ (\nu(x),\ 1-\mu(x)) & \text{if } x\in A,\ \mu(x)\le\nu(x)\\ (\mu(x),\ 1-\nu(x)) & \text{if } x\in A,\ \mu(x)>\nu(x)\end{cases}$$

$$((\mu I_A\wedge\nu I_A)(x),\quad((1-\mu)I_A\wedge(1-\nu)I_A)(x))=$$
$$=\begin{cases}(0,0) & \text{if } x\notin A\\ (\mu(x),\ 1-\nu(x)) & \text{if } x\in A,\ \mu(x)\le\nu(x)\\ (\nu(x),\ 1-\mu(x)) & \text{if } x\in A,\ \mu(x)>\nu(x)\end{cases}$$

Thus the set of kind $(\mu I_A,(1-\mu)I_A)$ is not closed with respect to operations $\wedge$ and $\vee$. If we want to keep closety, then operations will be defined as follows: $(X,Y)\vee(Z,T)=(X\vee Z,Y\wedge T)$

In this case, we have:

$$((\mu I_A\vee\nu I_A)(x),\quad((1-\mu)I_A\wedge(1-\nu)I_A)(x))=$$
$$=\begin{cases}(0,0) & \text{if } x\notin A\\ (\nu(x),\ 1-\nu(x)) & \text{if } x\in A,\ \mu(x)\le\nu(x)\\ (\mu(x),\ 1-\mu(x)) & \text{if } x\in A,\ \mu(x)>\nu(x)\end{cases}$$

$$((\mu I_A \wedge \nu I_A)(x), \quad ((1-\mu)I_A \vee (1-\nu)I_A)(x)) =$$

$$= \begin{cases} (0,0) & \text{if } x \notin A \\ (\mu(x),\ 1-\mu(x)) & \text{if } x \in A,\ \mu(x) \le \nu(x) \\ (\nu(x),\ 1-\nu(x)) & \text{if } x \in A,\ \mu(x) > \nu(x) \end{cases}$$

$\wedge$ and $\vee$ operations induce a following partial ordering relation between the pairs

$$(X,Y) \le (Z,T) \Leftrightarrow (X,Y) \vee (Z,T) = (Z,T) \Leftrightarrow$$

$$\Leftrightarrow (X,Y) \wedge (Z,T) = (X,Y) \Leftrightarrow (X(u) \le Z(u)) \& (Y(u) \ge T(u))$$

$$X,Y,Z,T \in [0,1]^A, u \in A.$$

The complement $(X,Y)^c$ of pair (X,Y) is defined in this way: $(X,Y) = (I_A \setminus X, I_A \setminus Y)$

Denote by a symbol L the lattice of fuzzy subsets of Ω and by symbols L^r the lattice of same subsets with reverse order. It is fairly straitforward to show that the following theorem holds:

<u>THEOREM 1.</u> *Pairs of kind* $(\mu I_A, (1-\mu)I_A)$ *for fixed* $A \subseteq \Omega$ *and any* $\mu \in [0,1]^\Omega$ *form complemented distributive lattice, which is a sublattice of* $L \times L^r$*, satisfying de Morgan's law; the complement is involutory and order reversing.*

Now suppose, that the values of membership function μ belong to the pseudo-boolean algebra $B = \langle B, \le \rangle$ /2/. As intuitionistic negation is not, in general, involutory, the unique splitting into contrary pair is impossible /3/. In this case, the construction defined below perhaps proved to be useful.

We introduce the "$\approx$" relation between elements of B and between their pairs as follows:

<u>DEFINITION 1.</u> $(a \approx b) \underset{df}{=} (a^* = b^*)$, $a,b \in B$. Here $(\ )^*$ denotes pseudo-complement.

2. $((a,b) \approx (c,d)) \underset{df}{=} ((a \approx c) \text{ and } (b \approx d))$, $a,b,c,d \in B$.

3. $(a,b)^* \underset{df}{=} (a^*, b^*)$, $a,b \in B$.

It is evident that "$\approx$" is an equivalence relation.

<u>THEOREM 2.</u> *If* $a_1 \approx a_2$, $b_1 \approx b_2$ *then*

$$((a_1,a_1^*) \vee (b_1,b_1^*)) \approx ((a_2,a_2^*) \vee (b_2,b_2^*)),\ (a_1,a_1^*)^* \approx (a_2,a_2^*)^*$$

PROOF.

$$(a_1,a_1^*)\vee(b_1,b_1^*)=(a_1\vee b_1,a_1^*\wedge b_1^*)=(a_1\vee b_1,a_2^*\wedge b_2^*)$$

$$(a_2,a_2^*)\vee(b_2,b_2^*)=(a_2\vee b_2,a_2^*\wedge b_2^*),\qquad (a_1\vee b_1)^*=a_1^*\wedge b_1^*=a_2^*\wedge b_2^*=(a_2\vee b_2)^*.$$

2. follows immediately from definitions.

Let $B'\subset B$ be the set of elements satisfying the following condition:

$$(a\wedge b)^*=a^*\vee b^*$$

THEOREM 3. *If* $a,b,c,d\in B'$, $a_1\approx a_2$, $b_1\approx b_2$ *then*

$$((a_1,a_1^*)\wedge(b_1,b_1^*))\approx((a_2,a_2^*)\wedge(b_2,b_2^*))$$

Proof by analogy with case 1. of theorem given above. Thus, we can consider the factor set $B'\times B'/_{\approx}$.

REFERENCES

1. F. Criado, T. Gachechiladze. *Fuzzy random events and their corresponding conditional probability measures.* Publicado en la Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales, de Madrid. Tomo LXXXIX, Cuaderno 1.° - 2.°
2. Е. Расева, Р. Сикорски. *Математика Метаматематики*. Москва. Наука. 1972.
3. R. Yager. *On the Measure of Fuzziness and Negation*. Part II. Inf. and Control. 44. 1980.

კონტრარული წყვილები ფსევდო–ბულის ალგებრაში

გ. კაშმაძე
კიბერნეტიკის კათედრა

განიხილება მესრული ოპერაციები ისეთი გახლეჩილი ქვესიმრავლეებისათვის, რომელთა მიკუთვნების ფუნქციები მნიშვნელობებს იღებენ ფსევდო–ბულის ალგებრიდან.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 27-32

ON AN APPROXIMATELY SOLUTION OF SAINT-VENANT'S PROBLEMS FOR A BEAMS WITH A PERTURBED LATERAL SURFACES

G. Khatiashvili
Computer Software and Information of Technologies

Abstract. First works on an indicated problems for an isotropic beams were given by Panov D.I.,Riz P.M., Rukhadze A.K and various authors. These results were generalized on a composed bodies and anisotrpic medium by various authors. In these works indicated problems were studied with help of a transformation of a system of coordinates and differential operators and boundary conditions were approximated with accuracy up to first power of a small parameter ν. As this takes place it is impossible to estimate a power of an approximation and give proof of this method for anisotropic medium is difficult, because a coefficients of elasticity in this method are varying. In this paper a solution of Saint-Ven ant's problems in a domain, occupying by a body similar to prismatic (cylindrical), with perturbed cylindrical surface, is represented as a series with respect of a small parameter ν, characterized a perturbation of a cylindrical surface. For each terms of a series are obtained the recurrent boundary problems of elasticity of Almansi-Michel's type for a cylindrical body. A class of surface is indicated, for which later on may be studied a question of a convergence of a double series with respect of a small parameters. Also this way gives a possibility of a solution of a problem with a required exactness. A first results on this direction were given by author of this article in papers [5,6] used methods considered in articles of A.N.Guz (1962) and I.N.Nemish (1976), where a method of a perturbation of a cross section of a surfaces of a canonical form was considered. This way as a base was used in another direction for a construction of algorithms for a solution of some problems of an elasticity, for a bodies similar to cylindrical by an arbitrary cross section. These results are given in the book [7]. A.N. Guz, I.N. Nemish and N.M. Bloshko created the methods of a perturbation of bodies boundary's form by its further generalization (see [2,3,4]).

1. Basic Equations

Let us consider a system of a cartesian coordinates $Ox_1x_2x_3$ and an elastic body occupying a domain Ω_*, bounded by planes

$$x_3 = 0,\ x_3 = l\ (l > 0) \tag{1}$$

and by lateral surface Γ_* given in a parametric form

$$x_j^* = f_j(t) + \nu P_j(z) q_j(t),\ (j = 1,2),\quad x_3^* = z, \tag{2}$$

where ν is a small parameter, $P_j(z)$ are a given polynomials with respect of a variable x_3 and t is a given parameter. We consider a particular case of an equations (2), when $q_j(t) = 1$ and $P_j(z) = -x_3^p (pml^p)^{-1}$. Thus, it will be considered a body bounded by planes (1) and by lateral surface

$$x_j^* = f_j(t) + \nu P(z),\quad (j = 1,2),\ x_3 = z, \tag{3}$$

where $0 < \nu < 1$ is a small parameter and m and p are integer numbers, which must be chosen so that it can be $\left|z^{p-1}(ml^p)^{-1}\right| < 1$; t is a natural coordinate taken on a curve γ of a boundary of a domain ω, wich is obtained by normal crossection of a cylindrical body, bounded by planes (1) and a surface

$$x_1 = f_1(t), \ \ x_2 = f_2(t). \qquad (4)$$

It is obvious that surface is obtained from surface (3) for $\nu = 0$ (or $x_3 = 0$). It must be remember that $0 \le x_3 \equiv z \le l$.

A cosinus of a normal $n^0(n_1^0, n_2^0, n_3^0)$ of the surface (4) will be given as

$$n_1^0 = n_0^{-1} f_2'(t), \ n_2^0 = -n_0^{-1} f_1'(t), \ n^0 = 0, \ n_0^2 = (f_1')^2 + (f_2')^2. \qquad (5)$$

As is known, a cosinus of a normal $n(n_1, n_2, n_3)$ to the surface (2) will be given by equalities

$$\begin{aligned} & n_j = B_j B^{-1} \ (j = 1,2,3); \ B_1 = (x_2^*)_t'(x_3^*)_z' - (x_2^*)_z'(x_3^*)_t', \\ & B_2 = (x_3^*)_t'(x_1^*)_z' - (x_1^*)_t'(x_2^*)_z', \ B_3 = (x_1^*)_t'(x_2^*)_z' - (x_1^*)_z'(x_2^*)_t', \\ & B^2 = B_1^2 + B_2^2 + B_3^2 > 0. \end{aligned} \qquad (6)$$

According to equations (3), we get

$$(x_j^*)_t' = f_t'(t), \ (x_j^*)_z' = -\nu x_3^{p-1}(ml^p)^{-1} \ (j = 1,2); \ (x_3^*)_t' = 0, \ (x_j^*)_z' = 1. \qquad (7)$$

Substitute these values in the expressions (6), we get

$$\begin{aligned} & n_0^{-1} B_1 = n_1^0, \ n_0^{-1} B_2 = n_2^0, \ n_0^{-1} B_3 = \nu(n_1^0 + n_2^0) z^{p-1}(ml^p)^{-1} \\ & B = \left[n_0^2 + \nu^2 (n_1^0 + n_2^0)^2 z^{2p-2}(ml^p)^{-2}\right]^{1/2}, \end{aligned} \qquad (8)$$

where $n_0 > 0$ is given by equality (5).

Thus, we consider Saint-Venant's problems for a body similar to a prismatic (cylindrical), bounded by planes (1) and by a lateral pertrubed cylindrical surface (3). The components of the stresses τ_{jk} must satisfy in a domain Ω. the equations of an equilibrium of an elasticity and a Hook's law (for an isotropic or an anisotropic mediums) and also must be satisfaid an "ends conditions" (for a lower end, when $x_3 = 0$)

$$\begin{aligned} & \iint_\omega \tau_{3j} d\omega = F_j \ (j = 1,2,3); \ \iint_\omega x_2 \tau_{33} d\omega = M_1, \ \iint_\omega x_1 \tau_{33} d\omega = M_2, \\ & \iint_\omega (x_1 \tau_{23} - x_2 \tau_{13}) d\omega = M_3, \end{aligned} \qquad (9)$$

where F_j and M_j are the given numbers.

On an "upper" end a resultant force and a resultant moment of an exterior forces, in general, must be equal to F_j and M_j respectively, but with opposite sign.

On each points of a lateral surface (3) the components τ_{jk} must satisfy the following boundary conditions:

$$\tau_{1j}n_1 + \tau_{2j}n_2 + \tau_{3j}n_3 = 0, \ (j = 1,2,3), \tag{10}$$

where a cosines n_j of a normal n are given by equalities (6).

Divide equalities (10) on n_0 and substitute values n_j, after a multiplication on B, by using of the equations (6), boundary conditions (10) takes the form

$$\tau_{1k}n_1^0 + \tau_{2k}n_2^0 - \nu\tau_{3k}\left(n_1^0 + n_2^0\right)P'(z) = 0 \quad (k = 1,2,3) \tag{10ν}$$

About of a fullfilment of an "ends conditions" (9) will be shown below.

2. An Approximately Solution

The solutions of the considered problems we seek as the series with respect of a small paramete ν

$$\tau_{jk} = \sum_{b=0}^{\infty} \nu^b \tau_{jk}^{(b)}(x_1, x_2, x_3). \tag{1}$$

Substitute these values in the condition (1.10ν), taking x_j^* (see(1.3)) instead of x_j. Therefore, in expression (1) each term will be expanded on surface (1.3) in a Taylor's serie

$$\tau_{jk}^{(b)}(x_1 + \nu P(z), x_2 + \nu P(z), x_3) = \left(\tau_{jk}^{(b)}\right)_0 + \sum_{\alpha=1}^{N} \frac{1}{\alpha!}[\nu P(z)]^\alpha \left(D^\alpha \tau_{jk}^{(b)}\right)_0 +$$
$$+ \frac{1}{(N+1)!}[\nu P(z)]^{N+1}\left(D^{N+1}\tau_{jk}^{(b)}\right)_{\xi_j}, \quad \xi_j(\Theta) = x_j + \Theta\nu P(z) \quad (j=1,2), \ x_3 = z, \tag{2}$$

where N is some integer positive number,

$$P(z) \equiv -z^p (mpl^p)^{-1}, \ D^\alpha \equiv (D_1 + D_2)^\alpha, \ 0 < \Theta < 1, \ (\tau_{jk}^{(b)})_0 \equiv (\tau_{jk}^{(b)})_{\nu=0}. \tag{3}$$

Using these expressions, the conponents (1) in points of a surface (1.3) may be expressed by the values in points of a cylindrical surface (1.4), with accuracy up to ν^{N+1}, in a form

$$\tau_{jk}(x_1 + \nu P(z), x_2 + \nu P(z), x_3) = \sum_{b=0}^{N}\sum_{\alpha=1}^{N-b} \frac{1}{\alpha!}\nu^b [\nu P(z)]^\alpha \left(D^\alpha \tau_{jk}^{(b)}\right)_0. \quad (j,k = 1,2,3).$$

For present purposes this expressions may be written in the form

$$\tau_{jk}(x_1+\nu P(z),x_2+\nu P(z),x_3)=\left(\tau_{jk}^{(0)}\right)_0+\sum_{\alpha=1}^{N}\nu^{\alpha}\left[\tau_{jk}^{(\alpha)}+\frac{1}{\alpha!}(P(z))^{\alpha}D^{\alpha}\tau_{jk}^{(0)}\right]_0+$$
$$+\sum_{b=1}^{N}\sum_{\alpha=1}^{N-b}\nu^{\alpha+b}\frac{1}{\alpha!}(P(z))^{\alpha}\left(D^{\alpha}\tau_{jk}^{(b)}\right)_0\Bigg]. \tag{4}$$

Substituting these expressions into the boundary conditions (1.10v), in each point of a cylindrical surface (1.4) for the values $\tau_{jk}^{(b)}$ the following boundary conditions are obtained:

$$\left(\tau_{1k}^{(0)}n_1^0+\tau_{2k}^{(0)}n_2^0\right)_0-\nu\left(n_1^0+n_2^0\right)P'(z)\left(\tau_{3k}^{(0)}\right)_0+\sum_{j=1,2}\left\{\sum_{\alpha=1}^{N}\nu^{\alpha}\left[\tau_{jk}^{(0)}+\frac{1}{\alpha!}(P(z))^{\alpha}D^{\alpha}\tau_{jk}^{(0)}\right]_0+\right.$$
$$\left.+\sum_{b=1}^{N}\sum_{\alpha=1}^{N-b}\nu^{\alpha+b}\frac{1}{\alpha!}(P(z))^{\alpha}\left(D^{\alpha}\tau_{jk}^{(b)}\right)_0\right\}n_j^0-$$
$$-P'(z)\left(n_1^0+n_2^0\right)\left\{\sum_{\alpha=1}^{N-1}\nu^{\alpha+1}\left[\tau_{3k}^{(\alpha)}+\frac{1}{\alpha!}(P(z))^{\alpha}D^{\alpha}\tau_{3k}^{(0)}\right]_0+\right. \tag{5}$$
$$\left.+\sum_{b=1}^{N-1}\sum_{\alpha=1}^{N-b-1}\nu^{\alpha+b+1}\frac{1}{\alpha!}(P(z))^{\alpha}\left(D^{\alpha}\tau_{3k}^{(b)}\right)_0\right\}=0,\quad (k=1,2,3).$$

In these expressions equating to zero the multipliers of the same power of a parameer ν, for the unknown components $\left(\tau_{jk}^{(b)}\right)_0$, in each points of a cylindrical surface (1.4) are obtained following boundary conditions:

$$\left(\tau_{1j}^{(0)}n_1^0+\tau_{2j}^{(0)}n_2^0\right)_0=0,\ \left(\tau_{1j}^{(1)}n_1^0+\tau_{2j}^{(1)}n_2^0\right)_0=-P(z)\left(n_1^0D^1\tau_{1j}^{(0)}+n_2^0D^1\tau_{2j}^{(0)}\right)_0+ \tag{6}$$
$$+\left(n_1^0+n_2^0\right)P'(z)\left(\tau_{3j}^{(0)}\right)_0,\ \left(\tau_{1j}^{(k)}n_1^0+\tau_{2j}^{(k)}n_2^0\right)_0=-\sum_{m=0}^{k-1}[(k-m)!]^{-1}\left\{P^{k-m}(z)\left(n_1^0D^{k-m}\tau_{1j}^{(m)}+\right.\right.$$
$$\left.\left.+n_2^0D^{k-m}\tau_{2j}^{(m)}\right)_0+(k-m)\left(n_1^0+n_2^0\right)P'(z)P^{k-m-1}(z)\left(D^{k-m-1}\tau_{3j}^{(m)}\right)_0\right\},\ (k=2,3,\dots,N).$$

From these recurrent exprssions is seen that components $\left(\tau_{ij}^{(0)}\right)_0$ must be a solution of Saint-Venant's (SV) problems for a cylindrical body "G" with a lateral surface (1.4). Therefore, the "ends conditions" (1.9) will be satisfied. In this case, as is well-known, for a homogeneous beam, only the components $\left(\tau_{13}^{(0)}\right)_0$, $\left(\tau_{23}^{(0)}\right)_0$ and $\left(\tau_{33}^{(0)}\right)_0$ are not equal to zero, i.e. it must be taken $\left(\tau_{11}^{(0)}=\tau_{22}^{(0)}=\tau_{12}^{(0)}=0\right)$. Therefore, in conditions (6) it will be taken only $\left(\tau_{j3}^{(0)}\right)_0\neq 0,\ (j=1,2,3)$.

It must be noted that the problems of exstension by longitudinal force and bending due couples of forces only $\left(\tau_{33}^{(0)}\right)_0 \neq 0$ and the expressions (6) greatly simplified.

In general an elementary analysis of the boundary conditions (6) shows that beginning from $k=2$ the components $\left(\tau_{ij}^{(k)}\right)_0$ must be the solutions of Almansi's and Michell's *(AM)*$_k$ problems studying in the book [8].

A solution of these three-dimensional problems are reduced to two-dimensional boundary problems in a plane Ox_1x_2 for the elliptic equations of a two, four and sixth powers, a solution of which may be estimated (include on a boundary of domain) in the way given in [1]. From the solutions of *(A-M)*$_k$ problems for body "*G*" on the end $x_3 = 0$ may be araised a resultant forces $F_j^{(k)}$ and a resultant moments $M_j^{(k)}$ of a couple-forces, for neutralized of which to the cimponents $\left(\tau_{ij}^{(k)}\right)_0$ must be added a solution $\left(\tau_{ij}^{S}\right)_k$ of *(SV)*$_k$ problems, corresponding to the given resultant forces - $F_j^{(k)}$ and resultant moments - $M_j^{(k)}$.

It will be remarked that this algorithm may be used not only in an elasticity, but in another problems of mathematical physics. For example, it may be used also in an heat conduction problems (see [6,7]), for which on (6) must be taken only conditions for $j=3$ and everywhere only $\left(\tau_{13}^{(0)}\right)_0 \neq 0$ and $\left(\tau_{23}^{(0)}\right)_0 \neq 0$.

In practice functions $P(z)$ are a linear, second or a third power polynomials of x_3. For instance, $P(z) = -x_3^2(4l)^{-2}$ (a slightly curved beam) and for a small parameter ν we may take number $(tan\,\alpha)_{x_3=l}\, l^{-2}$, where α is a corner in a plane x_1Ox_3 composed between Ox_1 axis and a tangent to a curve obtained by crossing of indicated plane and surface (1.3). This algorithm give in our disposal two small paraeter ν and $\nu^* = |P(z)| < 1$, what give a possibility to investigate a question of a double series, what will araise when $N \to \infty$.

It must be remarked, although an algorithm of the small parameter method for bodies similar to cylindrical has been constructed about 15 years ago [5,6], the idea of finding a perturbation function to estimate the remander of series with respect to he small parameter, made us come back to the issue.

REFERENCES

1. S. Agmon, A. Duglis, L. Nirenberg. *Estimates near the boundary for solution of elliptic partial differential equations satisfying generaL boundary conditions.* International Science Publishers, New Jork, 1959.
2. A.N. Guz, I.N. Nemish. *Statik of elastic bodies a non-canonical form.* Kiev, Naukova Dumka, 1987 (in Russian).
3. I.N. Nemish, N.M. Bloshko. *Stress state of an elastic cylinders with an intendet.* Kiev, Naukova Dumka, 1987.
4. I.N. Nemish. *Elements of mechaniks of piecewise homogeneous bodies with non-canonical interfaces.* Kiev, Naukova Dumka, 1989 (in Russian).
5. G.M. Khatiashvili. *On one way of an approximately solution of Saint-Venant's problems for a bodies similar to prismatic.* Prikladnaia Mechanika, t.XYII,4,1981,pp.10-15.
6. G.M. Khatiashvili. *Approximately solution of problems of a deformation for composed bodies similar to prismatic.* Pricladnaia Mechanika , t.XIX,7,1983,pp.71-76.
7. G.M. Khatiashvili. *Almansi-Michel's problems for homogeneous and composed bodies.* Tbilisi, Mecniereba,1983.
8. G.M. Khatiashvili. *Homogeneous and composed elastic cylinders.* Tbilisi, Mecniereba, 1991.

შეშფოთებული გვერდითი ზედაპირიანი დრეკადი ძელისათვის სენ–ვენანის მიახლოებითი ამოხსნის შესახებ

გ. ხატიაშვილი
კომპიუტერების მათემატიკური უზრუნველყოფისა და ინფორმაციული ტექნოლოგიების კათედრა

ნაშრომში განზოგადებულია რიგი ავტორების შედეგები დრეკადი იზოტროპული და ანიზოტროპული მასალებისაგან დამზადებული ძელისათვის მცირედ შეშფოთებული ცილინდრული გვერდითი ზედაპირით.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 33-38

INSUFFICIENT DATA AND WEIGHTED FUZZY EXPECTED INTERVAL OF POPULATION

G. Sirbiladze
Chair of the Random Processes Theory

Abstract. In the present work two new versions of Most Typical Value (MTV) are introduced – Generalized Weighted Means. The first WFEV g is generalized version of Weighted Fuzzy Expected Value (WFEV) [4] for any fuzzy measure g and, of course it coincides with WFEV in case of uniform probability distribution. Most Interesting is the second one – Weighted Fuzzy Expected Interval (WFEI) – which is the generalization of WFEV g, when Fuzzy Expected Value (FEV) [1-2] doesn't exist, but Fuzzy Expected Interval (FEI) [3] does. It is based on Friedman-Schneider-Kandel (FSK) principle [4] and using the interval analysis of FEI constructs new MTV, called WFEI. Corresponding converging iteration process is constructed also, which in this case is the interval version of Newton's iteration process and for convergence Moor's MV-Extensions of function are used taking into account generalized interval arithmetic [5].

Let X be finite set, $(\mathrm{X},\mathrm{B},g)$ – space of fuzzy measure, χ_A – membership function of A fuzzy subset: $\chi_A : \mathrm{X} \to [0;1]$. Let X set be divided into k groups so that:

$$X_1=\{x_1,\dots,x_{n_1}\},\ X_2=\{x_{n_1},\dots x_{n_2}\},\dots\ X_k=\{x_{n_{k-1}+1},\dots,x_{n_k}\}.$$

On sets ($\mathrm{X}=\bigcup_{i=1}^{k} X_i$) χ_A membership function is constant: $\chi_A(x)\equiv\chi_i$ for $\forall x\in X_i$. I.e. the fuzzy subset separates groups of population on X. For example, if X is n-dimensional sampling, from it we can get $k\le n$ dimensional variational sampling. If we take uniform discrimination in place of g, then fuzzy measure of group will equal to its frequency ($g(X_i)\equiv {n_i}/{n}$, $i=1,2,\dots,k.$).

On the set $\mathrm{X}=\{x_1,\dots,x_n\}$ there exist n! permutations. Denote any permutations as $\sigma=(\sigma(1),\sigma(2),\dots,\sigma(n))$. The set of all possible permutations by - S_n.

<u>DEFINITION 1</u> [6]: If $\sigma\in S_n$ is any arrangement, then following probability distribution:

$$P_\sigma(x_{\sigma(1)})=g(\{x_{\sigma(1)}\}),\ P_\sigma(x_{\sigma(2)})=g(\{x_{\sigma(1)},x_{\sigma(2)}\})-g(\{x_{\sigma(1)}\}),$$
$$\dots\dots\dots\dots\dots\dots\dots\dots \tag{1}$$
$$P_\sigma(x_{\sigma(n)})=1-g(\{x_{\sigma(1)},\dots,x_{\sigma(n-1)}\})$$

is called associated probability distribution with fuzzy measure g.

$\{\mathrm{P}_\sigma\}_{\sigma\in S_n}=\{P_\sigma(x_{\sigma(1)},\dots,P_\sigma(x_{\sigma(n)}\}_{\sigma\in S_n}$ is called the class of associated probabilities.

It's known, that for $\forall X_i \subset X$ set, $\exists \tau_i \in S_n$ arrangement such, that class of associated probabilities uniquely determines fuzzy measure $g(X_i) = P_{\tau_i}(X_i)$ on finite set [9]. Practically this is probability interpretation of fuzzy measure, which is essential in probability representation of fuzzy means.

DEFINITION 2: Suppose w is nonnegative, strictly decreasing function and $\lambda > 1$ is real number, then the solution of following equation:

$$S = \frac{\sum_{i=1}^{k} w(|\chi_i - S|) \cdot P_{\tau_i}{}^{\lambda}(X_i)\chi_i}{\sum_{i=1}^{k} w(|\chi_i - S|) \cdot P_{\tau_i}{}^{\lambda}(X_i)} \tag{2}$$

is called Weighted Fuzzy Expected Value of order λ with the attached weight function w. (MTV=WFEV $g(\lambda, w)$).

Lets recall two postulates of building WFEV[4], which we'll later call Friedman- Schneider - Kandel principle (FSK principle):

FSK PRINCIPLE: Mathematical expectation: $E^g_{w,\lambda}(\chi_A, \cdot)$ is invariant to the most typical value of population – MTV.

$$E^g_{w,\lambda}(\chi_A, MTV) = MTV. \tag{3}$$

The estimation of FSK principle is called the solution of equation (3) and as we mentioned is called WFEV g. By analogy with [4] we create iteration process for WFEI:

$$S_i = E^g_{w,\lambda}(\chi_A, S_{i-1}), \tag{4}$$

where S_0=FEV. If data on population groups isn't enough and FEV cannot be calculated, and values of membership compatibility are intervals, also it's possible interval representation of values of fuzzy measure $g(X_i)$ and this is more realistic. From statistic interval estimation of probabilities $g(X_i) = P_{\sigma_i}(X_i)$ we can estimate FEI and clearly (4) iteration process cannot work, because there doesn't exist S_0=FEV. In such conditions parameters in (2) are taken from intervals $\chi_i \in [\underline{\chi}_i; \overline{\chi}_i]$, $P^{\lambda}(\chi_i) \in [\underline{P}_i^{\lambda}; \overline{P}_i^{\lambda}]$. Also we can assume that solution of equation (2) is from interval FEI=$[\underline{fei}, \overline{fei}]$. (MTV∈FEI)

DEFINITION 3: If $\chi_i \in [\underline{\chi}_i;\overline{\chi}_i]$, $P^\lambda(\chi_i)\in[\underline{P}_i^\lambda;\overline{P}_i^\lambda]$, Suppose w is nonnegative, strictly decreasing function and $\lambda > 1$ is real number, and if exists unique solution of (2) in FEI, its called Weighted Fuzzy Expected Interval of order λ with the attached weight function w. Its denoted by WFEI $g(\lambda, w)$.

Evidently this principle takes into account FSK principle and for its estimation we'll use interval analysis [5]. Let's denote the right side function of equation (2) by $f_0(s)$. We'll introduce some interval extension of $f_0(s)$, where from analytical judgment $|\chi_i - s|$ will be changed by $|\chi_i - s|^2$.

$$F_0(s)=\frac{\sum_{i=1}^{k} w(|[\underline{\chi}_i;\overline{\chi}_i]-s|^2\cdot[\underline{P}_{\tau_i}^\lambda;\overline{P}_{\tau_i}^\lambda]\cdot[\underline{\chi}_i;\overline{\chi}_i]}{\sum w(|[\underline{\chi}_i;\overline{\chi}_i]-s|^2)[\underline{P}_{\tau_i}^\lambda;\overline{P}_{\tau_i}^\lambda]}, \quad (5)$$

where $s \subset$ FEI$=[\underline{fei},\overline{fei}]$ is any interval.

Clearly WFEI is null of function

$$f(s)=s-f_0(s) \quad (6)$$

Then interval extension of $f(s)$ will be: $F(s)=s-F_0(s), \forall s \subset$ FEI.

Consider function:

$$f(s)=s-\frac{\sum_{i=1}^{k} w(|\chi_i-s|^3)P_{\tau_i}^\lambda\chi_i}{\sum_{i=1}^{k} w(|\chi_i-s|^3)P_{\tau_i}^\lambda}$$

MV-Extension [5] of which $F(S)=F_{MV}(S)$ is:

$$F_{MV}(s)=f(m(FEI))+F_s'(s)(s-m(FEI)). \quad (7)$$

where F'_s is interval extension of:

$$f_s'(s)=1-\frac{2\sum_{i=1}^{k}\frac{\partial w}{\partial s}[\chi_i-s]P_{\tau_i}^\lambda\rho_1-2\sum_{i=1}^{k}\frac{\partial w}{\partial s}[\chi_i-s]P_{\tau_i}^\lambda\chi_i\rho_0}{\{\sum_{i=1}^{k} w(|\chi_i-s|^2)P_{\tau_i}^\lambda\}^2}, \quad (8)$$

where ρ_0 and ρ_1 are known numbers.

Clearly with interval extension its better to use generalized interval arithmetic. After everything develops according to Newton's method in [5,§5} with following denotions:

$$S_{i+1} = S_i \bigcap N(S_i),\ \ell im\ell(S_i) = 0 \text{ and } \ell im_{i\to\infty} S_i = WFEIg(\lambda, w) \qquad (9)$$

Clearly $S_0 \in FEI$. There exist sufficient conditions for convergence of (9).

Here we will state theorems without proofs, which is essential and unites all known and presented here weighted means, which retains correctness of generalization of statistical notions.

THEOREM 1: *If FEI=FEV, intervals of membership χ_i and associated probabilities $P^{\lambda}_{\tau_i}$ are intervals of one point then:*

$$WFEV\, g(\lambda, w) = WFEI\, g(\lambda, w)$$

Note, that for convergence of iteration process (9) the property of compression of f_s' on FEI is enough, which can be easily verified. E.g. when $w(t) = e^{-\beta t}, \lambda > 1$.

THEOREM 2: *If fuzzy measure g coincides with frequency distribution of population groups, then WFEI g (λ, w)= WFEI (λ, w) and for one-point intervals WFEI g (λ, w)= WFEV g (λ, w)= WFEV (λ, w).*

Can be stated, that in cases of insufficient data on population groups, process of fuzzy statistical estimation is distinguished by two stages: From little information follows generalization of fuzzy weighted estimator, which is formally formed by interval analysis, creates entropy growth of information, but mobile FSK principle enables entropy decrease of information, which is condensed in generalized fuzzy statistic, in new MTV of population, which is called Weighted Fuzzy Expected Interval (WFEI).

REFERENCES

1. Kandel A. *Fuzzy Statistics and Forecast Evaluation.* IEEE Trans. on Systems, Man and Cybernetics, SMG-8, No 5 (1978), 396-401.
2. Kandel A. *On the Control and Evaluation of Uncertain Processes.* IEEE Trans. on Automatic Control , vol AC-25, No.6 (1980), 1182-1187.

3. Schneider M., Kandel A. *Properties of the Fuzzy Expected Value and the Fuzzy Expected Interval in Fuzzy Environment.* Fussy Sets and Systems 28 (1988) 55-68.

4. M. Friedman, M. Schneider, A. Kandel. *The use of Weighted Fuzzy Expected Value (WFEV) in Fuzzy Expert Systems.* Fussy Sets and Systems 31 (1989) 37-45.

5. C.A. Kalmikov, U.N. Shokin, Z.Kh. Yulgashev. *Methods of Integral Analysis.* Nauka, Novosibirsk, Russia 1986.

6. I. Campos, C. Bolanos, *Representation of Fuzzy Measures Through Probabilities.* Fussy Sets and Systems 31 (1989) 23-36.

არამკაფიო მონაცემები და პოპულაციის შეწონილი საშუალო მნიშვნელობა

გ. სირბილაძე
შემთხვევით პროცესთა თეორიის კათედრა

ნაშრომში წარმოდგენილია პოპულაციის ყველაზე ტიპიური მნიშვნელობის (MTV) ორი ახალი ვარიანტი – განზოგადოებული შეწონილი საშუალოები. პირველი არის WFEV g, რომელიც შეწონილი არამკაფიო საშუალო მნიშვნელობის (WFEV [4]) განზოგადოებაა ნებისმიერი g არამკაფიო ზომის მიმართ პოპულაციაზე. იგი ემთხვევა WFEV-ს, როცა g თანაბარი ალბათური განაწილებაა. განსაკუთრებით საყურადღებოა მეორე, არამკაფიო საშუალო ინტერვალი, რომელიც თავის მხრივ WFEV g-ს განზოგადებაა, როდესაც არ არსებობს არამკაფიო საშუალო მნიშვნელობა (FEV [1-2]), მაგრამ არსებობს არამკაფიო საშუალო ინტერვალი (FEI [3]). იგი დაფუძნებულია FSK პრინციპზე და FEI–ში ინტერვალური ანალიზის [5] გამოყენებით აგებს ახალ MTV-ს, WFEI-ის სახელწოდებით. შექმნილია შესაბამისი იტერაციული კრებადი პროცესი FEI–ში, რომელიც ამ შემთხვევაში ნიუტონის იტერაციული პროცესის ინტერვალური ვარიანტია და სადაც კრებადობისთვის გამოიყენება ფუნქციის მურის MV–გაფართოება განზოგადოებული ინტერვალური არითმეტიკის გამოყენებით [5].

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 39-54

INTERPRETATION OF λ-ADDITIVE FUZZY DISCRETE MEASURES IN THE PROBLEMS OF FUZZY MEASURE RESTORATION FROM CORRESPONDING INSUFFICIENT DATA

G. Sirbiladze, N. Zaporozhets
Chair of Random Processes Theory

Abstract. In this paper we consider problems of fuzzy measure restoration from corresponding insufficient data on the finite set. An approach is located in the class of Choquet's second order capacities [1],[5]-[6] with nearest distance from λ-additive fuzzy measures [2], when the singleton's "fuzzy weights" are known. This essentially concerns certain frequency distributions, where the feature of additivity is doubtful. This follows from the fuzzy nature of data distribution, when the expert "appoints" data. This fact is an indisputable condition of fuzzy measure introduction, but insufficient for its construction.

Fuzzy measure of the optimal approximation is constructed in the class of Choquet's second order capacities. Measures of specificity, indices of uncertainty and estimations of approximations are calculated.

Some properties of the correctness of the approximation are proved.

INTRODUCTION

There are two classical approaches to data analysis. If experimental data is "sufficiently" exact then for their processing and estimation of general characteristics probabilistic-statistical methods can be used. If data is presented with sufficient "inaccuracy", then for their study the methods of theory of errors will be used. But there are cases when both methods of statistics and the theory of errors do not give satisfactory results.

When data is presented by intervals and their description is "vague" and characterized by overlapping and the receipt of data the expert is and in the intervened, it is clear that the nature of data are combined: parallel to probabilistic-statistical uncertainty there exists the possibilistic uncertainty, guarantees more or less adequate results.

Fuzzy statistics play an essential part in probability-possibility analysis and they are used very effectively in fuzzy expert decision-making systems. Non-additive but monotone measures (fuzzy measures) were first used in fuzzy statistics in 80S by M. Sugeno [3].

We consider problems of fuzzy measure restoration from corresponding insufficient data (the third section).

In [4] there is presented a problem of construction of the distance on fuzzy measures, which is reduced to the distance between probabilistic measures in the class of associated probabilities. This is the problem, considered in the second section. There are considered basic definitions with needed commentaries in the second section.

In the fourth section there is constructed the concrete example and its table interpretation.

1. PRELIMINARY CONCEPTS

Let $X=\{x_1,x_2,\dots,x_n\}$ is the finite reference set, $\mathcal{B}(X)$-algebra of all subsets of X, g-fuzzy measure $\mathcal{B}(X)$ in Sugeno's sense and $(X,\mathcal{B}(X),g)$-fuzzy measure space [3]

1°. Fuzzy measure $g_\lambda \in [0,1]^{\mathcal{B}(X)}$ $(\lambda>-1)$ is λ-additive fuzzy measure [2] if for $\forall A, B \in \mathcal{B}(X)$, $A \cap B = \varnothing$,

$$g_\lambda(A\cup B)=g_\lambda(A)+g_\lambda(B)+\lambda g_\lambda(A)\cdot g_\lambda(B). \tag{1}$$

It is easy to verify that $\forall A \in \mathcal{B}(X)$:

$$g_\lambda(A)=\frac{1}{\lambda}\left\{\prod_{x_i\in A}(1+\lambda\hat{g}_i)-1\right\}, \tag{2}$$

where $0<\hat{g}_i\equiv g\{x_i\}<1$; $\lambda>-1$ is a parameter with the following normalization condition:

$$\frac{1}{\lambda}\left\{\prod_{x_i\in X}(1+\lambda\hat{g}_i)-1\right\}=1.$$

Note that g_0 is probabilistic measure if $\sum_{x_i\in X}\hat{g}_i=1$.

2°. Dual fuzzy measures $g,g^* \in [0,1]^{\mathcal{B}(X)}$ are called respectively lower and upper Choquet's second order capacities [1], [6] if $\forall A,B\in\mathcal{B}(X)$:

$$\begin{gathered} g(A\cap B)+g(A\cup B)\geq g(A)+g(B), \\ g^*(A\cap B)+g^*(A\cup B)\leq g^*(A)+g^*(B), \end{gathered} \tag{3}$$

where $g^*(A)=1-g(\overline{A})$ (duality). Choquet's second order capacities are enough broad class of fuzzy measures. For example, λ-additive fuzzy measure g_λ is Choquet's second order capacity. It is easy verifiable that $g_\lambda^*=g_{-\lambda/(1+\lambda)}$. Let $\{\hat{g}_i\}$ *and* $\{\hat{g}_i^*\}, i=1,2,\dots,n$ denote "fuzzy weights" of singletons for g,g^* dual fuzzy measures respectively.

3°. For each $\sigma=(\sigma(1),\sigma(2),\dots,\sigma(n))\in S_n$ permutation of the finite set $\{1,2,\dots,n\}$ the probability functions [1], [8]:

$$\begin{gathered} P_\sigma(x_{\sigma(1)}) = g(\{x_{\sigma(1)}\}), \\ P_\sigma(x_{\sigma(2)}) = g(\{x_{\sigma(1)}, x_{\sigma(2)}\}) - g(\{x_{\sigma(1)}\}), \\ P_\sigma(x_{\sigma(i)}) = g(\{x_{\sigma(1)}, \ldots, x_{\sigma(i)}\}) - g(\{x_{\sigma(1)}, \ldots, x_{\sigma(i-1)}\})), \\ P_\sigma(x_{\sigma(n)}) = 1 - g(\{x_{\sigma(1)}, \ldots, x_{\sigma(n-1)}\})) \end{gathered} \tag{4}$$

are called the associated probabilities to the fuzzy measure g, where S_n is the permutations group of all natural number from 1 to n. It is proved [1] that if $g, g^* \in [0,1]^{\mathcal{B}(X)}$ are dual fuzzy measures then they have common associated probabilities class:

$$\{P_\sigma(\cdot)\}_{\sigma \in S_n} = \{P_\sigma(\cdot)\}_{\sigma \in S_n} \text{ and } \forall \sigma \in S_n : P_\sigma(\cdot) = P^*_{\sigma^*}(\cdot),$$

where σ^* is dual permutation of $\sigma(\sigma(i) = \sigma^*(n-i+1), i = 1,2,\ldots,n)$.

By (2) and (4) we may write down associated probabilities class for λ-additive fuzzy measure g_λ. $\forall \sigma \in S_n$

$$P_\sigma(x_{\sigma(i)}) = g_\lambda(\{x_{\sigma(i)}\}) \prod_{j=1}^{i-1} (1 + \lambda g_\lambda(\{x_{\sigma(i)}\})), \tag{5}$$

more suitable

$$P_\sigma(x_{\sigma(i)}) = g_\lambda(\{x_{\sigma(i)}\}) \prod_{j=1}^{i(\sigma)-1} (1 + \lambda g_\lambda(\{x_{\sigma(i)}\})), \tag{5'}$$

where $i = 1,2,\ldots,n;\ \sigma \in S_n;\ i(\sigma)$ is the location of x_i in σ permutation. (If $i(\sigma) = 1$ than $\prod_{j=1}^{0} \equiv 1$).

4°. Introduce the following notations. $\mathcal{M}(X) \subset [0,1]^{\mathcal{B}(X)}$-fuzzy measures on $\mathcal{B}$(X); $\mathcal{M}_c(X)$-Choquet's second order capacities on $\mathcal{B}$(X); $\mathcal{M}_\lambda(X) - \lambda$ additive fuzzy measures on $\mathcal{B}$(X); $\mathcal{R}$(X) -probability measures on B (X). It is clear

$\mathcal{R}(X) \subset \mathcal{M}_\lambda(X) \subset \mathcal{M}_c(X) \subset \mathcal{M}(X)$. We know [1] that if $g \in \mathcal{M}_c(X)$ then $\forall A \subseteq X$

$$g(A) = \min_{\sigma \in S_n} P_\sigma(A), \quad g^*(A) = \max_{\sigma \in S_n} P_\sigma(A)$$

5°. Let $T^m \equiv \{(y_1, y_2, \ldots, y_m) \in R^m \,/\, y_i \geq 0, i = 1,2,\ldots,m\}$. Let f be a function $f : T^m \to R^+$. f is called a function generatrix of distance, if the following five properties are satisfied:

(1) $f(y_1, y_2, \ldots, y_m) = 0 \Leftrightarrow y_1 = y_2 = \ldots = y_m = 0$,

(2) $y_i \le z_i, \forall i \Rightarrow f(y_1, y_2, ..., y_m) \le f(z_1, z_2, ..., z_m)$. f is monotone non-decreasing,

(3) $f(y_1 + z_1, y_2 + z_2, ..., y_m + z_m) \le f(y_1, y_2, ..., y_m) + f(z_1, z_2, ..., z_m)$. f is sub additive,

(4) $f(y, y, ..., y) = y$. f is idempotent,

(5) $f(y_{\sigma(1)}, y_{\sigma(2)}, ..., y_{\sigma(m)}), \forall \sigma \in S_m$. f is symmetric.

We rank the $n! \equiv m$ permutations of S_n with some criterion to number them, and thus to represent the class $\{P_\sigma(\cdot)\}_{\sigma \in S_n}$ as an n!-tuple $(P_1, P_2, ..., P_m)$. Let d be some distance on $\mathcal{R}(X)$ [4]. It is proved [4] that the function $D : \mathcal{M}(X) \times \mathcal{M}(X) \Rightarrow R^+$ defined as

$$D(g, g') = f(d(P_1, P_1'), d(P_2, P_2'), ..., d(P_m, P_m'))$$

is distance on $\mathcal{M}(X)$. The examples of function f:

$$f_m(y_1, y_2, ..., y_m) = \max_{1 \le i \le m}\{ y_i \},$$

$$f_q(y_1, y_2, ..., y_m) \equiv \left(\sqrt{\frac{1}{m} \sum_{i=1}^{m} y_i^q} \right)^{1/q}, \quad q \ge 1.$$

The examples of distance d:

$$d_m(P, P') = \max_{1 \le i \le m} |P(x_i) - P'(x_i)|,$$

$$d_q(P, P') \equiv \left(\sqrt{\frac{1}{m} |P_\sigma(x_i) - P_\sigma'(x_i)|^q} \right)^{1/q}, \quad q \ge 1,$$

$$d_S(P, P') = \max_{A \subseteq X} |P(x_i) - P'(x_i)|.$$

Let $D_2 \equiv D_{22}$ denotes the distance $(q = 2)$

$$D_{22}(g, g') = \sqrt{\frac{1}{n!} \sum_{\sigma \in S_n} \sum_{i=1}^{n} (P_\sigma(x_i) - P_\sigma'(x_i))^2}$$

and

$$D_{mq}(g, g') = \max_{\sigma \in S_n} \left(\sqrt{|P_\sigma(x_i) - P_\sigma'(x_i)|^q} \right)^{1/q}$$

6°. Given $g \in \mathcal{M}(X)$. The probability measure $P_g \in \mathcal{R}(X)$ is called nearest from fuzzy measure g if

$$D(g,P_g)=\min_{P\in\mathcal{R}(X)} D(g,P) \qquad (6)$$

It is known [1] that if $P\in\mathcal{R}(X)$ then associated probabilities class contains single probability distribution $P\equiv P_\sigma, \sigma\in S_n$. So the problem of minimizing may be reduced to the problem of minimizing the function D with respect to P:

$$D_2(g,P)=\sqrt{\frac{1}{n!}\sum_{\sigma\in S_n}\sum_{i=1}^{n}(P_\sigma(x_i)-P(x_i))^2}\Rightarrow min$$

$P\in\mathcal{R}(X)$. Applied well-known tools of analysis we receive the solution

$$P_g(x_i)=\frac{1}{n!}\sum_{\sigma\in S_n}P_\sigma(x_i) \qquad (7)$$

$i=1,2,\ldots,n$. If in (7) g is λ-additive fuzzy measure- g_λ and we'll foresee (5) then

$$P_{g_\lambda}(x_i)=\frac{\hat{g}_i}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{i(\sigma)-1}(1+\lambda\hat{g}_{\sigma(j)}), \qquad (8)$$

$i=1,2,\ldots,n$; if in (8) $i(\sigma)=1$ then addend is equal to 1. Here $\hat{g}_i\equiv g_\lambda(\{x_i\})$. The minimum distance D_2 between fuzzy measure g_λ and $\mathcal{R}(X)$ is

$$D_2(g_\lambda,\mathcal{R}(X))=D_2(g_\lambda,P_{g_\lambda})=$$

$$=\sqrt{\frac{1}{n!}\sum_{\sigma\in S_n}\sum_{i=1}^{n}\hat{g}_i^{\,2}\left[\prod_{j=1}^{i(\sigma)-1}\{1+\lambda\hat{g}_{\sigma(j)}\}-\frac{1}{n!}\sum_{\tau\in S_n}\prod_{k=1}^{i(\tau)-1}\{1+\lambda\hat{g}_{\tau(k)}\}\right]^2}. \qquad (9)$$

If $\lambda=0$, (when $\sum_{i=1}^{n}g_i=1$) g_0 is probability measure then $D_2\equiv 0$. This distance is called a degree of unspecific [4].

7°. For given $g\in\mathcal{M}(X)$

$$C(g)=min\{D(g,Bel_0),D(g^*,Pl_0)\} \qquad (10)$$

is called an induce of specificity [4], where Bel_0 and Pl_0 are dual fuzzy measures of the beliefe and plausibility of whole ignorance. $\forall A\subseteq X$

$$Bel_0(A)=\begin{cases}0 & if \quad A\neq X\\ 1 & if \quad A=X\end{cases}, \qquad Pl_0(A)=\begin{cases}0 & if \quad A\neq\varnothing\\ 1 & if \quad A=\varnothing\end{cases}.$$

If $C(g_\lambda) \approx 0$ then g_λ is near to Bel_0 or Pl_0 and g_λ hasn't the specificity. If $c >> 0$ then g_λ has a high degree of specificity. The associated probabilities class of Bel_0 is:

$$P_\sigma^{Bel_0}(x_{\sigma(i)}) = \begin{cases} 1 & if \quad i = n \\ 0 & if \quad i \neq n \end{cases},$$

$i = 1,2,\ldots,n;\ \sigma \in S_n$. Then

$$P_{Bel_0}(x_i) = \frac{1}{n!}\sum_{\sigma \in S_n} P_\sigma^{Bel_0}(x_{\sigma(i)}) = \frac{1}{n!}(n-1)! = \frac{1}{n}$$

and we receive the uniform probability distribution. Hence

$$C(g_\lambda) = \sqrt{\frac{1}{n!}\sum_{\sigma \in S_n}\sum_{i=1}^{n}\hat{g}_i^{\,2}\left[\prod_{j=1}^{i(\sigma)-1}\{1+\lambda\hat{g}_{\sigma(j)}\} - \frac{1}{n}\right]^2}. \qquad (11)$$

2. The Problem of Fuzzy Measure Restoring

In practice the subjective expert data is often performed only for singleton factors, because any measurements of multifactorial variants practically don't exist. For example: if four x_1, x_2, x_3, x_4 factors (symptoms) act on the illness y then by some expert (doctor) may be performed frequency distribution table (table1), where some "weights" are subjectively "appointed" but pair "fuzzy weights" almost don't exists.

Here we offer the method which restores dual fuzzy measures (g, g^*) with best approach to $\mathcal{M}(X)$ from $\mathcal{B}(X)$ in the sense of distance D_2 though with complimented condition. Let it is only known "fuzzy weights" of singletons:

$$0 < \hat{g}_i \equiv g(\{x_i\}) < 1\ ,\ i = 1,2,\ldots,n. \qquad (12)$$

Let

$$\mathcal{M}(X, \hat{g}_1, \hat{g}_2, \ldots, \hat{g}_n) = \{g \in \mathcal{M}(X) / g(\{x_i\}) = \hat{g}_i,\quad i = 1,2,\ldots,n\}$$

is the class of fuzzy measures of $\mathcal{M}(X)$ with coinciding values of measures on singletons.

TABLE 1

$A \subseteq X = \{x_1, x_2, x_3, x_4\}$	g
$\{x_1\}$	0.2
$\{x_2\}^{*)}$	0.3subj
$\{x_3\}$	0.4
$\{x_4\}$	0.2subj
$\{x_1, x_2\}$	?
$\{x_1, x_3\}$	?
$\{x_1, x_4\}$	?
$\{x_2, x_3\}$	?
$\{x_2, x_4\}$	?
$\{x_3, x_4\}$	?
$\{x_1, x_2, x_4\}$	?
$\{x_2, x_3, x_4\}$	?
$\{x_1, x_2, x_3\}$	?
$\{x_1, x_3, x_4\}$	?
$\{x_1, x_2, x_3, x_4\}$	1

TABLE1: insufficient expert frequency distribution of some illness with respect only to 4 symptoms in terms of the fuzzy measure g

*) Data with notion "subj" is appointed by the expert.

Analogously

$$\mathcal{M}_c(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n) = \mathcal{M}_c(X) \cap \mathcal{M}(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n)$$

is the class of second order Choquet's capacities with the same property and

$$\mathcal{M}_\lambda(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n) = \mathcal{M}_\lambda(X) \cap \mathcal{M}(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n)$$

is the same class for λ-additive measures. It is clear that

$$\mathcal{M}_\lambda(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n) \subset \mathcal{M}(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n)$$

$\lambda > -1$ is a free parameter of the distribution of λ-additive fuzzy measure $g \in \mathcal{M}_\lambda(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n)$ with normalization condition (3). If $\sum_{i=1}^{n} \hat{g}_i = 1$, then $\lambda_0 \equiv 0$ value is assumed (g_0 is probability measure), otherwise λ is the root of the following polinom:

$$\Pi(\lambda) = \left(\prod_{i=1}^{n} \hat{g}_i\right)\lambda^{n-1} + ... + \left(\sum_{i<j<k} \hat{g}_i \hat{g}_j \hat{g}_k\right)\lambda^2 + \left(\sum_{i<j} \hat{g}_i \hat{g}_j\right)\lambda + \sum_{i=1}^{n} \hat{g}_i - 1 \, . \quad (13)$$

Let $L \equiv \{\lambda_1, \lambda_2, ..., \lambda_l\}$ is the set of real roots of the polinom (13) ($\lambda > -1$).

Let $L \neq \varnothing$. Introduce the following short notations:

$$\mathcal{M}^L(X) = \{g_{\lambda_i} \in \mathcal{M}_\lambda(X; \hat{g}_1, \hat{g}_2, ..., \hat{g}_n) / \lambda_i \in L, i = 1,2,...,l\}$$

It is clear, that $\hat{g}_i^*$ values are not "freedom" (for $\forall \lambda \in L$):

$$\hat{g}_i^* = 1 - \frac{1}{\lambda}\left\{\prod_{\substack{j=1 \\ j \neq i}}^{n} (1 + \lambda \hat{g}_j) - 1\right\}, \quad i = 1,2,...,n$$

and if $\lambda > 0$ then $\hat{g}_i \leq \hat{g}_i^*$, $i = 1,...,n$; if $-1 < \lambda < 0$ then $\hat{g}_i \geq \hat{g}_i^*$, $i = 1,...,n$.

Analogously, we may construct $L^* = \{\lambda^* > -1 / \lambda^* = -\dfrac{\lambda}{1+\lambda}, \; \lambda \in L\}$ and $\mathcal{M}^{L^*}(X) \subset \mathcal{M}^*\left(X; \hat{g}_1^*, \hat{g}_2^*, ..., \hat{g}_n^*\right)$ classes.

The classes

$$\mathcal{R}^L(X) = \{P_{g_\lambda} \in \mathcal{R}(X) / \lambda \in L\}, \qquad \mathcal{R}^{L^*}(X) = \{P_{g_\lambda^*} \in \mathcal{R}(X) / \lambda^* \in L^*\}$$

are probability measures classes. We calculate the distances:

$$D_2(\mathcal{R}^L(X), \mathcal{M}^L(X)) = \min_{\lambda', \lambda'' \in L} D_2(P_{g_{\lambda'}}, g_{\lambda''}) = \min_{\lambda \in L} D_2(P_{g_\lambda}, g_\lambda) =$$
$$= \min_{\lambda \in L} \sqrt{\frac{1}{n!} \sum_{\sigma \in S_n} \sum_{i=1}^{n} \hat{g}_i^2 \left\{\prod_{j=1}^{i(\sigma)-1} \{1 + \lambda \hat{g}_{\sigma(i)}\} - \frac{1}{n!} \sum_{\tau \in S_n} \prod_{k=1}^{i(\tau)-1} \{1 + \lambda \hat{g}_{\tau(k)}\}\right\}^2}, \quad (14)$$

$$D_2(\mathcal{R}^{L^*}(X),\mathcal{M}^{\dot{L}}(X))=\min_{\lambda^*\in L^*}D_2(g^*_{\lambda^*},P_{g^*_{\lambda^*}})=$$

$$=\min_{\lambda^*\in L^*}\sqrt{\frac{1}{n!}\sum_{\sigma\in S_n}\sum_{i=1}^{n}\hat{g}_i^{\bullet 2}\left\{\prod_{j=1}^{i(\sigma)-1}\{1+\lambda^*\hat{g}^{\bullet}_{\sigma(j)}\}-\frac{1}{n!}\sum_{\tau\in S_n}\prod_{k=1}^{i(\tau)-1}\{1+\lambda^*\hat{g}_{\tau(k)}\}^{\bullet}\right\}^2}\,. \tag{14'}$$

Let these distances are reached on the fuzzy measures $g_{\hat{\lambda}}$ and $g^*_{\hat{\lambda}'}$:

$$D_2(\mathcal{R}^L(X),\mathcal{M}^L(X))=D_2(g_{\hat{\lambda}},P_{g_{\hat{\lambda}}}),\qquad D_2(\mathcal{R}^L(X),\mathcal{M}^{L^*}(X))=D_2(g^*_{\hat{\lambda}},P_{g^*_{\hat{\lambda}}}).$$

<u>DEFINITION 1:</u> *$(g_{\hat{\lambda}},g^*_{\hat{\lambda}})$ pair fuzzy measures are called λ-additive fuzzy approximation to $\hat{g}_i$, $i=1,\dots,n$ insufficient expert data.*

<u>DEFINITION 2:</u> $\left(P_{g_{\hat{\lambda}}},P_{g^*_{\hat{\lambda}}}\right)$ *pair probability measures are called probability approximation to $\hat{g}_i$, $i=1,\dots,n$ insufficient expert data.*

Notice that if $\sum_{i=1}^{n}\hat{g}_i=1$ and the problem of restoring of measure doesn't exist. If we know that g_0 is not probability measure then suppose $\lambda\neq 0$, $\mathcal{R}^L(X)\cap\mathcal{M}^L(X)=\varnothing$. It is easily checked that $(g_\lambda)^*=g-\dfrac{\lambda}{1+\lambda}\equiv g^*_{\lambda^*}$, $\lambda^*=-\dfrac{\lambda}{1+\lambda}$; When $\lambda\to 0$ in L then $\lambda^*\to 0$ in L^* and minima in (14), (14') are respectively reached on $\hat{\lambda}$, $\left|\hat{\lambda}\right|=\min_{\lambda\in L}|\lambda|$ and on $\hat{\lambda}^*$, because

$$\min_{\lambda^*\in L^*}\left|\lambda^*\right|=\min_{\lambda\in L}\frac{|\lambda|}{1+\lambda}=\frac{\left|\hat{\lambda}\right|}{1+\hat{\lambda}}=\left|\lambda^*\right|.$$ We receive $(\hat{\lambda})^*=\hat{\lambda}'$, $(g_{\hat{\lambda}})^*=g^*_{\hat{\lambda}}=g^*_{\hat{\lambda}'}$. Given result may be represented as a proposition:

<u>PROPOSITION 1:</u> *Probability approximation corresponds to dual fuzzy approximation measures and $D_2(\mathcal{M}^L,\mathcal{R}^L)=D_2(\mathcal{M}^{L^*},\mathcal{R}^{L^*})$.*

<u>PROPOSITION 2:</u> The *probability approximation pair is equal probability measures.*

<u>PROOF:</u> It is clear that $g_{\lambda_1}\le g_{\lambda_2}$ if $\lambda_1\le\lambda_2$ $(\lambda_1,\lambda_2\in L)$ and

$$g_{\hat{\lambda}} \leq P_{g_{\hat{\lambda}}}, \quad P_{g^{*}_{\hat{\lambda}}} \leq g^{*}_{\hat{\lambda}}.$$

λ-fuzzy measure $g_{\hat{\lambda}}, g^{*}_{\hat{\lambda}}$ are nearest to probability measures in the sense of distance D_2. Thus only probability distribution corresponds to the case $\lambda = 0$. $P_{g_{\hat{\lambda}}} = P_{g^{*}_{\hat{\lambda}}}$

$D_2(\mathcal{M}^L, \mathcal{R}^L) = D_2(\mathcal{M}^{L^*}, \mathcal{R}^{L^*})$.

In reality insufficient (similar to table 1) Data of dual fuzzy measures may be not single, but given by some experts $E_X = \{I_1, I_2, ..., I_E\}$.

<u>DEFINITION 3:</u> Data $\hat{g}^{\alpha}_{i}, i = 1,...,n;\ \ \alpha \in E_X$ (defined as (12)) are called insufficient expert data of the fuzzy measure g given by experts E_X.

Insufficient expert data produce $\{\mathcal{M}^{L_\alpha}, \mathcal{M}^{L^*_\alpha}, \mathcal{R}^{L_\alpha}, \mathcal{R}^{L^*_\alpha}\}, \alpha \in E_x$ classes from where we'll build probability approximations class $\{\hat{P}_\alpha / \alpha \in E_X\}$ and λ-additive fuzzy approximations class $\{(\overline{g}_\alpha, \overline{g}^{*}_\alpha) / \alpha \in E_X\}$.

<u>DEFINITION 4:</u> Dual fuzzy measures defined $\forall A \subseteq X$:

$$\tilde{g}(A) = \min_{\alpha \in E_X} \hat{P}_\alpha(A), \quad \tilde{g}^{*}(A) = \max_{\alpha \in E_X} \hat{P}_\alpha(A) \tag{15}$$

are called zero approach optimal approximation.

<u>DEFINITION 5:</u> Pair fuzzy measures defined as $\forall A \subseteq X$:

$$\overline{g}(A) = \min_{\alpha \in E_X} \overline{g}_\alpha(A), \quad \overline{g}'(A) = \max_{\alpha \in E_X} \overline{g}^{*}_\alpha(A) \tag{16}$$

are called first approach optimal approximation.

<u>DEFINITION 6:</u> Pair fuzzy measures defined as $\forall A \subseteq X$:

$$\overline{\overline{g}}(A) = \max_{\alpha \in E_X} \overline{g}_\alpha(A), \quad \overline{\overline{g}}^{*}(A) = \min_{\alpha \in E_X} \overline{g}^{*}_\alpha(A) \tag{17}$$

are called second approach optimal approximation.

<u>PROPOSITION 3:</u> *Pairs measures first and second approach optimal approximations are respectively dual fuzzy measures.*

<u>PROOF:</u> $\forall A \subseteq X$: $\overline{g}'(A) = \max_{\alpha \in E} \overline{g}^{*}_\alpha(A) = \max_{\alpha \in E}(1 - \overline{g}_\alpha(\overline{A})) = 1 - \min_{\alpha \in E} \overline{g}_\alpha(\overline{A}) = 1 - \overline{g}(\overline{A})$. i.e. $\overline{g}' = \overline{g}^{*}$. Analogously we'll receive $\overline{\overline{g}}' = \overline{\overline{g}}^{*}$.

PROPOSITION 4: *Between zero, first and second approach optimal approximations exists the following inequalities* $\forall A \subseteq X$:

$$1.\ \overline{g}(A) \le \overline{\overline{g}}(A);\quad \overline{\overline{g}}^{*}(A) \le \overline{g}^{*}(A),$$
$$2.\ \overline{g}(A) \le \widetilde{g}(A);\quad \widetilde{g}^{*}(A) \le \overline{g}^{*}(A),$$
$$3.\ \overline{\overline{g}}(A) \le \widetilde{g}^{*}(A);\quad \widetilde{g}(A) \le \overline{\overline{g}}^{*}(A).$$

PROOF: Consider $\forall A \subseteq X$. Then

$$1.\ \overline{g}(A) = \min_{\alpha\in E_X} \overline{g}_\alpha(A) \le \max_{\alpha\in E_X} \overline{g}_\alpha(A) = \overline{\overline{g}}(A);$$

$$\overline{\overline{g}}^{*}(A) = \min_{\alpha\in E_X} \overline{g}_\alpha^{*}(A) \le \max_{\alpha\in E_X} \overline{g}_\alpha^{*}(A) = \overline{g}^{*}(A),$$

$$2.\ \overline{g}(A) = \min_{\alpha\in E_X} \overline{g}_\alpha(A) \le \min_{\alpha\in E_X} P_{\overline{g}_\alpha}(A) = \widetilde{g}(A);$$

$$\widetilde{g}^{*}(A) = \max_{\alpha\in E_X} P_{\overline{g}_\alpha^{*}}(A) = \max_{\alpha\in E_X} P_{\overline{g}_\alpha}(A) \le \max_{\alpha\in E_X} \overline{g}_\alpha^{*}(A) = \widetilde{g}^{*}(A),$$

$$3.\ \overline{\overline{g}}(A) = \max_{\alpha\in E_X} \overline{g}_\alpha(A) \le \min_{\alpha\in E_X} P_{\overline{g}_\alpha}(A) = \max_{\alpha\in E_X} P_{\overline{g}_\alpha^{*}}(A) = \widetilde{g}^{*}(A),$$

$$\widetilde{g}(A) = \min_{\alpha\in E_X} P_{\overline{g}_\alpha}(A) = \min_{\alpha\in E_X} P_{\overline{g}_\alpha^{*}}(A) \le \min_{\alpha\in E_X} \overline{g}_\alpha^{*}(A) = \overline{\overline{g}}^{*}(A).$$

DEFINITION 7: Distances $D_2(g,\widetilde{g}) = D_2(g^{*},\widetilde{g}^{*})$, $D_2(g,\overline{g}) = D_2(g^{*},\overline{g}^{*})$ and $D_2(g^{*},\overline{\overline{g}}^{*}) = D_2(g,\overline{\overline{g}})$ are respectively called first and second approach optimal approximation errors.

There variants of optimal approximations, which are received from insufficient data of unknown fuzzy measure g, perform their "restored" faces. For comparison we consider the example where fuzzy measure g, will be known, We'll restore them their "fuzzy weights" of single sets (insufficient data) and estimate errors.

3. THE EXAMPLE

Consider one example when fuzzy dual measures (g, g^{*}) are known (case for one expert). Let $X = \{x_1, x_2, x_3\}$ and dual fuzzy measure $g \le g^{*}$ are "almost" uniform probability measures (table2),

TABLE2

$A \subseteq X = \{x_1, x_2, x_3\}$	g	g^*
$\varnothing$	0	0
$\{x_1\}$	1/4	1/3
$\{x_2\}$	1/3	1/3
$\{x_3\}$	1/3	1/3
$\{x_1, x_2\}$	2/3	2/3
$\{x_1, x_3\}$	2/3	2/3
$\{x_2, x_3\}$	2/3	1/4
$\{x_1, x_2, x_3\}$	0	0

but their associated probabilities class are given in table 2'

TABLE 2'

σ	$P_\sigma(x_{\sigma(1)})$	$P_\sigma(x_{\sigma(2)})$	$P_\sigma(x_{\sigma(3)})$	$P^*_\sigma(x_{\sigma(1)})$	$P^*_\sigma(x_{\sigma(2)})$	$P^*_\sigma(x_{\sigma(3)})$
1,2,3	1/4	5/12	1/3	1/3	1/3	1/3
1,3,2	1/4	5/12	1/3	1/3	1/3	1/3
2,1,3	1/3	1/3	1/3	1/3	1/3	1/3
2,3,1	1/3	1/3	1/3	1/3	5/12	1/4
3,1,2	1/3	1/3	1/3	1/3	1/3	1/3
3,2,1	1/3	1/3	1/3	1/3	5/12	1/4

The equation (13) has the following face:

$$0.02(7)\lambda^2 + 0.27(7)\lambda - 0.008334 = 0$$

from where $\lambda = \hat{\lambda} = 0.029928$. Then $\lambda^{*} = -\hat{\lambda}/(1+\hat{\lambda}) = -0.0290583$. $g_{\hat{\lambda}} = g_{0.029928}$

$\overline{g}^{*} = \hat{g}_{0.029928}$; the associated probabilities class of $\overline{g}$ is performed in table 3:

TABLE 3

σ	$P_{\sigma}(x_{\sigma(1)})$	$P_{\sigma}(x_{\sigma(2)})$	$P_{\sigma}(x_{\sigma(3)})$
1,2,3	0.25	0.3360771	0.4139229
1,3,2	0.25	0.3360771	0.4139229
2,1,3	0.3333333	0.2527458	0.4139229
2,3,1	0.3333333	0.3366473	0.3300194
3,1,2	0.3333333	0.2527438	0.4139229
3,2,1	0.3333333	0.3366473	0.3300194

We'll set distribution table of fuzzy measures $g, g^{*}, \overline{g}, \overline{g}^{*}$ with errors (table 4): (here $\overline{g} = \overline{\overline{g}}$, $\overline{g}^{*} = \overline{\overline{g}}^{*}$)

TABLE 4

$A \subseteq X = \{x_1, x_2, x_3\}$	$\overline{g} = g_{0.0299084}$	$\overline{g}^{*} = g_{-0.0290583}$	$\lvert g(A) - \overline{g}(A)\rvert$	$\lvert g^{*}(A) - \overline{g}^{*}(A)\rvert$
$\{x_1\}$	1/4	0.3300185		0.0000185
$\{x_2\}$	0.3333333	0.4139569~0		~0.0806
$\{x_3\}$	0.3333333	0.4139569~0		0.0806
$\{x_1, x_2\}$	0.5860771	0.6666678	0.0805	~0.0000078
$\{x_1, x_3\}$	0.5960771	0.6666678	0.0805	0.00000078
$\{x_2, x_3\}$	0.6699806	0.7500764	0.003319	0.0000764
$\{x_1, x_2, x_3\}$	≈ 1	≈ 0		≈ 0

The first approach optimal approximation error is

$$D_2(g,\overline{g}) = D_2(g^*,\overline{g}^*) = 0.0108278 .$$

For setting $\tilde{g},\tilde{g}^*$ we have one expert, therefore $\tilde{g}$ and $\tilde{g}^*$ are probability measures and $\tilde{g} = P_{g_{\hat{\lambda}}} = P_{g^*_{\hat{\lambda}^*}} = \tilde{g}^* = P_{g_{0.029928}}$ of which distribution is (tabie 5):

TABLE 5:

σ/P	$P(x_1)$	$P(x_2)$	$P(x_3)$
	0.2775876	0.3612063	0.3612061

but the distribution table with errors is (table 6):

TABLE 6:

$A \subseteq X$	$g = g^* = P_{g_{0.029928}}$	$\lvert g(A) - \tilde{g}(A)\rvert$	$\lvert g^*(A) - \tilde{g}^*(A)\rvert$
$\{x_1\}$	0.25	0	0.08
$\{x_2\}$	0.31	0.02	0.02
$\{x_3\}$	0.42	0.09	0.09
$\{x_1,x_2\}$	0.56	0.1	0.1
$\{x_1,x_3\}$	0.67	0.01	0.01
$\{x_2,x_3\}$	0.73	0.07	0.07
$\{x_1,x_2,x_3\}$	1	0	0

The zero approach optimal approximation error is

$$D_2(g,\tilde{g}) = D_2(g^*,\tilde{g}^*) = 0.0046453$$

As calculations shows (tables 2-6) estimations given by the approximations are enough "high", maintain precisions of approaches. The zero approach optimal approximation is more exact than first one, because here fuzzy measures (g,g^*) are "near" or "similar" with probability measure and given by one expert.

REFERENCES

1. L.M. de Campos and M.J. Bolanos, *Representation of fuzzy measures through probabilities*, Fuzzy Sets and Systems 31 (1989) 23-36
2. I. Tsukamoto, M.M. Gupta, P.N. Nikiphoruk. *On the dencity of λ-fuzzy measure*. Fuzzy set and Possibility Theory. Recent Developments Toronto, Sydney, Paris, Frankfurt. Edited by R.R. Yager, Iona College New York, Oxford 1986.
3. M. Sugeno, Theory of fuzzy integrals and applications, Ph. D. Thesis, Tokio Inst. Of Technology, (1974).
4. L.M. de Campos. M.T. Lamata and S. Moral, *Distances between Fuzzy measures through associated probabilities*, Some applications-Fuzzy sets and Systems 35 (1990) 57-71.
5. G.Sirbiladze, *Choquet's capacities on the field of insufficient expert data*. Rep. of Enlarged Session of Vekua's Istitute of Appl. Mathem. vol 14 N8 (1999) 31-37.
6. G. Choquet, *Theoy of capacities*, Ann. Inst. Fourir 5 (1953) 131-295.
7. A. Kandel, *On the control and evolation of uncertain processes*. IEEE Trans. on Automatic control, vol. AC-25, N6 (1980) 1182-1187.
8. G.Sirbiladze, *Fuzzy Subset Construction through the associated Probabilities*, Bulletin of the Georgian Academy of Sciences, vol. 163,3(2001) 436-440.

λ ადიტიური დისკრეტული არამკაფიო ზომების ინტერპრეტაციები არასაკმარისი მონაცემებიდან არამკაფიო ზომის აღდგენის ამოცანებში

გ. სირბილაძე, ნ. ზაპოროჟეცი
შემთხვევით პროცესთა თეორიის კათედრა

ცნობილია, რომ თუ მონაცემები წარმოდგენილია ინტერვალებით, ქმნიან ე.წ. კონსონანტურ ტანს, მათი განაწილება ბუნდოვანია, ხასიათდება გადაფარვებით და ამ მონაცემების აღწერასა და მიღებაში ჩარეულია ექსპერტი, რომელიც მონაცემთა ობიექტური აღწერის პარალელურად „ერევა" მონაცემთა შეფასებებში, მაშინ მონაცემთა ბუნება კომბინირებულია და არსებობს ე.წ. ალბათურ–შესაძლებლობითი განუზღვრელობა. ცხადია ასეთ შემთხვევებში მხოლოდ ალბათურ–შესაძლებლობითი ანალიზი განაპირობებს დამაკმაყოფილებელი შედეგების მიღებას.

ალბათურ-შესაძლებლობით ანალიზში ერთ-ერთი მნიშვნელოვანი ადგილი უკავია არამკაფიო სტატისტიკას, რომელიც ეფექტურად გამოიყენება გადაწყვეტილების მიღების დამხმარე არამკაფიო ექსპერტულ სისტემებში. არაადიტიური, მაგრამ მონოტონური (არამკაფიო) ზომები არამკაფიო სტატისტიკაში პირველად გამოყენებული იქნა მ. სუჯენოს [3] მიერ. არამკაფიო ზომის აგება უმნიშვნელოვანესი ამოცანაა არამკაფიო ანალიზში. ამ სტატიაში ჩვენ ვეხებით ასეთ ამოცანას: არამკაფიო ზომის აღდგენა მისი ექსპერტული არასაკმარისი მონაცემებიდან (მესამე ნაწილი).

პრაქტიკაში სუბიექტური მონაცემების აღწერისას ექსპერტული მონაცემების წარმოდგენები ხშირად მხოლოდ „ერთ ელემენტიან" ფაქტორებს გააჩნიათ, ან თითქმის მხოლოდ მათ, რადგან ეს ყველაფერი დაკავშირებულია კომბინირებული ვარიანტების ფაქტორებზე ანალოგიურ გაზომვებთან, შეფასებებთან, „დაშვებებთან" და ა.შ., რაც პრაქტიკულად არ გვხვდება.

ჩვენ ამ სტატიაში გთავაზობთ მეთოდს, რომელიც უცნობი დუალური (g, g^*) არამკაფიო ზომებს აღადგენს D_2 მანძილის აზრით ე.წ. λ-ადიტიური არამკაფიო ზომების კლასზე საუკეთესო მიახლოებით. თუმცა ერთი დამატებითი პირობით: ცნობილია მხოლოდ ერთ ელემენტიანი „არამკაფიო" წონები:

$$0 \le g_i \equiv g(\{x_i\}) \le g^*(\{x_i\}) \equiv g^* \le 1.$$

აქედან გამომდინარე აგებულია ოპტიმალური აპროქსიმაციის არამკაფიო ზომა – შოკეს მიერ რიგის ტევადობა. გამოთვლილია მისი სპეციფიციურობისა და განუზღვრელობის ინდექსები. ასევე აპროქსიმაციის ცდომილება. მტკიცდება (კორექტულობის) თეორემა დუალური ზომების ოპტიმალური აპროქსიმაციაზე.

მესამე ნაწილში [4] წარმოდგენილია არამკაფიო ზომათა ველზე მანძილის აგების ამოცანა, რომელიც ე.წ. ასოცირებულ ალბათობათა კლასებზე მანძილის აგებაზე დაიყვანება.

მეორე ნაწილში ძირითადად წარმოდგენილია საბაზისო ცნებები, ხოლო მეოთხე ნაწილში წარმოდგენილია საილუსტრაციო მაგალითი.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 55-60

ВЫЧИСЛЕНИЕ ОСОБОГО ОПТИМАЛЬНОГО УПРАВЛЕНИЯ В КВАЗИЛИНЕЙНЫХ УПРАВЛЯЕМЫХ СИСТЕМАХ СО СМЕШАННЫМИ ОГРАНИЧЕНИЯМИ

З. Цицадзе
Кафедра теории управления

Абстракт. Рассмотрена квазилинейная задача оптимального управления со скалярным управлением при наличии смешанных ограничений. Приведены необходимые условия оптимальности в форме аналога принципа максимума.

Исследован особый случай, когда функция управления не определяется однозначно из принципа максимума. Построена процедура вычисления одномерного особого управления, и с применением этой процедуры решена конкретная задача.

В некоторых задачах оптимального управления использование принципа максимума Понтрягина (см.[1]) сопряжено с необходимостью исследовать особые управления (см.[2]). Для простоты рассмотрим следующую задачу со скалярным управлением :

$$\int_{t_0}^{t_1} h(x(t))dt \to min \ , \tag{1}$$

при ограничениях

$$\frac{dx}{dt} = f_0(x) + uf_1(x) \ , \tag{2}$$

$$g(x,u) \le 0, \tag{3}$$

$$x(t_0) = x_0, x(t_1) = x_1, \tag{4}$$

где $x \in E^n$, $f_0 \in E^n$, $f_1 \in E^n$, $g \in E^m$, $u \in R$.

В случае, когда $u = u(t)$ -кусочно непрерывная функция, $x = x(t)$ -кусочно гладкая вектор-функция, а вектор-функции f_0, f_1, g и скалярная функция h являются непрерывными и достаточное число раз непрерывно дифференцируемыми относительно своих аргументов, процедура определения оптимального особого управления предложена в работе [3]. Этой процедурой можно пользоваться и в случае, когда $u(t) \in L_1[t_0,t_1]$, $x(t) \in W^n_{1,1}[t_0,t_1]$, $f_0(x)$ -выпуклая вектор-функция своего аргумента, $f_1(x)$ -постоянный ненулевой вектор, $h(x)$ -скалярная выпуклая функция аргумента x, а вектор-функция g линейна по u и выпукла по x. Действительно, при сделанных предположениях можно воспользоваться теоремой1 из [4], откуда следует справедливость следующих необходимых условий оптимальности:

если $(x(t),u(t))$ оптимальное решение задачи (1)-(4), то существуют такие абсолютно непрерывные на интервале $[t_0,t_1]$ функции $\psi^1(t),...,\psi^n(t)$, постоянная $\psi^0 \le 0$ и функции $\mu^\alpha(t) \in L_\infty[t_0,t_1]$, $\alpha=\overline{1,m}$, которые почти всюду на $[t_0,t_1]$ удовлетворяют условиям:

$$\frac{d\psi}{dt}=-\frac{dH}{dx}, \tag{5}$$

$$\mu^\alpha(t)\ge 0, \tag{6}$$

$$\mu^\alpha(t)g^\alpha(x(t),u(t))=0\ ,\quad \alpha=\overline{1,m}; \tag{7}$$

$$\psi(t)f_1u(t)=\max_{u\in\{u|g(x(t),u)\le 0\}}\psi(t)f_1u, \tag{8}$$

$$\frac{\partial H}{\partial u}=0, \tag{9}$$

$$(\psi^0,\psi(t))\ne(0,0), \tag{10}$$

где

$$\psi(t)=(\psi^1(t)...\psi^n(t)),$$

$$H=\psi^0h+\psi(t)(f_0(x)+uf_1)-\mu(t)g=\psi^0h+\sum_{i=1}^n\psi^i(t)(f_0^i(x)+uf_1^i)-\sum_{\alpha=1}^m\mu^\alpha(t)g^\alpha.$$

В случае, когда

$$\mu^\alpha(t)\equiv 0,\ \psi(t)f_1\equiv 0,\ t_0\le t\le t, \tag{11}$$

условия (7)-(9) не дают возможность определения $u(t)$, т.е. возникает особое управление. Следуя [3], обозначим $H_0(x,\psi)=\psi_0h(x)+\psi(t)f_0(x)$, $H_1(\psi)=\psi(t)f_1$. Тогда $H=H_0(x,\psi)+H_1(\psi)u-\mu g$. Так, как (2) можно переписать в виде

$$\frac{dx}{dt}=\frac{\partial H}{\partial\psi},$$

то в силу свойств скобок Пуассона с учётом (11) имеем:

$$\frac{d}{dt}H_1(t)=\langle H_1,H\rangle=\langle H_1,H_0\rangle+u\langle H_1,H_1\rangle-\sum_{\alpha=1}^m\mu^\alpha\langle H_1,g^\beta\rangle=0,$$

почти всюду на $t_0\le t\le t_1$. Таким же образом получаем:

$$\frac{d^2}{dt^2}H_1 = -\frac{d}{dt}\langle H_0, H_1\rangle = -\langle\langle H_0, H_1\rangle, H\rangle = \langle H_0, \langle H_0, H_1\rangle\rangle + \\ + u(t)\langle H_1, \langle H_0, H_1\rangle - \sum_{\alpha=1}^{m}\mu^\alpha(t)\langle g^\alpha, \langle H_0, H_1\rangle = 0. \quad (12)$$

почти всюду на $t_0 \le t \le t_1$. Здесь коэффициент при u(t) , вообще говоря, не равен нулю. Поэтому для точек t, $t_0 \le t \le t_1$, где он отличен от нуля, учитывая (11), можно определить особое управление $u(t)$ по формуле:

$$u(t) = -\frac{\langle H_0, \langle H_0, H_1\rangle\rangle}{\langle H_1, \langle H_0, H_1\rangle\rangle}. \quad (13)$$

Если коэффициент при управлении $u(t)$ в (12) равен нулю на отрезке $\tau \subset [t_0, t_1]$, то для получения особого управления на отрезке τ продолжаем вычислять последовательные полные производные по t от функции H_1. Все они должны равняться нулю на отрезке τ. Ясно, что управление $u(t)$ явно может появиться лишь в выражении производной чётного порядка, в силу чего особое управление необходимо равно:

$$u(t) = -\frac{\langle H_0, \langle H_0, ..., \langle H_0, H_1\rangle\rangle ...\rangle}{\langle H_1, \langle H_0, ..., \langle H_0, H_1\rangle\rangle ...\rangle}, \quad (14)$$

для п.в. $t \in [t_o, t_1]$, в которых выражение $H_2 = \langle H_1, \langle H_0, ..., \langle H_0, H_1\rangle\rangle ...\rangle$ подсчитанное вдоль особых траекторий $[x(t), u(t)]$, отлично от нуля.

Для вычисления особого управления на множестве нулей функции H_2, заметим, что формула (14) получена для случая, когда в выражении $\bar{H} = H_0 + H_1 u$ коэффициент при u на отрезке τ равен нулю, что не позволило сразу вычислить особое управление. Теперь аналогичная ситуация возникла с выражением

$$0 \equiv \frac{d^{2m}}{dt^{2m}}H_1 = \langle H_0, \langle H_0, ..., \langle H_0, H_1\rangle\rangle ...\rangle + u(t)\langle H_1, \langle H_0, ..., \langle H_0, H_1\rangle\rangle ...\rangle,$$

из которого нельзя найти управление на множестве нулей функции H_2.

Изложенную выше для $H_1 = 0$ процедуру вычисления особого управления можно применить и к изучению этого случая. Следующий пример иллюстрирует эффективность описанной выше процедуры: пусть требуется минимизировать $\frac{1}{2}\int_0^T x^2 dt$ при ограничениях: $\dot{x} = u$, $|u| \le \sqrt{1-x^2}$, $|x| < 1$, $x(0) = x_0 > 0$, $x(T) = x_1 > 0$, T – фиксировано, $x = x(t)$ абсолютно непрерывная , а $u = u(t)$ интегрируемая на $[0,T]$ функции.

Необходимые условия оптимальности имеют вид:

$$\psi_0 = -1,$$

$$\dot{\psi} = x + \mu_1 \frac{x}{\sqrt{1-x^2}} + \mu_2 \frac{x}{\sqrt{1-x^2}},$$

$$\mu_1\left(u - \sqrt{1-x^2}\right) = 0,$$

$$\mu_2\left(-u - \sqrt{1-x^2}\right) = 0, \quad \mu_i \ge 0, i = 1.2,$$

$$\psi(t)u(t) = \max_{U \in \{U \,|\, |U| \le \sqrt{1-x^2(t)}\}} \psi(t)u,$$

$$\psi = \mu_1 - \mu_2,$$

где $\psi = \psi(t)$ – абсолютно непрерывная, а $\mu_i = \mu_i(t), i = 1,2$, существенно ограниченные на $[0,T]$ функции.

Поскольку $H = -\frac{1}{2}x^2 + \psi u - \mu_1\left(u - \sqrt{1-x^2}\right) - \mu_2\left(-u - \sqrt{1-x^2}\right)$, то на особом участке $\tau \subset [0,T]$ из (13) имеем:

$$u(t) = -\frac{\langle -\tfrac{1}{2}x^2, x\rangle}{\langle \psi, x\rangle} = \frac{0}{1} = 0.$$

Легко видеть, что с помощью найденного особого оптимального управления без принципиальных затруднений находится оптимальное решение в рассматриваемом примере, если только x_0, x_1 и T подобраны таким образом, что задача имеет допустимое решение.

ЛИТЕРАТУРА

1. Л.С. Понтрягин, В.Г. Болтянский, Р.В. Гамкрелидзе, Е.Ф. Мищенко. *Математическая теория оптимальных процессов.* М.Физматгиз, 1961, 479 с.

2. Р. Габасов, Ф.М. Кириллова. *Особые оптимальные управления.* М. Наука, 1973, 256 с.

3. З.А. Цинцадзе. *Вычисление особых управлений в оптимальных задачах со смешанными ограничениями.* Сб.: Оптимальные задачи в системах с переменной структурой . Изд-во ТГУ, Тбилиси,1985,с 155-175.

4. Z. Tsintsadze. *The Lagrange principle of taking restrictions off and its applications to linear optimal control problems in the presence if mixed restrictions and delays.* Memoirs on DEMPH,Vol.11, 1997, pp 105-128.

განსაკუთრებული ოპტიმალური მართვის გამოთვლა კვაზიწრფივ შერეულშეზღუდვებიან სამართ სისტემებში

ზ. ცინცაძე
მართვის თეორიის კათედრა

განხილულია ოპტიმალური მართვის კვაზიწრფივი ამოცანა სკალარული მართვით და შერეული შეზღუდვებით. მოყვანილია ოპტიმალურობის აუცილებელი პირობები მაქსიმუმის პრინციპის ფორმით.

გამოკვლეულია განსაკუთრებული შემთხვევა, როდესაც მაქსიმუმის პრინციპიდან ცალსახად არ განისაზღვრება სამართი ფუნქცია. აგებულია ერთგანზომილებიანი განსაკუთრებული მართვის გამოთვლის პროცედურა და ამ პროცედურის გამოყენებით ამოხსნილია კონკრეტული ამოცანა.

SPECIFIC OPTIMAL CONTROL COMPUTATION IN QUASI-LINEAR CONTROL SYSTEMS WITH MIXED LIMITATIONS

Z. Tsintsadze
Control Theory Chair

The linear optimal control problem with scalar control and mixed restrictions is considered. The necessary conditions of optimality with the form of the maximum principle are given.

The special case, when the control function is not defined uniquely from the maximum principle is researched. The procedure of specific control finding is constructed and by using this procedure the concrete problem is solved.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 61-66

НЕКОТОРЫЕ ВОПРОСЫ ЭЛЕМЕНТАРНЫХ ЧАСТИЦ С ТОЧКИ ЗРЕНИЯ ТЕОРИИ ТРЕХМЕРНЫХ МНОГООБРАЗИЙ

Д. Вашакмадзе
Кафедра Теоретической Физики Физического Факультета

1. Основные определения*)

В современной геометрии и топологии узлы играют фундаментальную роль – например, в изучении трехмерных многообразий.

Исходя из определения узла, как области ветвления некоторого трехмерного многообразия M_1^3 при разветвленном накрытии многообразия M_0^3, обобщим определение частицы – узла, рассматривая последний как топологическую особенность, а именно, область ветвления физического пространства. Полагая глобальную геометрию пространства гомеоморфной 3- мерному гиперболическому пространству H3, рассмотрим действие на нем введенной в работах ([3,4,7]) Универсальной группы U: Для всякого замкнутого и ориентируемого многообразия M3 существует конечноиндексная подгруппа G ≤ U, такая, что M3 гомеоморфно фактор – пространству H3/G.

Фундаментальным многогранником, соответствующим этой Универсальной группе, является правильный гиперболический додекаэдр, имеющий двугранные углы, равные 900, где отождествление происходит по некоторой группе G⊆Isom E3, а факторпространство есть S3 с сингулярным множеством Σ, состоящим из зацепленных окружностей – Боромеевых колец. Тогда, каждое замкнутое 3-х мерное многообразие является разветвленным накрытием сферы S3 над Боромеевыми кольцами с индексами ветвления 1, 2, и 4; Универсальная группа U порождается поворотами на $\pi/2$ вокруг скрещивающихся ребер додекаэдра.

Генераторы группы U имеют следующее представление:

$$A=\frac{\sqrt{2}}{2}\begin{bmatrix}1-iR+iR^2 & -iR-iR^2-iR^3\\ 1-2iR+iR^3 & 1+iR-iR^2\end{bmatrix},$$

$$B=\frac{\sqrt{2}}{2}\begin{bmatrix}1-R+R^2 & 1-R+R^2\\ -R-R^2+R^3 & 1+R-R^2\end{bmatrix},$$

*) Настоящая работа является продолжением и расширением статьи [8].

$$C = \frac{\sqrt{2}}{2}\begin{bmatrix} 1+R-iR^2 & -i-2iR-iR^3 \\ -1-R+R^2 & 1-R+iR^2 \end{bmatrix}.$$

Одним из важных свойств группы U является тот факт, что для семейства плоскостей, образуемых левоинвариантными относительно действия U гранями додекаэдров, любые две плоскости или не пересекаются, или пересекаются под прямыми углами.

В дальнейшем, под U – квантизацией пространства будем подразумевать факторизацию H3/U.

2. Основные Соответствия

Ниже постулируются некоторые соответстствия, которые в частности будут использованы в дальнейшем.

(1) Распространению электромагнитной волны в вакууме соответствует топологическая U – квантизация данной области пространства.

Учитывая существование т.н. исключительного изоморфизма (см [5]) Ψ:PSL(2,C) →PSO1(IR,q1)=PSO(1,3), где q1(x0,x1,x2,x3)= $x_0^2 - x_1^2 - x_2^2 - x_3^2$, уравнения Маквелла, инвариантные относительно автоморфизмов 4-мерного псевдоевклидового пространства, можно рассматривать как уравнения инвариантные относительно автоморфизмов $\varphi \subset PSL(2,C)$ гиперболического 3-мерного пространства.

Тогда (1) подразумевает, что наложение условия инвариантности относительно действия группы U приведет к одновременному квантованию электромагнитного поля и пространства.

Перечислим некоторые следствия (1):

I Согласно определению группы U, фотоны могут находится лишь на U – инвариантном Евклидовом подпространстве H3;

II Присутствие во всем пространстве Реликтового излучения можно интерпретировать как следствие глобального топологического квантования H3, имевшего место в некоторый момент существования неквантованного H3;

III Вычислим объем единичного “кванта” пространства – гиперболического додекаэдра, приближенно считая додекаэдр сферой с радиусом r = α, где α – полуось додекаэдра, R / α ≈1.27 см. [4]

$$V_{hyp} = 4\pi\left(\frac{r(1+r^2)}{(1-r^2)^2} - \frac{1}{2}ln\frac{1+r}{1-r}\right)$$

Сравнивая Vhyp с объемом соответствующей сферы в E3, получим Vhyp/Veuc≈44. Это означает, что объем "наблюдаемого" евклидового пространства на самом деле составляет около 3% всего гиперболического пр-ва H3.

Как известно, для достижения критической плотности, необходимой для объяснения наблюдаемой евклидовости Вселенной, стало необходимым, помимо "темной массы", введение также "темной энергии" вакуума (квантового поля, названного "квинтэссенцией"), они вместе обеспечивают недостающие ≈95% всей массы Вселенной. Однако, согласно (1), "наблюдаемая" Вселенная будет всегда Евклидова, ненулевая же энергия вакуума, (позволяющая приписать всему пространству евклидову метрику), в геометрической инпретации соответствует топологически квантованной гиперболической структуре самого 3-х мерного пространства. Траектория фотона, локализованного в пространстве, например, в атоме, принимает форму 3-х зацепленных между собой взаимноортогональных Боромеевых колец, так как евклидовы прямые, соответствующие осям вращения 3-х генераторов универсальной группы, при накрывающем отображении H3/U →S3/B бесконечное количество раз обматываются вокруг Боромеевых колец (области ветвления).

(2) За массу частицы принимается гиперболический объем соответствующего гиперболического многообразия, являющийся возрастающей функцией сложности многообразия (см[2]).

(3) Переносчикам слабых взаимодействий – частицам W± , Z0, в отличие от не-полевых частиц, соответствуют Универсальные узлы.

Слабые распады вида A→BF (например n0→p+W-) можно определить как перестраивание многообразия $M_0^{(3)} \cong S^3 \setminus k_A$ в многообразие $M_0^{(3)} \cong S^3 \setminus k_B$, происходящее благодаря существованию регулярного разветвленного накрытия многообразиями $M_A^{(3)}$ и $M_B^{(3)}$ многообразия $M_0^{(3)} \cong S^3 \setminus k_{univ}$, или ,эквивалентно:

H3/ГA→H3/ГB+ H3/ГF, ГA≤ГA≤Гf.

Постулируя в общем групповой характер описания взаимодействие между отдельными частицами, основываясь на интерипретации частиц в терминах накрывающих пространств, рассмотрим лептоны и промежуточные базоны как элементы некоторой группы

$$\{G^{*}\}=\{e,\nu_{e},\overline{e},\overline{\nu}_{e},\overline{\mu},\overline{\nu}_{\mu},\tau,\nu_{\tau},\overline{\tau},\overline{\nu}_{\tau},W^{-},W^{+},Z^{0}\}.$$

Тогда соотношения вида $[g(i)][g(k)(-1)]=K$, где K подгруппа G^{*}, разобьет G^{*} на классы эквивалентности $\{\alpha\}$, $\{\beta\}$ и $\{g\}$, относительно подгруппы F={Z0,W-,W+}

$$\alpha_i\alpha_j^{-1}=C_{ij},$$

$$\beta_i\beta_j^{-1}=C_{ij},$$

$$\gamma_i\gamma_j^{-1}=C_{ij},$$

где $\{\alpha_i\}=\{e,\gamma_e\}$, $\{\beta_i\}=\{\mu,\nu_\mu\}$ $\{\gamma_i\}=\{\tau,\nu_\tau\}$, $C_{ij}=\begin{pmatrix} Z_0 & W^{+} \\ W^{-} & Z_0^0 \end{pmatrix}$.

Согласно теореме Э. Нетер, данное разбиение индуцирует натуральный гоморфизм группы G* на группу Γ= G*/F, где ядро гомоморфазма есть подгруппа полей, в то время как неединичные элементы группы Γ являются гомоморфными образами поколений лептонов.

Следовательно, сохранение лептонного числа можно записать как групповой гомоморфизм Γ= G*/F, где группа Γ может быть, например, группой гомологий для некоторого разветвленного многообразия с гомологическими классами, представленными элементами из {G*}.

(4) Возмущение накрывающей группы и период полураспада.

Предположим, что протон и нейтрон являются разными накрывающими одного и того же $M_0^{(3)}$, однако в накрывающей группе нейтрона имеется возмущение (см. [1]), т.е. в присутствии n>1 нейтронов единичное ветвление универсальной накрывающей функции нейтрона над M0 захватывает слои от разных экземпляров $S^3\backslash k_{(n^0)}$, что можно проиллюстрировать аналогией с Диким Узлом Канторовского вида (см. напр. [6]).Тогда, для N>>1, за первый период $\tilde{\tau}$ ($\tilde{\tau}$-время жизни нейтрона) распад захватит 2^k нейтронов и число оставшихся частиц будет $1+2+\ldots 2^{k-1}=2^k-1\approx 2^k$.

Заметим, что еще в 1957 году для объяснения вероятности природы распада элементарных частиц, Хью Эверетт использовал понятие разветвленной волновой функции частицы, введя "многомировую" интерпретацию квантовой механики, где для всех моментов времени, когда нейтрон доолжен распасться, существует 1 копия Вселенной, где это реально происходит.

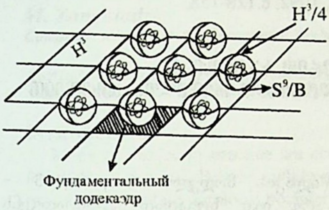

Рис. 1. Действие группы U на пространстве H(3)

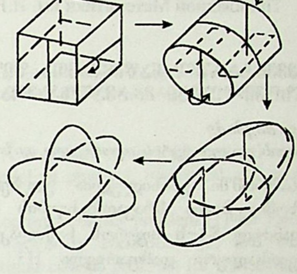

Рис. 2. Переход скрещенных ребер в Боромеевы кольца при отождествлении граней прямоугольного додекаэдра.

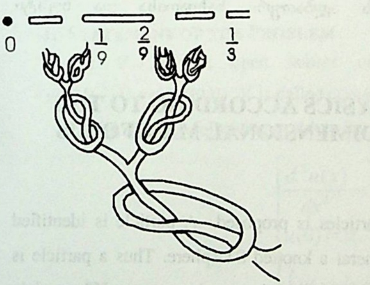

Рис. 3. Ветвление волновой функции нейтрона

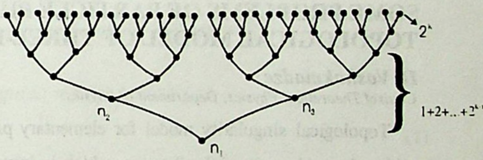

Рис. 4. N-частичная волновая функция (нейтронов)

ЛИТЕРАТУРА

1. В. Абикоф. *Вещественно-аналитическая теория пространства Тейхмюллера*. М.:Мир 1985.
2. Б. Н. Апанасов. *Геометрия дискретных групп и многообразий*. М. : Наука, 1991
3. H. M. Hilgen, M. T. Lozano, J. M. Montesion. *On the universal group of the Borromean rings*. Lect. Notes. Math. 1350. Berlin : Springer Verlag. 1989. pp1-13.
4. N. M. Hilden, M. T. Lozano, J. M. Montesion, W.C. Whitten. *On universal group and three-dimensional manifolds*. Invent. Mat., Vol. 87, pp 441-456 (1987).
5. J. Elstrodt, F. Grunewald, *Mennicke Groups Acting on Hyperbolic Space*. Springer, 1997 (Springer monographs in Mathematics)

6. A. Sossinsky, *Nodi Genesi di una teoria matematica* Bollati Boringhieri 2000
7. W. Thurston. *The Geometry and Topology of three –dimensional manifolds*. Princeton Press Preprint 1996.
8. Д.Т. Вашакмадзе. *Теория Узлов и Физика Элементарных Частиц*. Труды Института Прикладной Математики им. И.Н. Векуа., т. 44, 1992, с.128-158.

ელემენტარულ ნაწილაკთა ფიზიკის ზოგიერთი საკითხი 3 – განზომილებიან მრავალსახეობათა ტოპოლოგიური მოდელის მიხედვით

დ. ვაშაყმაძე
ფიზიკის ფაკულტეტის თეორიული ფიზიკის კათედრა

ნაშრომში განხილულია ელემენტარული ნაწილაკის მოდელი, როგორც 3 – განზომილებიანი მრავალსახეობის განშტოების არე. იგი ზოგადად წარმოადგენს გაკვანძულ S1-ს სფეროს. ნაწილაკს მიეწერება S3\k–ს ჯგუფური თვისებები, ხოლო უნივერსალური დამფარველი H3 სივრცის ვექტორი H3/U, სადაც U არის ე.წ. უნივერსალური ჯგუფი, გაიგივებულია დაკვანტულ ფიზიკურ სივრცესთან G^*, სადაც ურთიერთქმედების გავრცელება ხდება U – ინვარიანტულ ევკლიდურ ქვესივრცეზე. განხილულია ნახევრადდაშლის ალბათური ხასიათისა და სუსტი დაშლის ტოპოლოგიური მოდელები.

SOME PROBLEMS OF PARTICLE PHYSICS ACCORDING TO THE TOPOLOGICAL MODEL OF THREE-DIMENSIONAL MANIFOLDS

D. Vashakmadze
Chairof Theoretical Physics, Department of Physics

Topological singularity model for elementary particles is proposed. A particle is identified with a branching set of the 3-space, which is ingeneral a knotted S1-sphere. Thus a particle is charachterized by group properties of S3\k, while universal covering space H3 modulo U(universal group) is identified with the quantized physical space, where photon-subspace G^* corresponds to U (left)-invariant euclidean singularity submanifold.

Weak decays are considered in terms of branched covering over universal knots, while for the interpretation of the half-decay period, Kantor Wild Knot -type branching is introduced for the N-particle wave function.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 67-72

THE OPTIMAL CONTROL PROBLEM FOR THE SECOND ORDER ORDINARY DIFFERENTIAL EQUATION WITH INTEGRAL BOUNDARY CONDITION AND QUADRATIC FUNCTIONAL

M. Zangaladze
Computer Software and Informational Technologies Chair

Abstract. In this paper the optimal control problem for the second order ordinary differential equation with non-local boundary conditions is considered. The necessary and sufficient condition for optimality has been obtained.

INTRODUCTION

Many processes in practice are controlled and it is important to find the optimal resolution for their realization. Besides, while mathematical modeling physical, biological and ecological processes we obtain non-local boundary problems.

Creado, Meladze and Odisehlidze considered the optimal control problem [1] for Helmholtz equation with Bitsadze-Samarski's type non-local boundary conditions [2] and quadratic functional. In the present paper is considered the optimal control problem for the second order ordinary differential equation with another type of non-locality – integral boundary conditions [3].

1. STATEMENT OF THE PROBLEM

Let V be an open subset of R and Ω be a set of control functions: $v:[0,1]\to V, v\in L_2([0,1])$, V is called domain of controls.

Let us consider following problem for each fixed $v\in\Omega$ in $[0,1]$ interval:

$$\begin{cases} \dfrac{d^2u(x)}{dx^2}-qu(x)=f(x)+a(x)v(x), \\ u(0)=\alpha, \\ \displaystyle\int_0^1 u(x)dx=\beta, \end{cases} \tag{1}$$

where $x\in[0,1], \alpha,\beta\in R, a\in L_\infty([0,1]), f\in L_2([0,1]), 0<q=const$. It is known, that the solution of problem (1) exists, is unique and belongs to space $W_2^2([0,1])$ [6].

Let $I(v)$ be the following quadratic functional:

$$I(v)=\int_0^1\left[b_1(x)u^2(x)+b_2(x)v^2(x)\right]dx, \tag{2}$$

where $b_1,b_2\in L_\infty([0,1])$ are given functions.

Now let's state the following optimal control problem: find the function $v_0\in\Omega$, whose corresponding solution of problem (1) together with v_0 results in the minimal functional value.

2. ADJOINT EQUATION

To obtain conditions of optimality we follow the scheme developed in the works [4][5].

Assume, that $v_0 \in \Omega$ is an optimal control, $v_\varepsilon \in \Omega$ is arbitrary admissible control and u_0, u_ε are corresponding solutions of problem (1). Let's take the following notations:

$$\tilde{v} \equiv v_\varepsilon - v_0, \quad \tilde{u} \equiv u_\varepsilon - u_0. \tag{3}$$

If we'll consider problem (1) correspondingly to (u_0, v_0) and $(u_\varepsilon, v_\varepsilon)$, then we come to the following problem for $\tilde{u}$:

$$\begin{cases} \dfrac{d^2\tilde{u}(x)}{dx^2} - q\tilde{u}(x) = a(x)\tilde{v}(x), \quad x \in (0,1), \\ \tilde{u}(0) = 0, \\ \int_0^1 \tilde{u}(x)dx = 0. \end{cases} \tag{4}$$

For a certain v_0 and v_ε let us consider the following difference:

$$\tilde{I} \equiv I(v_\varepsilon) - I(v_0) = \int_0^1 b_1(x)u_\varepsilon^2 dx + \int_0^1 b_2(x)v_\varepsilon^2(x)dx -$$
$$\int_0^1 b_1(x)u_0^2(x)dx - \int_0^1 b_2(x)v_0^2(x)dx = \tag{5}$$
$$\int_0^1 \left[b_1(x)\tilde{u}^2(x) + b_2(x)\tilde{v}^2(x)\right]dx + 2\int_0^1 \left[b_1(x)u_0(x)\tilde{u}(x) + b_2(x)v_0(x)\tilde{v}(x)\right]dx.$$

Let $\psi \in W_2^2([0,1])$ and $\psi \neq 0$. If we multiply (4) on ψ, then integrate obtained expression on the interval [0,1] and take into account (5) equality, we shall have:

$$\tilde{I} = \int_0^1 \psi(x)\left[\frac{d^2\tilde{u}}{dx^2} - q\tilde{u}\right]dx - \int_0^1 \psi(x)a(x)\tilde{v}(x)dx +$$
$$\int_0^1 \left[2b_1(x)u_0(x)\tilde{u}(x) + 2b_2(x)v_0(x)\tilde{v}(x)\right]dx + \int_0^1 \left[b_1(x)\tilde{u}^2(x) + b_2(x)\tilde{v}^2(x)\right]dx. \tag{6}$$

Let us make the fölowing transformation for constructing the adjoint equation: two times using partially integration formula and fact that $\tilde{u}(0) = 0$, the first member of equation (6) will transform into following:

$$\int_0^1 \psi(x)\frac{d^2\tilde{u}}{dx^2}dx = \int_0^1 \tilde{u}(x)\frac{d^2\psi}{dx^2}dx + \psi(1)\tilde{u}'(1) - \psi(0)\tilde{u}'(0) - \psi'(1)\tilde{u}(1). \tag{7}$$

Integrating (4) equation in [0,1] interval we obtain:

$$\tilde{u}'(1) = \tilde{u}'(0) + q\int_0^1 \tilde{u}(x)dx + \int_0^1 a(x)\tilde{v}(x)dx = \tilde{u}'(0) + \int_0^1 a(x)\tilde{v}(x)dx. \tag{8}$$

Placing (8) equation in (7) expression, we shall have:

$$\int_0^1 \psi(x)\frac{d^2\tilde{u}}{dx^2}dx = \int_0^1 \tilde{u}(x)\frac{d^2\psi}{dx^2}dx + (\psi(1)-\psi(0))\tilde{u}'(0) - \psi'(1)\tilde{u}(1) + + \psi(1)\int_0^1 a(x)\tilde{v}(x)dx. \tag{9}$$

Replacing (6) into (9) we shall see, that if ψ_0 is the solution of the following problem:

$$\begin{cases} \dfrac{d^2\psi(x)}{dx^2} + q\psi(x) = -2b_1(x)u_0(x), \quad x \in (0,1), \\ \psi(0) = \psi(1), \\ \psi'(1) = 0, \end{cases} \tag{10}$$

then functional $\tilde{I}$ expressed in (6) will be as follows:

$$\tilde{I} = \int_0^1 [(\psi(1)-\psi(x))a(x) + 2b_2(x)v_0(x)]\tilde{v}(x)dx + \int [b_1(x)\tilde{u}^2(x) + b_2(x)\tilde{v}^2(x)]dx. \tag{11}$$

<u>THEOREM 1</u>: Let $q = const > 0$ *and* $b_1(x) \in L_\infty([0,1])$, $u(x) \in W_2^2([0,1])$ *are given functions, then the solution of the problem (10) exisis, is unique, belongs to space* $W_2^2([0,1])$ *and could be written as follows:*

$$\psi(x) = c_0 e^{\sqrt{q}x} + c_1 e^{-\sqrt{q}x} + \frac{1}{2\sqrt{q}}\int_0^x \left(c_0 e^{-\sqrt{q}(t-x)} - c_0 e^{\sqrt{q}(t-x)}\right) f(t)dt + k\frac{e^{-\sqrt{q}x} - e^{-2\sqrt{q}}e^{\sqrt{q}x}}{1-e^{\sqrt{q}}} +$$

$$+\left(k\frac{\left(e^{\sqrt{q}} - e^{-\sqrt{q}}\right)^2}{2\left(e^{\sqrt{q}} + e^{-\sqrt{q}}\right)} - \frac{\left(c_0 e^{\sqrt{q}} - c_1 e^{-\sqrt{q}}\right)\left(e^{\sqrt{q}} - e^{-\sqrt{q}}\right)}{e^{\sqrt{q}} - e^{-\sqrt{q}}} + \frac{2}{e^{\sqrt{q}} + e^{-\sqrt{q}}} - \right.$$

$$\left. - \frac{\left(e^{\sqrt{q}} - e^{-\sqrt{q}}\right)^2}{2\sqrt{q}\left(e^{\sqrt{q}} - e^{-\sqrt{q}}\right)}\int_0^1 f(t)dt\right)\frac{e^{\sqrt{q}x} - e^{-\sqrt{q}x}}{e^{\sqrt{q}} - e^{-\sqrt{q}}},$$

where

$$f(x) = -2b_1(x)u(x),$$

$$k = \frac{4\left(c_0 e^{2\sqrt{q}} + c_1 e^{-2\sqrt{q}} - 1\right) + \frac{1}{\sqrt{q}}\left(e^{\sqrt{q}} + e^{-\sqrt{q}}\right)\int_0^1\left(e^{-\sqrt{q}(t-1)} - e^{\sqrt{q}(t-1)}\right)f(t)dt}{2\left(1 + e^{\sqrt{q}} + e^{-\sqrt{q}}\right) - e^{2\sqrt{q}} - e^{-2\sqrt{q}}} -$$

$$- \frac{e^{2\sqrt{q}} + e^{-2\sqrt{q}} - 2}{2\left(\sqrt{q}\left(1 + e^{\sqrt{q}} + e^{-\sqrt{q}}\right) - e^{-2\sqrt{q}} - e^{2\sqrt{q}}\right)}\int_0^1 f(t)dt.$$

And c_0 *and* c_1 *are defined as follows:*

$$\begin{cases} c_0 = -c_1 \\ c_1 = \dfrac{1}{2\sqrt{q}\left(e^{\sqrt{q}} - e^{-\sqrt{q}}\right)} \int_0^1 \left(e^{\sqrt{q}(1-t)} - e^{-\sqrt{q}(1-t)}\right) f(t)dt. \end{cases}$$

3. Necessary and Sifficient Condision for optimality of Solution

<u>THEOREM 2</u>. *Let functional be given by the formula (2), $b_2(x) > 0$ and ψ_0 is solution of the problem (10), then the couple (u_0, v_0) is optimal if, and only if, the following condition is satisfied:*

$$(\psi_0(1) - \psi_0(x))a(x) + 2b_2(x)v_0(x) = 0$$

almost everywhere in the [0,1] interval.

The confirmation of this theorem is the same as in [1] ,where the confirmation of necessary and sufficient condition for optimal control problem with another type of non-locality is presented.

4.Conceqence

Let's consider the following optimal control problem:

$$\begin{cases} \dfrac{d^2u(x)}{dx^2} - q(x)u(x) = f(x) + a(x)v(x), \\ u(0) = \alpha, \\ \int_0^1 q(x)u(x)dx = \beta, \end{cases} \tag{12}$$

where $x \in [0,1], \alpha, \beta \in R, a \in L_\infty([0,1]), f \in L_2([0,1]), 0 < q \in L_\infty([0;1])$. It is known, that the solution of problem (1) exists, is unique and belongs to space $W_2^2([0,1])$ [6].

Let $I(v)$ be the following quadratic functional:

$$I(v) = \int_0^1 \left[b_1(x)u^2(x) + b_2(x)v^2(x)\right]dx, \tag{13}$$

where $b_1, b_2 \in L_\infty([0,1])$ are given functions and (u, v) searched couple, satisfy (12) and will assign minimal value to (12) functional.

Above considered problem will transform to the following system:

$$\begin{cases} \dfrac{d^2u(x)}{dx^2} - q(x)u(x) = f(x) + \dfrac{a^2(x)}{2b_2(x)}(\psi(x) - \psi(1)), \quad x \in (0,1), \\ u(0) = \alpha, \\ \int_0^1 q(x)u(x)dx = \beta; \end{cases} \tag{14}$$

$$\begin{cases} \dfrac{d^2\psi(x)}{dx^2} + q(x)\psi(x) = -2b_1(x)u(x), \quad x \in (0,1), \\ \psi(0) = \psi(1), \\ \psi'(1) = 0. \end{cases}$$

The adjoint problem of (12)-(13) problem is corrected and also the already received necessary and sufficient condition of optimality is truthful for it.

REFERENCES

1. F. Criado, G. Meladze, N. Odishelidze. *An optimal control problem for Helmholtz equation with non-local boundary conditions and quadratic functional*, Rev. R. Acad. Cienc. Exact. Fis. Nat, (Esp), vol.**91**, 1997, N 1, pp 65-69.
2. D.G. Gordeziani (1971). *On some boundary problems solvability for one variant of thin shells theory*, Papers of Academy of Sciences of USSR, 215,(7), 1289-1292.(Russian).
3. J.R. Cannon. *The solution of the heat equation subject to the specification of energy*, Quart.Appl.Math.21(1963).155-160.
4. V.L. Plotnikov (1971). *The necessity and sufficient conditions of optimality and uniqueness conditions of optimize functions for control systems of general form*, Papers of Academy of Sciences,199, (2),275-278.(Russian).
5. V.L. Plotnikov (1973). *The necessity and sufficient conditions of optimality and uniqueness conditions of optimize functions for control systems of general forms*, Proceedings of Academy of Sciences of USSR,series math.,36, (3),652-679..(Russian).
6. N.I. Ionkin. *Solution of boundary value problem in heat conduction theory with non-local* boundary conditions, Diff.Uravn.13(1977), 294-394.

ოპტიმალური მართვის ამოცანა ინტეგრალური ტიპის სასაზღვრო პირობით მეორე რიგის ჩვეულებრივი დიფერენციალური განტოლებისა და კვადრატული ფუნქციონალისთვის

მ. ზანგალაძე
კომპიუტერების მათემატიკური უზრუნველყოფის კათედრა

ნაშრომში განხილულია ოპტიმალური მართვის ამოცანა მეორე რიგის ჩვეულებრივი დიფერენციალური განტოლებისთვის ინტეგრალური სასაზღვრო პირობით და მიღებულია ოპტიმალურობის აუცილებელი და საკმარისი პირობა.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 73-76

კომპიუტერული მეცნიერებები • COMPUTER SCIENCES

МАТЕМАТИЧЕСКИЕ ВОПРОСЫ КОНЦЕПТУАЛЬНО-ФРАКТАЛЬНОГО АНАЛИЗА

Г. Бесиашвили
ТГУ, лаборатория физической кибернетики

Абстракт. В статье рассмотрен метод концептуально-фрактального анализа в распознавании образов. Определены понятия расстояния, подобия, инвариантности и фрактальная мера подобия в пространстве Хаусдорфа.

ВВЕДЕНИЕ

На сигнальных полях образы (изображения, речевые) реализуются отображением $f: S \rightarrow R^n$; и они имеют концептуально-фрактальную природу [1,5].

Множество реализации образов (изображения, речевых сигналов) составляют последовательности в R^n, предельные изображения (концепты) воспринимают глаз, ухо [1, 2, 3, 4].

Концептуально-фрактальный анализ в своих методах для анализа, классификации, распознавания и понимания образов пользуется известными определениями расстояния, подобия, инвариантности и самоподобия в пространстве Хаусдорфа [4, 5].

1. МЕТРИКА ХАУСДОРФА [4, 6]

Допустим, $x \in X$, $A \subset X$, тогда расстояние между x и A определяется как

$$d(x,A)=inf\{d(x,a):a \in A\}.$$

Если $A \subset X$, $\varepsilon > 0$, тогда для A обозначим ε-окрестность

$$A_\varepsilon=\{ x \in X\ d(x,A)< \varepsilon\}, \quad A \subset A_\varepsilon$$

Допустим $F: X \rightarrow X$, тогда $Lip\ F$ определяется как

$$LipF = \sup_{x \neq y} \frac{d(F(x),F(y))}{d(x,y)}.$$

Если $Lip\ F=\lambda$, тогда $d(F(x), F(y)) \leq \lambda\ d(x,y))$ для любых x, $y \in X$. Когда $Lip\ F<1$, F - сжимающее.

Допустим, B - совокупность замкнутых, ограниченных непустых подмножеств в X, a b - непустых компактных подмножеств.

δ-обозначим Хаусдорфовую метрику на B.

$$\delta(A,B)=sup\{d(a,B),\ d(b,A): a\in A;\ b\in B\}.$$

$$\delta(A,B)<\varepsilon \text{ если } A\subset B\varepsilon \text{ и } B\subset A\varepsilon.$$

δ-метрика на B.

Некоторые элементарные свойства δ:

$$\delta(F(A),\ F(B))\leq Lip\ (F)\ \delta(A,B),$$

$$\delta\left(\bigcup_{i\in I} Ai \bigcup_{i\in I} Bi\right)\leq sup\,\delta(Ai\,,Bi).$$

Если $(B,\ \delta)$ полное метрическое пространство $K\subset X$ компактно, тогда $b\cap\{A:\ A\subset K\}$ компактно.

Мера m на X множестве есть отображение

$$m: P(x)=\{A: A\subset X\}\rightarrow[0,\ \infty],$$

такое $m\ (\varnothing)=0$

$$m\left(\bigcup_{i=1}^{\infty} E_i\right)\leq\sum_{i=1}^{\infty} m(E_i) \qquad E_i\subset X,$$

если $A\subset B$, тогда $m\ (A)\leq m\ (B)$.

Допустим $\kappa\geq 0$ некоторое действительное фиксированное число, для любого $\delta>0$ и $E\subset X$ обозначим

$$H^k_\delta(E)=inf\left\{\sum_{i=1}^{\infty}\alpha_k 2^{-k}(diamE_i)^k:\ E\subset\bigcup_{i=1}^{\infty}E_i,\ diamE_i\leq\delta\right\}\text{m}$$

$$H^k(E)=\lim_{\delta\to 0}H^k_\delta(E)=\sup_{\delta\geq 0}H^k_\delta(E).$$

$H^k(E)$ называется Хаусдорфовая κ-размерная мера E, α_κ-соответствующая константа нормировки [4].

Фрактальная мера определяется покрытием фрактала сеткой из квадратов или сферами, так что компактные множества удовлетворяют этим требованиям.

Подобие [4, 6], допустим $S=\{S_1,\dots\ S_n\}$ совокупность конечных образов $S: X\rightarrow X$ является подобием, если $d(S(x),\ S(y))=rd(x,y)$ для любых x, y и некоторых фиксированных r.

$\mu_r: R^n \to R^n$ гомотетический $\mu_r(x)=r(x)$ $r \geq 0$, $\tau_b: R^n \to R^n$ перевод $\varepsilon_b(x)=x-b$, если $S: R^n \to R^n$ подобие, тогда имеет место $S=\mu_r \circ \tau_b \circ O$

Для некоторых μ_r-гомотетии, ε_b-перевода и O-ортонормальных преобразований.

Допустим для произвольного $A \subset X$ (инвариантность [4, 6]),

$$S(A) = \bigcup_{i=1}^{n} S_i A .$$

выполняются условия $S^0(A)=A$, $S^1(A)=S(A)$, $S^p(A)=S(S^{p-1}(A))$ для всех $p \geq 2$. Если $A=S(A)$, тогда A-инвариантно.

Обозначим $A_{i_1 \dots i_p} = S_{i_1 \dots i_p}(A)$ $S^p(A) = U_{i_1,\dots,i_p} A_{i_1,\dots,i_p}$, $diam\ A_{i_1,\dots,i_p} \leq r_{i_1},\dots,r_{i_p}$, $diam(A) \to 0$, когда $p \to \infty$ при условии, что A ограничено.

Если K-замкнутое, ограниченное и инвариантное на S и

$$K = \bigcup_{i=1}^{n} K_i ,$$

тогда K-компактно. Если A непустое ограниченное множество, тогда $d(A_{i_1,\dots,i_p}, K_{i_1,\dots,i_p}) \to 0$, когда $p \to \infty$, $S^p(A) \to K$ в Хаусдорфовой метрике.

Если B совокупность замкнутых, ограниченных подмножеств в X, b-совокупность компактных подмножеств $S: B \to B$ и $g: b \to b$ (принцип сжимающих отображений [6])

$$\delta(S(A),S(B)) = \delta\left(\bigcup_i S_i(A), \bigcup_i S_i(B)\right) \leq$$

$$\leq \max_{1 \leq i \leq n} \delta(S(A),S_i(B)) \leq \left(\max_{1 \leq i \leq n} r_i\right) \delta(A,B).$$

В Хаусдорфовой метрике S-сжимающее отображение на B (соответственно на b). Неподвижная точка – аттрактор в физическом смысле. Если (X,d) полное метрическое пространство, $S=\{S_1,\dots S_n\}$ совокупность сжимающих отображений, дополнительно мы предполагаем существование множества $\rho=\{\rho_1,\dots\rho_n\}$, таких $\rho_i \in (0,1)$ и

$$\sum_{i=1}^{n} \rho_i = 1$$

для S_i подобных $Lip S_i = r_i$.

D - такое положительное число, для которого выполняется

$$\rho_i = r_i^D \qquad\qquad \sum_{i=1}^{n} r_i^D = 1$$

D - размерность подобия на S по Мандельброту.

Из-за ограничения размера статьи невозможно рассмотреть вопросы самоподобия концептуально-фрактальных структур в пространстве Хаусдорфа.

ЛИТЕРАТУРА

1. В.В. Чавчанидзе – *«Аналитические эвристики исскуственного интеллекта при формировании понятий, опознании образов и классификации обьектов»* - № 2080-70 деп. Тбилиси, 1970;
2. B.B. Mandelbrot – *"Fractal Geometry of Nature"* – New-York, 1982;
3. M. Barnsley, L. Huard – *"Fractal image compresson"* – 1990;
4. S.E. Hutchinson – *"Fractal and Self Similarity" - "Indians University Mathematics Journal"* – v.30., n. 5., 1981;
5. В.В. Чавчанидзе, Г.М. Бесиашвили – *«Нечёткие концептуально-фрактальные модели речевых сигналов»* - Труды конференции «Концептуальные и компьютерные модели речи», Тбилиси, 1997;
6. А.Н. Колмогоров, С.В. Фомин – *«Элементы теории функции и функционального анализа»*, Москва, 1976.

ႩႭႬႺႤႮႲႳႠႪႳႰ ႴႰႠႵႲႠႪႳႰႨ ႠႬႠႪႨႦႨႱ ႫႠႧႤႫႠႲႨႩႳႰႨ ႱႠႩႨႧႾႤႡႨ

გ. ბესიაშვილი
ფიზიკური კიბერნეტიკის პრობლემური ლაბორატორია

სტატიაში განხილულია კონცეპტუალურ–ფრაქტალური ანალიზის მეთოდი სახეთა გამოცნობაში. განსაზღვრულია მანძილის, მსგავსობის, ინვარიანტობის ცნებები და მსგავსობის ფრაქტალური ზომა ჰაუსდორფის სივრცეში.

THE MATHEMATICS QUESTIONS OF CONCEPTUAL FRACTAL ANALYSIS

G. Besiashvili
Problem Laboratory of Physical Cybernetics

This paper describes the method of conceptual fractal analysis in pattern recognition. Definition distance, similarity, invariant and fractal similarity dimension in Hausdorff space.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 77-82

МНОГОПЛАНОВАЯ И МНОГОАЛЬТЕРНАТИВНАЯ ЛИНЕЙНАЯ РЕГРЕССИОННАЯ МОДЕЛЬ ЭКСПЕРИМЕНТА

Н. Бокучава
Проблемная лаборатория физической кибернетики

Абстракт. Представленную работу можно рассматривать как попытку унификации имеющихся достижений в научном направлении по планированию эксперимента и формирования обобщенной линейной информационно-статистической многоплановой и многоальтернативной регресионной модели эксперимента.

ВВЕДЕНИЕ

В научных исследованиях основной задачей экспериментатора является отыскание (определение) наилучшего плана эксперимента, позволяющего с наименьшими усилиями и затратами (временными, техническими, финансовыми и др.) получить наибольшую информацию об исследуемом объекте. В процессе поиска такого оптимального плана экспериментатору приходится рассматривать несколько вариантов (моделей) планирования эксперимента с последующим определением некоторого количественного критерия предпочтительности выбора наилучшего из них.

1. ОБОБЩЕННАЯ ЛИНЕЙНАЯ РЕГРЕССИОННАЯ МОДЕЛЬ ЭКСПЕРИМЕНТА

Обозначим через X_j $(j=\overline{1,K})$ число выбранных экспериментатором базисных матриц с x_{jir} независимыми элементами $(i=\overline{1,N_j}, r=\overline{1,m_j}, m_j<N_j)$ и через Y_j соответствующие им матрицы столбцы наблюдений с y_{ji} элементами наблюдений. Тогда, следуя [1], линейные регрессии для каждой конкретной базисной матрицы Xj могут быть записаны следующим образом

$$Y_j = X_j\beta_j + \varepsilon_j \tag{1}$$

где

$Y_j=[y_{j1}, y_{j2},\dots,y_{jN_j}]^T$, $X_j=\lfloor X_{j1}, X_{j2},\cdots,X_{jm_j}\rfloor$, $X_jr=[x_{j1r}, x_{j2r},\dots,x_{jN_jr}]^T$,

$\beta_j=[\beta_{j1},\beta_{j2},\dots,\beta_{jm_j}]^T$ - вектор неизвестных параметров, $\varepsilon_j=[\varepsilon_{j1},\varepsilon_{j2},\dots,\varepsilon_{jN_j}]^T$ - вектор ненаблюдаемых ошибок . *)

Следует отметить, что обычно вектор ненаблюдаемых ошибок представляет собой комбинацию "модельных" и "измерительных" ошибок. Вводя теперь обозначения,

---

*) Заглавными и строчными жирными буквами обозначены матрицы и векторы соответственно

$$X=\begin{bmatrix} X_1 & & & \\ & X_2 & & O \\ O & & \ddots & \\ & & & X_K \end{bmatrix}, \quad Y^{(l)}=\begin{bmatrix} Y_1 & & & \\ & Y_2 & & O \\ O & & \ddots & \\ & & & Y_K \end{bmatrix}$$

$$\beta^{(l)}=\begin{bmatrix} \beta_1 & & & \\ & \beta_2 & & O \\ O & & \ddots & \\ & & & \beta_K \end{bmatrix}, \qquad \varepsilon^{(l)}=\begin{bmatrix} \varepsilon_1 & & & \\ & \varepsilon_2 & & O \\ O & & \ddots & \\ & & & \varepsilon_K \end{bmatrix}$$

мы можем записать $j=\overline{1,k}$ систем уравнений регрессии (1) в виде одного объединенного уравнения

$$Y = X\beta + \varepsilon. \tag{2}$$

Если при этом для каждой конкретной базисной матрицы X_j существует $H^{(l)}\ (l=\overline{1,M})$ альтернативных гипотез подбора неизвестных параметров β_j, то, следуя (2), многоальтернативную регрессионную модель можем записать в следующей форме

$$Y^{(l)} = X\beta^{(l)} + \varepsilon^{(l)} \tag{3}$$

Исходя из многопланового и многоальтернативного регрессионного уравнения (3) следует, что вектор ошибок $\varepsilon^{(l)}$ зависит от вектора параметра $\beta^{(l)}$, и следовательно, от удачного выбора которого и будет зависеть качество линейной регрессионной модели. В качестве критерия наилучшей оценки параметра $\beta^{(l)}$ воспользуемся оценкой МНК. Тогда, обозначая эту оценку через $\hat{\beta}^{(l)}$, при условии $E\left(\varepsilon^{(l)}\right)=0$, будем иметь

$$\hat{\beta}^{(l)} = \min_{\forall \beta_j^{(l)T} \in R^{m_j}} \left(\varepsilon^{(l)T}\varepsilon^{(l)}\right) = \min_{\forall \beta_j^{(l)T} \in R^{m_j}} \left[Y^{(l)} - X\beta^{(l)}\right]^T \left[Y^{(l)} - X\beta^{(l)}\right],$$

откуда с учетом условия

$$\frac{\partial}{\partial \beta^{(l)}}\left(\left[Y^{(l)} - X\beta^{(l)}\right]^T \left[Y^{(l)} - X\beta^{(l)}\right] \right) = 0 \tag{4}$$

и правила дифференцирования произведения матриц [2]

$$\frac{\partial}{\partial \beta^{(l)}}\left(A\left(\beta^{(l)}\right)B\left(\beta^{(l)}\right)\right) = \frac{\partial A\left(\beta^{(l)}\right)}{\partial \beta^{(l)}} B\left(\beta^{(l)}\right) + A\left(\beta^{(l)}\right)\frac{\partial B\left(\beta^{(l)}\right)}{\partial \beta^{(l)}}. \tag{5}$$

Для $\hat{\beta}^{(l)}$ получим

$$\hat{\beta}^{(l)} = \left(X^T X\right)^{-1} X^T Y^{(l)} \tag{6}$$

3. Определение наилучшей функции регрессии.

После того, как определены оценки $\hat{\beta}^{(l)}$ (следовательно и $\hat{\beta}^{(l)}$), встает вопрос определения наилучшей функции регрессии. Обозначим через

$$\eta^{(l)} \equiv \eta(X, \beta^{(l)}) = X\beta^{(l)} \tag{7}$$

функции регрессии уравнения (3), где

$$\eta^{(l)} = \begin{bmatrix} \eta_1^{(l)} & & & \mathrm{O} \\ & \eta_2^{(l)} & & \\ & & \ddots & \\ \mathrm{O} & & & \eta_K^{(l)} \end{bmatrix}, \quad \eta^{(l)}{}_j = [\eta_{j1}^{(l)}, \eta_{j2}^{(l)}, \ldots, \eta_{jN_l}^{(l)}]^T$$

и разложим ее в ряд Тейлора по параметрам $\beta_j^{(l)}$ в окрестности $\hat{\beta}_j^{(l)}$, ограничиваясь первыми производными, тогда

$$\eta(X, \beta^{(l)}) = \eta(X, \hat{\beta}^{(l)}) + \left.\frac{\partial \eta^{(l)}}{\partial \beta_1^{(l)}}\right|_{\beta_1^{(l)} = \hat{\beta}_1^{(l)}} (\beta_1^{(l)} - \hat{\beta}_1^{(l)}) + \left.\frac{\partial \eta^{(l)}}{\partial \beta_2^{(l)}}\right|_{\beta_2^{(l)} = \hat{\beta}_2^{(l)}} (\beta_2^{(l)} - \hat{\beta}_2^{(l)}) + \cdots$$

$$\cdots + \left.\frac{\partial \eta^{(l)}}{\partial \beta_K^{(l)}}\right|_{\beta_K^{(l)} = \hat{\beta}_K^{(l)}} (\beta_K^{(l)} - \hat{\beta}_K^{(l)}) = \eta(X, \hat{\beta}^{(l)}) + \sum_{j=1}^{k} X_j \Delta\beta_j^{(l)} \tag{8}$$

где

$$\Delta\beta_j^{(l)} = \beta_j^{(l)} - \hat{\beta}_j^{(l)}, \quad \frac{\partial \eta^{(l)}}{\partial \beta_j^{(l)}} = X_j \; .$$

откуда

$$\Delta\eta^{(l)} = \eta(X, \beta^{(l)}) - \eta(X, \hat{\beta}^{(l)}) = \sum_{j=1}^{k} X_j \Delta\beta_j^{(l)} \; . \tag{9}$$

Условие $\Delta\eta^{(l)} < \delta$, где δ наперед заданное сколь угодно малое положительное число, будем именовать условием отсева тех функции регрессии, которые не удовлетворяют этому неравенству; постепенным добавлением в разложении (8) производных высших порядков отсев будем продолжать до тех пор, пока не будет выявлена наилучшая (единственная) функция регрессии.

Если после добавления в (8) членов высших производных вопрос однозначного определения наилучшей функции регрессии, с учетом критерия отсева, остается не решенным, то экспериментатору следует:

а) продолжить накопление информации относительно измеряемых (наблюдаемых) переменных, не меняя уже избранного плана эксперимента, путем проведения повторных наблюдений;

б) изменить план эксперимента с помощью увеличения или уменьшения числа элементов базисной матрицы, тем самым изменяя соответственно и размерность параметрического пространства $\beta^{(l)}$, провести дополнительные измерения.

Рассмотрим подробно каждый из этих вариантов.

Вариант а. Вследствии того, что на каждое повторное измерение будут влиять т.н. "скрытые параметры", связанные с "модельными" и "измерительными" ошибками, недоступные контролю со стороны экспериментатора и меняющиеся со временем (например: износ установки, психологический настрой исследователя, изменение внешней среды и др.), для исключения большой разрозненности результатов повторных измерений следует, насколько это возможно:

1) внести определенные коррективы в пространство параметров $\beta^{(l)}$, 2) ограничиться разумным числом повторных измерений.

Обозначим через $\mu^{(p)}_{\eta^{(l)}_j} = \left[\mu^{(p)}_{\eta^{(l)}_{j1}}, \mu^{(p)}_{\eta^{(l)}_{j2}}, \cdots, \mu^{(p)}_{\eta^{(l)}_{jN_j}} \right]$ и $f^{(p)}\left(\eta^{(l)}_{j_1}, \eta^{(l)}_{j_2}, \cdots, \eta^{(l)}_{j_{N_j}} \right)$ соответственно средние значения и плотности распределения векторов $\left(\eta^{(l)}_j \right)^T$ при повторных измерениях ($p = \overline{1,c}$ - разумное число повторных измерений). Тогда, исходя из известного факта теории регрессии [2] о нормальном распределении параметров $\beta^{(l)}$ и вектора $\varepsilon^{(l)}$ и принципа максимума информационной энтропии [3] плотности $f^{(p)}\left(\eta^{(l)}_{j_1}, \eta^{(l)}_{j_2}, \cdots, \eta^{(l)}_{j_{N_j}} \right)$ могут быть представлены в следующем виде *):

$$f^{(P)}\left(\left(\eta^{(l)}_j \right)^T \right) = \frac{1}{(2\pi)^{N_j/2} \left| \Lambda^{(p)}_{\eta^{(l)}_j} \right|^{1/2}} exp\left(-\frac{1}{2} \left[\eta^{(l)}_j - \mu^{(p)}_{\eta^{(l)}_j} \right]^T \Lambda^{(p)^{-1}}_{\eta^{(l)}_j} \left[\eta^{(l)}_j - \mu^{(p)}_{\eta^{(l)}_j} \right] \right) \quad (10)$$

где

---

*) $\Lambda^{(p)^{-1}}_{\eta^{(l)}_i}$ и $\Lambda^{(s)^{-1}}_{\eta^{(l)}_i}$ соответственно обратные матрицы матриц $\Lambda^{(p)}_{\eta^{(l)}_i}$ и $\Lambda^{(p)}_{\eta^{(l)}_i}$.

$$\left[\eta_j^{(l)} - \mu_{\eta_j^{(l)}}^{(p)}\right]^T = \left[\left(\eta_{j_1}^{(l)} - \mu_{\eta_{j_1}^{(l)}}^{(p)}\right)\left(\eta_{j_2}^{(l)} - \mu_{\eta_{j_2}^{(l)}}^{(p)}\right)\cdots,\left(\eta_{j_{N_j}}^{(l)} - \mu_{\eta_{j_{N_j}}^{(l)}}^{(p)}\right)\right],$$

$$\Lambda_{\eta_j^{(l)}}^{P} = cov\left\{\left[\eta_j^{(l)} - \mu_{\eta_j^{(l)}}^{(p)}\right]\left[\eta_j^{(l)} - \mu_{\eta_j^{(l)}}^{(p)}\right]^T\right\}.$$

Исходя из (10), выбирая для варианта а в качестве критерия выбора наилучшей функции регрессии меру различающей информации [3]

$$J\left(p : s, \eta_j^{(l)^T}\right) = \int f^{(p)}\left(\eta_j^{(l)^T}\right) ln \frac{f^{(p)}\left(\eta_j^{(l)^T}\right)}{f^{(S)}\left(\eta_j^{(l)^T}\right)} d\eta_j^{(l)^T} \quad \left(p, s = \overline{1,c};\ p \neq s\right), \tag{11}$$

которая, с учетом известного соотношения из матричной алгебры [4] $b^T A b = \mathrm{Sp}\left(Abb^T\right)$, где $A \in M_{n\times n}$, $b \in R^n$, принимает следующий вид:

$$J\left(p : s, \eta_j^{(l)^T}\right) = \frac{1}{2} ln \frac{\left|\Lambda_{\eta_j^{(l)}}^{(S)}\right|}{\left|\Lambda_{\eta_j^{(l)}}^{(P)}\right|} + \frac{1}{2} Sp\left\{\Lambda_{\eta_j^{(l)}}^{(P)}\left(\Lambda_{\eta_j^{(l)}}^{(S)^{-1}} - \Lambda_{\eta_j^{(l)}}^{(P)^{-1}}\right)\right\} + \frac{1}{2} Sp\left\{\Lambda_{\eta_j^{(l)}}^{(S)^{-1}}\left(\mu_{\eta_j^{(l)}}^{(P)} - \mu_{\eta_j^{(l)}}^{(S)}\right)\left(\mu_{\eta_j^{(l)}}^{(P)} - \mu_{\eta_j^{(l)}}^{(S)}\right)^T\right\} \tag{12}$$

будем отдавать предпочтение той функции регрессии, для которой (12) принимает максимальное значение. Если таковыми окажутся несколько $\eta_j^{(l)}$, то в этом случае следует последовательно увеличивать количество повторных измерений (в разумном объеме) до получения однозначного ответа, в противном случае надо переходить на вариант б.

Вариант б. В этом варианте возможно:

1) не изменяя числа столбцов в исходных базисных матрицах X_j, постепенно увеличивать число строк от N_j до $N_j + n_{j_1}^{*}$ $(n_{j_1}^{*} = \overline{1, n_{j_1}}, n_j \leq N_j)$; в результате этого размерности векторов β_j останутся прежними, а размерности векторов $\eta_j^{(l)}$ увеличатся на то же самое число $n_{j_1}^{*}$, предоставляя тем самым дополнительную информацию о функциях регрессии;

2) не изменяя числа строк в исходных базисных матрицах X_j, постепенно увеличивать число столбцов от m_j до $m_j + n_{j_2}^{*}$ $(n_{j_2}^{*} = \overline{1, n_{j_2}}, n_{j_2} \leq m_j)$; хотя в этом

случае размерности векторов $\eta_j^{(l)}$ остаются прежними (т.е. неизменными), исследователь получает дополнительную информацию о функциях регрессии за счет появления дополнительных слагаемых в каждом из $\eta_j^{(l)}$.

Проводя повторные наблюдения, поиск наилучшей функции регрессии прекращаем на том случае варианта б, для которого информационный критерий $max\, J(p:s)$, соотношения (12), дает единственное решение для $\eta_j^{(l)}$.

ЛИТЕРАТУРА

1. Н. Бокучава, Н. Николадзе, Г. Мамасахлисов. *Информационно статистическая модель планирования эксперимента.* Тр. ТГУ. Том 330(19), 1998.
2. С.М. Ермаков, А.А. Жигловский. *Математическая теория оптимального эксперимента.* Москва, "Наука", 1987.
3. Н. Бокучава, Н. Николадзе. *О принципах информационного моделирования.* Тр. ТГУ. Том 330(19), 1998.
4. Ф.Р. Гантмахер. *Теория матриц*. Москва, Наука, 1967.

ექსპერიმენტის მრავალგეგმიანი და მრავალალტერნატივიანი წრფივი რეგრესიული მოდელი

ნ. ბოკუჩავა
ფიზიკური კიბერნეტიკის პრობლემური ლაბორატორია

განზოგადოებული ენტროპიული და ინფორმაციული ზომების ექსტრემალური პრინციპებიდან გამომდინარე, ჩამოყალიბებულია ექსპერიმენტის მრავალგეგმიანი და მრავალალტერნატივიანი განზოგადოებული წრფივი რეგრესიული მოდელის შესაბამისი პარამეტრების შეფასებისა და საუკეთესო წრფივი რეგრესიული ფუნქციის შერჩევის ინფორმაციული კრიტერიუმები.

A MUTI-PLAN AND MULTI-ALTERNATIVE LINEAR REGRESSION MODEL OF AN EXPERIMENT

N.Bokuchava
The problem laboratory of Physical Cybernetics

Proceeding from extreme principles of entropy and informational measures, the informational criteria are formulated for estimating the parameters and choosing the best linear regression function for a multi-plan and multi-alternative generalized linear regression model of an experiment.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 83-90

СИНТЕЗ КРИПТОГРАФИЧЕСКОГО МЕТОДА ПОСРЕДСТВОМ МАТРИЦ НАД КОНЕЧНЫМИ ПОЛЯМИ

Р. П. Мегрелишвили, Д. З. Булавришвили, Т.В. Гнолидзе, М. Т. Махатадзе, В. А. Тогонидзе, Г. К. Хуцишвили
Проблемная лаборатория физической кибернетики

Абстракт. Исследуется отличный от метода Виженера [1] матричный подход к построению симметричных криптографических систем защиты информации.

ВВЕДЕНИЕ

Криптографические методы, основанные на специальных матричных структурах, образуют отличную от методов Виженера систему, которая менее распространена и изучена [1,2]. Обычно, в обеих системах, слова, подлежащие шифрованию, представляются в виде векторов $a \in V_n$ - n-мерного векторного пространства над полем $GF(2)$ или многочленом $a(x)$ из алгебры классов вычетов многочленов A_n по модулю $f(x)$ над тем же конечным полем.

Криптограмма получается умножением вектора a на специальную матрицу A порядка n, а дешифрация зашифрованной информации b осуществляется умножением вектора b на A^{-1} - обратную для A матрицу, т.е.

$$aA=b; \qquad bA^{-1}=a\,. \tag{1}$$

При таком подходе, как и собственно для любых симметричных систем, основную проблему составляют вопросы формирования множества ключей – множества матриц (не поддающихся перебору в реальном масштабе времени, что определяет криптостойкость системы), а также – скорость осуществления шифрации-дешифрации и др.

Основная цель настоящей работы – синтез методов, осуществляющих алгоритмически несложное формирование и представление классов специальных невырожденных матриц порядка n (и обратных для них матриц), удовлетворяющих требованиям криптостойкости.

1. ОБЩИЕ МЕТОДЫ ФОРМИРОВАНИЯ КРИПТОГРАФИЧЕСКИХ МАТРИЧНЫХ КЛЮЧЕЙ

Известны методы нахождения обратных для A матриц A^{-1} [3]. Построение обратной для $A=(a_{ij})_i^n$ (если она несингулярна) матрицы A^{-1} возможно, например в виде:

$$A^{-1} = \begin{bmatrix} \frac{A_{11}}{|A|} & \frac{A_{21}}{|A|} & \dots & \frac{A_{n1}}{|A|} \\ \frac{A_{12}}{|A|} & \frac{A_{22}}{|A|} & \dots & \frac{A_{n2}}{|A|} \\ \dots & \dots & \dots & \dots \\ \frac{A_{1n}}{|A|} & \frac{A_{2n}}{|A|} & \dots & \frac{A_{nn}}{|A|} \end{bmatrix},$$

где A_{ij} - алгебраическое дополнение элемента a_{ij} матрицы A.

Несмотря на относительную простоту проведения операций над полями $GF(2)$, метод, связанный с реализацией (2), не может оказаться приемлемым для криптографического пользователя главным образом из-за невозможности представления обратных матриц в явном виде без выполнения достаточно сложных вычислений.

Необходимое решение не дается и посредством произведения матриц:

$$E_k E_{k-1} \dots E_1 A = I, \\ E_k E_{k-1} \dots E_1 = A^{-1}, \tag{3}$$

где A^{-1} - левая обратная матрица для матрицы A; $E_1,\dots,E_k$ - элементарные матрицы, с помощью которых матрицу A можно привести к каноническому виду и, следовательно, к единичному виду.

Для вычисления элементов x_1 , ... , x_n i-го столбца матрицы A^{-1}, исходя из равенства $A A^{-1} = I$, используют также решение системы уравнений:

$$a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kn}x_n = \begin{cases} 0, & \text{если } k \neq i; \\ 1, & \text{если } k = i; \end{cases} \tag{4}$$

где $k = 1, \dots, n$, и, как и прежде, $|A| \neq 0$.

Из алгебраической теории кодирования известно [4], что в алгебре A_n многочленов над полем $GF(q)$ по модулю многочлена $f(x)$ могут быть заданы классы базисных матриц G (размерности $(k \times n)$) и H (размерности $((n-k) \times n)$), которые удовлетворяют условию:

$$G H^T = 0, \tag{5}$$

где H^T - транспонированная матрица H.

Пространства строк матриц G и H являются идеалами в A_n. Для таких матриц задаются производящие многочлены соответственно $g(x)$ и $h(x)$ $(g(x)\cdot h(x)=f(x))$, которые формируют строки матриц G и H.

Аналогично вышесказанному, квадратные матрицы порядка n (и обратные для них) возможно записать в виде:

$$A^{-1}=\begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ 0 & a_1 & a_2 & \dots & a_{n-2} & a_{n-1} \\ 0 & 0 & a_1 & \dots & a_{n-3} & a_{n-2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_1 & a_2 \\ 0 & 0 & 0 & \dots & 0 & a_1 \end{bmatrix}, \tag{6}$$

где строки матриц (6), аналогично идеалам многочленов, образуют компоненты некоторого вектора $a\in V_n$, т.е. предполагается, что матрица A преобразуется вектором $a=(a_1,\dots,a_n)$, а матрица A^{-1} составляется отличным от а определенным вектором $b=(b_1,\dots,b_n)$.

Для фиксированного вектора a и матрицы (6) с помощью известного (например (4)) метода нетрудно определить значения компонентов вектора b для произвольного значения n. Например, прибегая к математической индукции можно показать, что при произвольном $n>1$ матрица вида

$$A=\begin{bmatrix} 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}, \tag{7}$$

с производящим вектором $a=(a_1,\dots,a_n)$ (где $a_i=1$, если $i\le 2$ и $a_i=1$, если $i>2$) в качестве своей обратной имеет матрицу:

$$A^{-1}=\begin{bmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & 1 & \dots & 1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}, \tag{8}$$

где $b=(b_1,...,b_n)$, $b_i=1$ $(1 \le i \le n)$.

Аналогично для производящих векторов $a=(a_1,a_2,...,a_n)$ $(a_i=1,\ i\le 3;\ a_i=0,\ i>3)$ и $b=(b_1,b_2,...,b_n)$ $(a_i=1,\ i=3k+1,\ i=3k+2;\ a_i=0,\ i=3k)$, соответственно получаем:

$$A=\begin{bmatrix} 1 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \tag{9}$$

$$A^{-1}=\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & \cdots & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & \cdots & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 1 & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

(заметим, что в записи (9) $n=3k$, и $k \ge 0$ - целое число).

Главной задачей при построении матриц вида (8), (9) является не создание метода построения обратной матрицы, основанной на операциях вычисления, а выявление простого метода (или функции) соответствия для нахождения матрицы, обратной для матрицы (6).

Высокая криптостойкость требует построения множества ключей высокой мощности (например, $N=10^{30}$) из которого выбирается конкретный ключ, т.е. - конкретная матрица. Если в матрице (6) представить строки в определенном порядке, то и в обратной для нее матрице необходимо в том же порядке переставлять столбцы, т.е. из заданной матрицы можем получить $n!$ ключевых матриц. Аналогично можно в матрице A переставлять столбцы, что в целом для одной первичной матрицы (при фиксированных производящих a и b) составить множество ключей порядка $(n!)^2$.

Идеальные криптографические системы, к сожалению, построить невозможно. Выигрыш в криптостойкости чаще всего приводит к потере быстродействия или понижению значимости других характеристик, хотя известные различия в криптосистемах, несомненно, оправдываются неодинаковыми условиями их практического применения.

Преимущество матричных методов перед методами Виженера в принципе проявляется в том, что разовое вскрытие криптограммы не вызывает вскрытия самого ключа системы. Это достигается за счет понижения быстродействия. Однако именно потери в быстродействии окупаются качественно отличной и повышенной криптостойкостью данных систем.

2. Синтез криптографических матриц на основе алгебраических структур кодирования

Построение вышерассмотренных матриц принимает более целенаправленный характер с привлечением к решению задачи некоторых специальных структур алгебраической теории кодирования [4]. Как уже было указано, элементы $a=(a_1,\dots,a_n)\in V_n$ и

$$a(x)=\sum_{i=0}^{n} a_i x^i \in A_n$$

подразумеваются эквивалентными объектами. Известно также, что в алгебре A_n для любого идеала I существует единственный нормированный многочлен $g(x)$ наименьшей степени, такой, что класс вычетов $\{g(x)\}$ принадлежит идеалу I и, наоборот, каждый нормированный многочлен $g(x)$, который делит $f(x)$, образует определенный идеал I, в котором $g(x)$ есть нормированный многочлен минимальной степени такой, что класс вычетов $\{g(x)\}\in I$.

Известна следующая Теорема.

<u>ТЕОРЕМА 1.</u> *Пусть, $f(x)$ - многочлен степени n, $f(x)=g(x)h(x)$, а $h(x)$ - многочлен степени k. Тогда в алгебре многочленов по модулю $f(x)$ класс вычетов $\{g(x)\}$ имеет размерность k. Следовательно, степень многочлена $g(x)$ равна*

$$r=n-k. \tag{10}$$

Справедлива также следующая Теорема.

<u>ТЕОРЕМА 2</u>. *Предположим, что $f(x)$, $g(x)$ и $h(x)$ нормированные многочлены и $f(x)=g(x)h(x)$. Тогда класс вычетов $\{a(x)\}$ принадлежит нулевому пространству, порожденному $h(x)$ тогда и только тогда, когда он принадлежит идеалу, порожденному многочленом $g(x)$.*

Из вышеприведенного следует:

СЛЕДСТВИЕ 1. *Пусть, $f(x)=g(x)h(x)$, где $f(x)$ - степени n, а $g(x)$ - степени r многочлены, тогда*

$$G H^T = 0,$$

где G и H порождаются соответственно многочленами $g(x)$ и $h(x)$.

Циклический сдвиг компонентов вектора $g(x)$ на i позиций представляет собой вектор $g(i)=(g_i,\dots,g_{n-i})$, т.е. i-тый сдвиг многочлена $g(x)=1+xg_1+\dots+x^{n-1}g_{n-1}$ приводит к многочлену $g(x^{(i)})=x^i g(x) \bmod (x^n-1)$.

Предположим, что $g(x)h(x)=x^n-1$, а $g(x)$ и $h(x)$ порождают соответственно идеалы I и I'. Тогда

$$G=\begin{bmatrix} g_0 & g_1 & \dots & g_r & 0 & \dots & 0 & \dots & 0 \\ 0 & g_0 & \dots & g_{r-1} & g_r & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & g_0 & \dots & g_r \end{bmatrix}, \qquad (11)$$

$$H=\begin{bmatrix} h_0^* & h_1^* & \dots & h_k^* & 0 & \dots & 0 & \dots & 0 \\ 0 & g_0 & \dots & h_{k-1}^* & h_k^* & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & h_0^* & \dots & h_k^* \end{bmatrix}, \qquad (12)$$

и для произвольных $g(x^{(i)})$ и $h(x^{(i)})$ справедливо равенство:

$$g(x^{(i)})\, h(x^{(i)}) \equiv 0 \bmod (x^n-1), \qquad (13)$$

где $i, j \in \{1,\dots,n\}$. С учетом того, что над полем $GF(2)$ произведения многочленов и векторов не совпадают, для любого $g\in I$

$$gH^{*T}=0, \qquad (14)$$

где H^* матрица образуется вектором h^*, содержащим компоненты h, записанные в обратном порядке следования.

Следует подчеркнуть (что важно для последующих выводов) справедливость (13) и (14), исходящей из замкнутости идеалов I и I^* относительно векторных циклических сдвигов.

Рассмотрим соответствующие матрице (6) квадратные (порядка n) матрицы, которые порождаются многочленами $g(x)$ и $h(x)$ (т.е. многочленами, с помощью коэффициентов которых образуются матрицы (11) и (12)):

$$A_1 = \begin{bmatrix} g_0 & g_1 & \dots & g_r & 0 & \dots & 0 \\ 0 & g_0 & \dots & g_{r-1} & g_r & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & g_0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} h_0 & h_1 & \dots & h_k & 0 & \dots & 0 \\ 0 & h_0 & \dots & h_{k-1} & h_k & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & h_0 \end{bmatrix}, \qquad (15)$$

где j-ый столбец матрицы A_2 представляет собой вектор $h'(j)$ в алгебре многочленов по модулю $x^n - 1$, i-тые компоненты которого суть компоненты вектора $h^*(x)\, x^{i+j-1}$, при $i \le j$ и $h'_i = 0$, если $i > j$.

Исходя из вышеизложенного (учитывая условие (10)) следует, что

$$g(i)h'(j)^T = \begin{cases} 0, & \text{если } i \ne j; \\ 1, & \text{если } i = j; \end{cases} \qquad (16)$$

где $h'^{,T}$ - вектор-столбец, т.е. транспонированный вектор h'.

Следовательно доказана теорема:

ТЕОРЕМА 3. *Пусть $g(x)$ и $h(x)$ - многочлены соответственно степени r и k над полем $GF(2)$ в алгебре многочленов по модулю x^n-1 такие, что $g(x)h(x)=x^n-1$, а A_1 и A_2 - матрицы порядка n, которые порождаются многочленами $g(x)$ и $h(x)$ (15), тогда A_1 и A_2 взаимообратные, т.е.*

$$A_1A_2=I, \quad A_2A_1=I,$$

где I - единичная матрица.

Заметим, что разработаны и известны конструктивные методы построения многочленов $g(x)$ и $h(x)$ в алгебре многочленов по модулю x^n-1 с условием $g(x)h(x)=x^n-1$, которые создают необходимые предпосылки для конструктивной реализации метода, исходящего из теоремы 3.

Л И Т Е Р А Т У Р А

1. К Шеннон. *Работы по теории информации и кибернетике*. М.: ИЛ, 1963.
2. B. Schneier. *Applied cryptography. John Wiley and Sons*, Inc. New-York, 1996.
3. Ф. Р. Гантмахер. *Теория матриц*. М.: Наука, 1967.
4. Т. Касами, Н. Токура, Е. Ивадари, Я. Инагаки. *Теория кодирования*. М.: Мир, 1978.

კრიპტოგრაფიული მეთოდის სინთეზი სასრულ ველზე განსაზღვრული მატრიცების მეშვეობით

რ. მეგრელიშვილი, დ. ბულავრიშვილი, თ. გნოლიძე, მ. მახათაძე, ბ. ტოგონიძე, გ. ხუციშვილი
ფიზიკური კიბერნეტიკის პრობლემური ლაბორატორია

დამუშავებულია კოდირების ალგებრულ სტრუქტურებზე დაფუძნებული ინფორმაციის დაცვის მატრიცული მეთოდი სიმეტრიული კრიპტოგრაფიული სისტემებისათვის. გამოკვლეულია სასრულ ველებზე განსაზღვრული *n* რიგის მატრიცების შებრუნებული მატრიცების სინთეზის საკითხები.

SYNTHESIS OF A CRYPTOGRAPHIC METHOD WITH MATRICES DEFINED OVER THE FINITE FIELDS

R. Megrelishvili, D. Bulavrishvili, T.Gnolidze, M.Maxatadze, B. Togonidze, G. Xutsishvili
Problem Laboratory of Physical Cybernetics

An information security matrix method for symmetrical systems is developed based on algebraic structures of coding theory. The synthesis of matrices opposite to *n*-dimensional matrices defined over finite fields.

Proceedings of Javakhishvili Tbilisi State University
Applied Mathematics and Informatics
Vol. 342 (20), (2000)
pp. 91-96

NEW TECHNOLOGIES OF DESIGN OF SOME BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

T. Vashakmadze), E. Gordeziani*), A. Muradova**), T. Zarqua*)*
**) Computer Software and Informational Technologies chair **) I. Vekua Institute of Applied Mathematics*

Abstract. The technology of design of two-point boundary value problems for ordinary differential equations, containing also boundary layer effects is elaborated, using [19,23,25,26]. The proposed methods essentially refine and enlarge a class of algorithms for solving aforesaid problems. From these methods there follow also classical methods, including methods of Collatz, Henrici, Marchuk, SchrÖder, Tikhonov-Samarskii, finite elements and exponential fitted methods. Then the program part is realized in the form of package of applied programs consisting of control program and modules. For fulfilling this work we followed the manual [2.12] with its software that was kindly given to us by Gilbert. Some parts of this technology are systematically inculcated in teaching processes and not only in the basic courses and also in student's course and diploma works at Iv. Javakhishvili Tbilisi State University, Vekua Institute of Applied Mathematics, University of Delaware. Is created the program package on Turbo Pascal7.0 for solving the boundary value problems for the second order ordinary linear differential equations (fourth issue), [25].

The contents of the report besides the scientific side present an effective manual, realizing purposes, which are stipulated in teaching processes for high school and in practice.

INTRODUCTION

A purpose of the present paper is to suggest a new technology of design of a class of boundary value problems (BVPs) for the second order ordinary differential equations, introduced in educational processes of a number of universities. We will present here also the manuscript-manual as the enlarged version of this report, using essentially the structure of the books [2,6,12].

We note also, that a class of studied BVPs, presented below is chosen for an illustration of the methodology, however, a more general case is considered in [22,24,25].

Let us consider BVPs for the second order non-linear ordinary differential equations

$$\left(k(x)y'(x)\right)' = f(x,y(x),y'(x)),\ \ k(x)>0,\ \ 0<x<1, \tag{1}$$

with the boundary conditions

$$\begin{aligned} y(0)-k_1y'(0)=\alpha,\ \ k_1\geq 0,\\ y(1)+k_2y'(1)=\beta,\ \ k_2\geq 0. \end{aligned} \tag{2}$$

The problems connected with our technology of design are studied for the following subclasses of BVPs (1)-(2):

1.Picard-Banach type conditions are satisfied; 2.The maximum principle is fulfilled.

These subclasses of BVPs, having uniqueness solutions, are important for practice also. This problem of solvability is studied, in particular, in [3,16,17,19,20] and [1,3,5,6,10,11] correspondingly.

We note, that for constructiong Tikhonov-Samarskii schemes [18] it is necessary to compute multiply integrals while by the works of Volkov [28] for getting p-th order ($p>2$) of exactness with respect to h of three-point schemes it is necessary to compute the derivatives of $p-2$ order from the given data of BVP (1)-(2).

1. BVPs of the Picard-Banach Type

For this class the numerical methods, presented, for instance, in the works [3,17], are usually supported on a construction of a difference analogue of the Green function and the solution is found using the fixed-point theorem. The corresponding iterative scheme has the form:

$$y_i^{(m)} = \sum_{k=1}^{N} g_{ik} f_i^{(m-1)} + l_i(\alpha,\beta), \; i = 1,2,\dots,N, \tag{3}$$

where $h = 1/N$ is a step of the mesh, g_{ik} are values of the discrete Green function, $l_i(\alpha,\beta)$ are corresponding functionals, satisfying the boundary conditions (2), $y_i^{(m)}$ are values of te unknown function y in x_i-mesh point on the m-th iteration. An exactness if schemes according to [3,17] have second order, if the iteration process (3) is convergent and of $y \in C^4[0,1]$. From the expression (3) it evidently follows that for constucting approximate solution it is necessary to do $\asymp N^2 \ln N$ arithmetical operations (as is well known that $\asymp \ln N$ operations is the number of iterations).

In the works of Vashakmadze [19,20] there are considered problems of numerical solutions of the Picard-Banach type subclasses of BVPs. The remainder terms of the corresponding schemes are $O(N^{-p+2})$, if $y \in C^p[0,1], (p \geq 4)$. For $= 4$ the result with respect to an order of convergence is similar to those of classical methods [3,17], but the order of an arithmetic operations is minimal $\asymp N \ln N$. This order for an estimate of arithmetical formulae are presented in details in [19,20,23] or in the aforesaid manuscript.

2. BVPs Satisfying the Maximum Principle

The constructions of an approximate solution of this subclass by the finite-difference or varational-difference (i.e., Finite Elemet) methods represents classical part of numerical analysis and are studied, for example, in [1,3,5,6,7,10,11,13,14,16,28, etc]. If for the most of these works the coresspondingly schemes have the second order of accuracy, in the monographs having the fourth order of approximation are also investigated.

We remind that for this subclass satisfying the maximum principle the following conditions are true: the function f is independent of y' and $f_y = \partial f / \partial y \geq 0$ are fulfiled.

In the most well known manuals referenced above this case is investigated, when the left hand side is approximated by the three-point scheme or the veriational difference method, giving the three point template. As is known these schemes have the second order of exactness on h, if $y \in C^4(0,1)$ or the fourth order with respect to h, if the unknown function $y(x)$ is continuously differentiable up to the sixth order. For the linear BVPs, when $k(x) \equiv 1$, are investigated in [18,28] the three-point schemes of the high degree of exactness for this subclass.

In the works of Vashakmadze [19-21,22], when initial BVPs are linear and the weight $k(x)$ is a positive differentiable function on $(0,1)$, the corresponding schemes are constructed by the special class of spline-finctions named as (P) and (Q) formulae, different from the correspoinding systems of the coordinate functions constructed in [4,9,13,15,etc]. (P) and (Q) formulae have also an arbitrary order of exactness depending on the smoothness of $y(x)$ and requiring neither calculating the multiply integrals, nor computing derivatives from the given data of initial BVPs (1)-(2) unlike works [18,28]. For non-linear problems in case $f_y \leq \pi^2 - \varepsilon$ the Belman-Kalaba iterative scheme [1,25] is applied. The suggested schemes for $p = 2$ coincide with the results of Henrichi [10].

In [26], when BVPs (1)-(2) is linear with $k = const$ small positive parameter, using (P) and (Q) formulae, we investigated this problem. At first we considered this problem with a view of the theory of differential equations, according to the work of Viskik and Lusternik [27]. Then we constructed high accuracy multi-point schemes, created the corresponding software and did the numerical experiments. The process of comparision with methods from the monograph of Doolan, Miller and Schilders [7] had been done. The scheme of [26] is also true in a more general case, when $k^{-1}(x)$ non-negative function is integrable with (2) boundary conditions, using data of [21,25].

3. Description of Programm Complexes

There is created the programm package, written in the programming language Turbo Pascal 7.0 for the resolution of the second order ordinary differential equations.

The programm modules are written on the base of new, high accuracy algorithms developed in [19], [21], [23] or [25] and are intended to solve the following problems:

$$y''(x) = f(x, y(x)), \qquad (a)$$

$$y''(x) - q(x)y(x) = f(x), \quad 0 < x < 1, \qquad (b)$$

$$y''(x) = f(x, y(x), y'(x)), \quad 0 < x < 1, \qquad (c)$$

$$\varepsilon y''(x) - q(x)y(x) = f(x), \quad 0 < x < 1, \qquad (d)$$

with boundary comditions (2) when $K_\alpha = 0$,

$$y''(x) - q(x)y(x) = f(x), \quad 0 < x < 1, \qquad (e)$$

with boundary conditions (2), where $y(x)$ is unknown function and $q(x), f(x), f(x, y(x)), f(x, y(x), y'(x))$ satisfy conditiones given in [25], ε is a small positive parameter.

Package consists of five units, each of them separately solving (*a*), (*b*), (*c*), (*d*) and (*e*) problems and one main program, called MAIN.PAS, which enables choice of which problem is to be solved.

Each module uses the program, written by T. Zarqua, for reading the function from screen and counting its value in requested value of arguments. Thus, it is possible to enter the prescribed $q(x), f(x), f(x, y(x))$, and $f(x, y(x), y'(x))$ functionsfrom the screen using the keyboard. The unit is caleed GAMOTVLA.TPU.

The first module PROGRAM1.TPU is soving problem (a) usinf (P) formulae from [25]. Procedure PRO1 is executing its numerical resolution.

The second module is PROGRAM2.TPU. It is counting approximate solution of the problem (b) by means of algorithms elaborated in [25]. The main procedure is PRO2.

The third one is solving problem (c), PROGRAM3.TPU, containing procedure PRO3.

The fourth is for the resolution (d) is named PROGRAM4.TPU, procedure is PRO4.

All these units require input data: boundary conditions $y0$ and $y11$ values; s numbers for the calculation of boundary knots, k, n - number of points, tt number of knots for the calculation of approximate value of integrals in formula for b_{ij} and c_{ij} coefficients ([25]); Output is value of approximate solution $y[i]$, $i = 1,2,..2ks$.

The fifth program module PROGRAM5.TPU is solving (e) problem, the main procedure is MP_MET MP-Met calls the procedures LIJ, BIJ, FUN_Q_F, AIJ, SOLSYST, GRAPHIC. LIJ computes the Lagrange polynomials. BIJ computes coefficients b_{ij} as $b[i, j]$. FUN_Q_F computes $q(x)$ and $f(x)$. AIJ forms the matrice of coefficients a_{ij} of the multi-point method from [25] which has a tape structure. The matrix a_{ij}, $i, j = 2...n$, is written to memory of the computer in the rectangle form as $a[i]^\wedge[j]$, $j = 2,..,2s+3, i = 2,..,n$. Beginning with row $s+3$ the elements of the matrix are stored in memory of the computer beginning with the first column, i.e. $a_{i,i-s}$, $i = s+3,..,n = s+1$ and $a_{i,n-2s+2}$, $i = n-s+2,..,n$ will be stored it the first column. SOLSYS solves the obtained algebraic system by the Gauss exception method. As a consequence values $f0[i], i = 2,..,n$ are obtained. GRAPHIC constructs graphics of the obtained solution $y[i], i = 2..n$ with the boundary conditions $y0$ and $y1$.

The corresponding programm package represents complete, independent product ready for users.

REFERENCES

1. Bellman R.E., Kalaba R.E., *Quasilinearization of Nonlinear Boundary Value Problems*, Amer. Elsener Publ. Com.Inc. Co. N-Y, 1965;

2. Ben-Israel A., Gilbert R.P., *Computer-Supportes Calculus,* Springer, Wien-N-Y, 2002. (in appear);
3. Berezin I.S., Shidkov N.P., *Methodi Vichislkenii,*Moscow, "Nauka", Vol.2, 1960, ch. IX.;
4. de Boor C., *A Practical Guide to Splines,* Springer, B-H-N-Y, 1979;
5. Collatz L.,*The Numerical Treatment of Differential Equations*, Springer, B-H-N-Y, 1966, ch.III, ch. IV;
6. Engeln-Muellges G., Reutter F., *Formelsammlung zur Numerischen Mathematic mit C-Programmen,* B.I. Wissenschaft-Verlag, Manheim-Wien-Zurich, 1990, ch.16, ch. P16;
7. Doolan E.P., Miller J.H., Schilders W.H.A., *Uniform Numerical Methods for Problems with Initial and Boundary Layers,* Boole Press, Dublin, 1980, part II;
8. Farrel P.A.,O'Riordan E., Miller J.H., Shishkin G.I.,*Parameter Uniform Fitted Method for Quasilinear Differential Equations with Bopundary Layers,* Com. Meth. in Aool. Math., Vol.1, #2, 2001,pp. 154-153;
9. Fix G., Strang G., *An Analysis of the Finite Element Method,* Englewood, Gliffs, N-Y, 1973;
10. Henrichi P., *Discrete Variable Methods in Ordinary Differential Equations,* Wiley, N-Y, L-S, 1968, part III;
11. Keller H.B., *Numerical Methods for Two- Point Boundary Value Problems, SIAM,* , Brasdell. Massachusette, T-L, 1968;
12. Koepf W., Ben-Israel A., Gilbert R.,*Mathematik mit Derive,* Vieweg Verlag, Brounschweig, 1999;
13. Marchuk G.I., *Methods of Numerical Mathematics,* Moscow, Nauka, 1989, Ch.1(In Russian);
14. Mikeladze Sh., *New Methods for the Integrating of Differential Equations,* M-L, 1951;
15. Mikhlin S.G., *To the Variational Difference Method for One-dimensional Boundary Value Problems,* Soviet. Dok., Vol.198, 1971,pp. 39-41;
16. Picard P.I., *Lessons sur Quelques Problems aux limites de la Theorie des Equations* Differentialles, Gauthier Villas, Paris, 1930;
17. Schroder J., Ueber das Differentzverfahren *bei nichtlinearen Randwertaufgaben,* I. ZAMM, 36, 1956, pp.319-351, II pp. 443-455;
18. Tikhonov A.N., Samarskii A.A., *Homogeneous Difference Schemes,* J. of Vichislitelnoi Matematiki I Matematicheskoi Fyziki, I. Vol.1, #1, 1961, pp. 5-61, II. Vol.1. #3, 1961, pp. 425-440, III. Vol.2, #5, 1962, pp. 812-132;
19. Vashakmadze T., *On Numerical Solution of Boundary Value Problems,* J. of Vichislitelnoi Matematiki I Matematicheskoi Fyziki, Vol.4, #4, 1964, pp. 623-637;
20. Vashaknadze T., *A Generalized Finite Difference Method,* Fifferential Equations, Vol. 2, #5, 1966, pp. 615-618;
21. Vashakmadze T., *To Numerical Solution of Boundary Value Problems,* Bulletin of Georgian SSR of AS, Vol. 48. pp. 519-524;

22. Vashakmadze T., *On Some Optimal Algorithms* , Proccedings of the Institute of Applied Mathematics, Vol. 1,1969,pp. 516-517 (In Russian);

23. Vashakmadze T., *The Generlized Factorization Method and its Application for Numerical Solution of Some Problems of Mathematical Physics,* Procee. Symp. Continuum Mech. and Related Problems of Analysis, Tbilisi: Metsniereba, Vol. 1, 1972, pp.39-45;

24. Vashakmadze T. (Editor), Approx*imate Methods of Solution of Some Problems of Mathematical Physiscs,* Tbilisi University Press, 1975 (In Russian);

25. Vashakmadze T., *The Theory of Anisotropic Elastic Plates,* Kluwer Acadenic Publishers, Dortecht-L-Boston, 1999, ch.III;

26. Vashakmadze T., Muradova A., *To an Analysis of Numerical Methods of Boundary Value Problems with a Small Parameter,* J. Applied Mathematics and Informatics, Vol. 1, #1, 1996, pp. 180-192 (In Russian);

27. Vishik M., Lusternik L., *Regular Degeneration and Boundary Layer for Linear Differential Equations with a Small Parameter*, Uspekhi Matematicheskikh Nauk, XII,5(77), 1957, pp. 3-122;

28. Volkov E., *Effective Error Estiomates for Difference Solutions of BVP for OD,* proceedings Steklov MIAS, vol. CXII, issue 1, M.: Nauka, 1971, pp. 141-151;

ჩვეულებრივი დიფერენციალური განტოლებებისათვის ზოგიერთი სასაზღვრო ამოცანის გათვლის ახალი ტექნოლოგიების შესახებ

თ. ვაშაყმაძე), ე. გორდეზიანი*), ა. მურადოვა**), თ. ზარქუა*)*

**) კომპიუტერების მათემატიკური უზრუნველყოფისა და ინფორმაციული ტექნოლოგიების კათედრა; **) ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტი*

ნაშრომი ეყრდნობა ძირითადად თ. ვაშაყმაძის შრომებს ჩვეულებრივი დიფერენციალური განტოლებებისათვის ზოგიერთი სასაზღვრო ამოცანის მიახლოებითი ამოხსნის მიმართულებით. ეს შრომები ანზოგადებს და აზუსტებს რიგ ავტორთა შედეგებს, რომლებიც წარმოდგენილია სახელმძღვანელოებში.

შემოთავაზებულ ალგორითმებზე დაყრდნობით შექმნილია და რეალიზებულია კომპიუტერზე პროგრამული პროდუქცია. ალგორითმები და პროგრამული ნაწარმი წარმოადგენს ჩვეულებრივი დიფერენციალური განტოლებებისავის ზოგიერთი სასაზღვრო ამოცანის გათვლის ახალ ტექნოლოგიას. პროგრამული პაკეტი არის სრული დამოუკიდებელი მომხმარებლისთვის მზა პროდუქცია.

სარჩევი
CONTENTS

გამოყენებითი მათემატიკა • APPLIED MATHEMATICS

კომპიუტერული მეცნიერებები • COMPUTER SCIENCES

გამომცემლობის რედაქტორები: ე. აბუაშვილი
მ. ინასარიძე

კორექტორები: ნ. დიხამინჯია
რ. კვაჭანტირაძე
ნ. ჩახაია

ხელმოწერილია დასაბეჭდად 23.VII.02
საბეჭდი ქაღალდი 60×84/16
პირ. ნაბეჭდი თაბახი 12.5
სააღრ.-საგამომცემლო თაბახი 6.13

შეკვეთა №2 ტირაჟი 120

ფასი სახელშეკრულებო

თბილისის უნივერსიტეტის გამომცემლობა
380028, თბილისი, ი. ჭავჭავაძის გამზ. 14
თბილისის უნივერსიტეტის სარედაქციო-
სადუმლიკაციო კომპიუტერული სამსახური
თბილისი, ი. ჭავჭავაძის გამზ. 1

PROCEEDINGS
of Javakhishvili
TBILISI STATE UNIVERSITY

AIMS AND SCOPE

The Journal *Proceedings of Javakhishvili Tbilisi State University* publishes original research articles on Applied Mathematics, Computer Sciences, Linguistics and their application to science. Subject areas are very vast and include initial-boundary value problems for ordinary and partial differential equations and integral equations, appearing in applied mathematics and mathematical physics, theoretical contributions, numerical algorithms and computer simulations etc. all the areas concerning applied mathematics, computer sciences and linguistic theme. The journal will also concentrate on the interaction between different fields of mentioned disciplines. In this connection the journal will appreciate survey papers for publication.

NOTES FOR CONTRIBUTORS

Manuscripts, as Word files, may be sent to the Editor-in-Chief by e-mail: **vasha@viam.hepi.edu.ge.** Besides the electronic version of the manuscript, one hard copy with signature of one author should be submitted to the Editor-in-Chief: **I. Vekua Institute of Applied Mathematics, University St. #2, Tbilisi, Georgia.**

Submission of manuscript is welcome provided that:

- this manuscript, or any translation of it, has not been copyrighted or previously published and is not under consideration for publication elsewhere

Manuscripts should be written in English, Georgian, German, French, Italian, Russian, Spanish languages. Each article should have abstracts in Georgian and English languages. The length of paper is normally limited to 10 standard Word pages (format: **A4**, font size: **11pt**, Top: **3.46cm**, Bottom: **4.25cm**, Inside: **3.3cm**, Outside: **2.7cm**). Publication of the papers that exceed this limit will depend on a special decision of the Editors.