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INITIALS OF UNIFIED FIELD THEORY

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The monograph deals with studying of the problem raised by Einstein – on the united nature of gravitational and electromagnetic fields. On the basis of generalization of Riemannian geometry and tensor analysis a system of differential equations relative to potentiality of gravitation and electromagnetic field (GEH) has been obtained which is not a generalization of Einstein and Maxwell equations. The systems of Einstein and Maxwell equations (for curved spaces), mentioned in the monograph, have been used for calculation of characteristic parameters (tensors of energy-pulse and current) of the matter according to the solution of GEH field differential equations.

The solution of differential equation of GEH field in case of central symmetry has been constructed in the monograph, which is free of singularity in central point (main requirement of Einstein) and in asymptote coincides with solution of Schwarzschild solution and Coulomb potential. The solution comprises of four arbitrary constants. According to various values of these constants three concrete tasks have been studied: from Astrophysics and from nuclear and elementary-particle physics.

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The Monograph is dedicated to blessed
memory of my wife, Mrs. Lamara

P R E F A C E

This Monograph is the result of the author's many year scientific creative work in the field of Unified Field Theory.

The work has been ready yet at the beginning of the last century, but the processes developed in Georgia of those times have prevented its timely publication.

The Monograph has been translated by mrs. **M. Machavariani** for what I'm vary thankful to her. As well I express my gratitude to Nino Chulukhadze for computer realization of the Monograph.

It should be especially noted the greatest merit of my colleague and friend, professor **Rafael Chikovani** in the matter of organization of publication of the Monograph and financial provision. His attempt and direct participation made it possible to publish the Monograph in the shortest period of time for what I declare Him my cordial gratitude with love and respect.

R. Gogsadze

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INTRODUCTION

It is known that hypothesis of unified character of gravitational and electromagnetic fields, as well the field structure of the matter, belong to Einstein. This idea can be practically realized in two stages. On the first stage of investigation it is necessary to generalize the laws of gravitational and electromagnetic fields and formulate the unified field, theory comprising (in definite meaning of this word) of laws of gravitational and electromagnetic fields. Such theory as it was repeatedly noted by Einstein himself, should be free of singularities in four-dimension world. The limitation which is superimposed on the structure of the unified theory, is the necessity in its elasticity, capability to respond to the questions, characteristic for the material world. However, one such a requirement can't provide the singularity of unified field theory. Various unified theories, existence of which is possible by their possibilities should be identical, transforming, in definite meaning, one into another.

On the second stage, the investigations should be realized in compliance between the conceptions of unified field theory and conceptions of modern physics by means of which the investigator percieves the outer world.

It should be noted that main difficulties on the way of realization of this program are met on the second stage of investigation, i.e. in the process of establishing the relation (relation between the scheme of unified theory and outer world, as it takes place, e.g. in quantum physics.

The recent work is dedicated to one of the possible versions of practical fulfillment of the mentioned program.

The main principle on the basis of which we'll try to perform an investigation of the first part of the planned program, is adopted from the evolution of physics and consists of the following: If on this stage of development the definite physical theory is covariant in relation to some group of transformation, than the analysis of evolution of physical science shows that the requirement of covariance relative to more common group of transformation results to necessity of introduction of new notions and widening and perfection of old theory. E.g. demand of covariance of laws of mechanics relative to Lorenz group of transformation resulted to creation of mechanics of special relativity theory and requirement of relativity of continuous (holonomic) groups - to creation of mechanics of general relativity theory and relativistic theory of gravitation field. In this connection for unified description of gravitational and electromagnetic fields in the present work there is introduced a conception of continuous (holonomic) groups, a corresponding nonholonomic geometry has been constructed, which, in future is used for construction of unified field theory, covariant relative to nonholonomic group of transformation. Such a procedure is quite identical to that, which was early applied by Einstein during creation of common relativity theory and relativistic theory of gravitational field and is capable to reveal by means of completely in analogy the physical essence of new concepts, connected with nonholonomic group of transformation.

The established links (relations) between the concepts of unified field theory and usual physical notions are independent principal task, which, in the present work has been realized while applying the equations of relativistic theory of gravitational field and the equations of electromagnetic field in the curved space, belonging to Einstein. In comparison to old classical theory, in which these equations have been applied for determination of the metric tensor components g_{ij} and electromagnetic field potential φ_i along to given values of the components of energy pulse tensor T_i^k and density of electric current j^i, in the proposed theory parameter T_i^k and j^i are determined by values of components g_{ij} and φ_i, representation solving the system of equation of unified field theory. Such a procedure of establishing the relation between the mentioned parameters logically are not strongly substantiated, and it may be justified only by the degree of coincidence of obtained theoretical results with the corresponding experimental data.

With this purpose in this work there has been considered an unified field of central symmetry. The corresponding system of relativity equations g_{ij} and φ_i has been solved and is proved that these parameters are free of singularity in central point $r = 0$ and in the whole four-dimensional space. The asymmetric behavior of these very parameters have been investigated as well while $r \to \infty$ and it has been shown that g_{ij} and φ_i $r \to \infty$ coincide with the known classical values.

Further, the received theoretical results have been used for approximated description of proton structure of atomic cores and the Sun, identifying them by corresponding unified fields of central symmetry. Comparison of obtained results with corresponding empiric data shows that the offered field model of the matter with the precision of sufficient degree describes the structure of proton, heavy atomic cores and the Sun.

C H A P T E R 1

ELEMENTS OF NONHOLONOMIC GEOMETRY

Main elements of nonholohomic geometry which are stated in the second paragraph of this Chapter are the direct generalization of proper items of Riemannian geometry. The best way, by means of which it is possible to realize the mentioned generalization, in our opinion, is an apparatus of absolutely differential calculations and tensor analysis. This circumstance conditions introduction in this chapter (in the first paragraph) of a known material from the Riemannian geometry and tensor analysis, which, as we consider, contributes to natural transition from Riemannian (holonomic) geometry to nonholonomic one.

1.1. SOME MAIN ELEMENTS OF RIEMANNIAN GEOMETRY

1.1.1. ELEMENTS OF TENSOR ALGEBRA [1]

Space – is the aggregate of points. The notion of points is elementary (limiting), not determined by more elementary notions. For addressing the points of the space there can be used four acting numbers x^0, x^1, x^2, x^3. The numbers x^0, x^1, x^2, x^3 are called the coordinates of the main elements (points) of space.

Determination carried out here is not connected with the material phenomena and, that is why is of abstract character; space-independent reality, in which run the material phenomena.

And the character of the space (see below) is closely related with material phenomena; only thanks to material phenomena the main characteristics-metric features – space in which we live, can be determined.

A priori it can be applied the following proposition: the space cannot have several various metric features at the same time. Than, with allowance that these features with single meaning are connected with material phenomena, running in the space, it becomes clear, that the independent, as regards to classical physics, phenomena – mechanical, gravitational, electromagnetic, etc. are unified.

Addressing of space points can be realized by completely arbitrary way, only following one of the main requirements – monosemanticity of correspondence between points and their addresses.

Let x'^k – other addresses (coordinates) of points of considered spaces. The coordinates x^k *and* x'^k must be located in interunambiguity functional dependence, i.e.:

$$x'^k = x'^k(x^0, x^1, x^2, x^3), \quad k = 0, 1, 2, 3 \qquad (1.1.1.1)$$

Values of x^k *and* x'^k determine various systems of addresses, or the various systems of coordinates, and the equations (1.1.1.1) express the law of transformation of coordinates (law of readdressing of points) during transition of one system of coordinates into another.

In the space, for which there has not yet been determined the metrics, it can't be realized the geometrical constructions, i.e. it can't be judged the distances and directions, however, in such a space, by means of method of point grouping, having the definite similar features, there may be introduced the geometrical notions, useful during solution of various problems. For example, totality of points having the similar x^k coordinates (k - fixed number from multitude of 0,1,2,3) constitutes the three-dimensional hyper surface. Let's generally call the three-dimensional hyper surface as a totality of four-dimensional space, coordinates x^k of which are represented by the equations:

$$x^k = x^k(p^1, p^2, p^3), \quad k = 0, 1, 2, 3 \qquad (1.1.1.2)$$

where, p^1, p^2 and p^3 are some parameters having the definite domain of variation.

Completely similarly, the equations:

$$x^k = x^k(p^1, P^2), \quad k = 0, 1, 2, 3 \qquad (1.1.1.3)$$

determine the two-dimensional hypersurface, and the equities:

$$x^k = x^k(p^1), \quad k = 0, 1, 2, 3 \qquad (1.1.1.4)$$

determine the one-dimensional hypersurface or, that is the same – the line.

The hypersurface $x^k = 0,\ x^i_{\min} < x^i < x^i_{\max}\quad i \neq k$ is called the k -*th* coordinate hypersurface (k - fixed number); in case, when i - fixed number, these terms determine the i -*th* coordinate line.

By means of coordinates of main elements (points) of four-dimensional space it is possible to form some characteristic elements, in particular, so called infinitesimal controlled vector. If $M_1(x^0, x^1, x^2, x^3)$ and $M_2(x^0 + dx^0,\ x^1 + dx^1, x^2 + dx^2, x^3 + dx^3)$ – two points of space, coordinates of which differ from each other by infinitesimals of dx^0, dx^1, dx^2, dx^3. Let's call the totality of these values as the infinitesimal contravariant vector, and the separate elements of dx^k – components of this vector.

It is obvious that in other system of coordinate the considering infinitesimal contravariant vector is characterized by the components dx'^k. Relation between infinitesimals dx^k *and* dx'^k can be established by using (1.1.1.1). Really, from this equity after differentiation we'll have:

$$dx'^k = \frac{\partial x'^k}{\partial x^p} dx^p \qquad (1.1.1.5)$$

On the basis of transformation law, during transition from one system of coordinates into another, it is possible to introduce generally a determination of controlled vector.

If some physical phenomenon which runs in considered four-dimensional space in various systems of coordinates x^k and x'^k is characterized by totality of values A^k *and* A'^k correspondingly, and if between these values there exists a relation of the type (1.1.1.5), i.e. there takes place the following equity:

$$A'^k = \frac{\partial x'^k}{\partial x^p} A^p \qquad (1.1.1.6)$$

than we'll say that A^k constitutes a contravariant vector with the components A^0, A^1, A^2, A^3.

If A^k and B^k – are the contravariant vectors, than from them it is possible to form totality of the values $A^k \cdot B^k$, number of which is equal to 16. In the system of coordinates x'^k to these values according to (1.1.1.) correspond:

$$A'^k \cdot B'^l = \frac{\partial x'^k}{\partial x^p} \frac{\partial x'^l}{\partial x^q} A^p B^q \qquad (1.1.1.7)$$

In connection to transformation law we are introducing a tensor of second order. If some physical phenomenon in various systems of coordinates x^k and x'^k are characterized by totality of the values A^{kl} and A'^{kl} correspondingly and if there takes place the transformation law of the type (1.1.1.7)

$$A'^{kl} = \frac{\partial x'^k}{\partial x^p} \frac{\partial x'^l}{\partial x^q} A^{pq} \qquad (1.1.1.8)$$

than the totality of values A^{kl} will be called a contravariant tensor of second order.

Similarly, the totality of values $A^{k_1 k_2 \dots k_n}$ constitutes a contravariant tensor of *n-th* order, if the corresponding components in the system of coordinate x'^k are determined by the following equities:

$$A'^{k_1 k_2 \dots k_n} = \frac{\partial x'^{k_1}}{\partial x^{p_1}} \frac{\partial x'^{k_2}}{\partial x^{p_2}} \dots \frac{\partial x'^{k_n}}{\partial x^{p_n}} A^{p_1 p_2 \dots p_n} \qquad (1.1.1.9)$$

A physical value u is called a scalar, if its value in any point of the space does not depend on choice of coordinates, i.e. if

$$u(x^0, x^1, x^2, x^3) = u'(x'^0, x'^1, x'^2, x'^3) \qquad (1.1.1.10)$$

From u it is possible to form so called covariant vector $\frac{\partial u}{\partial x^k}$. Accordingly, in the system of coordinates x'^k this vector will have the following appearance $\frac{\partial u'}{\partial x'^k}$. Relationship between these vectors is established by means of application of (1.1.1.10) and (1.1.1.1), in particular:

$$\frac{\partial u'}{\partial x'^k} = \frac{\partial x^p}{\partial x'^k} \frac{\partial u}{\partial x^p} \qquad (1.1.1.11)$$

In connection to this transformation law let's introduce a notion of covariant vector. The totality of physical values A_i is called a covariant vector, if its components in the coordinate system x'^k *are* determined by the following equities:

$$A'_i = \frac{\partial x^p}{\partial x'^i} A_p \qquad (1.1.1.12)$$

It's obvious that if A_i and B_i are the covariant vectors, than according to (1.1.1.12) the following equities are valid:

$$A'_i B'_j = \frac{\partial x^p}{\partial x'^i} \frac{\partial x^q}{\partial x'^j} A_p B_q \qquad (1.1.1.13)$$

In this connection, A_{ij} *is* called a covariant tensor of second order, if its components in the system of coordinates x'^k are determined by the following equities:

$$A'_{ij} = \frac{\partial x^p}{\partial x'^i} \frac{\partial x^q}{\partial x'^j} A_{pq} \qquad (1.1.1.14)$$

and $A_{i_1 i_2 \ldots i_m}$ is a covariant tensor of n-th order, if the following transformation law is valid:

$$A_{i_1 i_2 \ldots i_n} = \frac{\partial x^{p_1}}{\partial x'^{i_1}} \frac{\partial x^{p_2}}{\partial x'^{i_2}} \ldots \frac{\partial x^{p_n}}{\partial x'^{in}} A_{p_1 p_2 \ldots p_n} \qquad (1.1.1.15)$$

If A^k is contravariant, and B_i - is a covariant vector, than it is obvious that

$$A'^k B'_i = \frac{\partial x'^k}{\partial x^p} \frac{\partial x^q}{\partial x'^i} A^p B_q \qquad (1.1.1.16)$$

Reducing these equities by indices i and k and with allowance that:

$$\frac{\partial x'^k}{\partial x^p} \frac{\partial x^q}{\partial x'^k} = \delta^q_p , \qquad (1..1.1.17)$$

we'll obtain:

$$A'^k B'_k = A^k B_k \qquad (1.1.1.18)$$

i.e. $A^k B_k$ is invariant.

In connection to the transformation law (1.1.1.16) let's introduce a notion of mixed tensor of second order. A_i^k is called a mixed tensor of second order if there takes place the following transformation law:

$$A_i'^k = \frac{\partial x'^k}{\partial x^p} \frac{\partial x^q}{\partial x'^i} A_q^p \qquad (1.1.1.19)$$

as well $A_{i_1 i_2 \ldots i_m}^{k_1 k_2 \ldots k_n}$ is called m times covariant and n times contravariant mixed tensor of $m+n$ order, if

$$A'^{k_1 k_2 \ldots k_n}_{i_1 i_2 \ldots i_m} = \frac{\partial x'^{k_1}}{\partial x^{p_1}} \frac{\partial x'^{k_2}}{\partial x^{p_2}} \ldots \frac{\partial x'^{k_n}}{\partial x^{p_n}} \frac{\partial x^{q_1}}{\partial x'^{i_1}} \frac{\partial x^{q_2}}{\partial x'^{i_2}} \ldots \frac{\partial x^{q_m}}{\partial x'^{i_m}} A^{p_1 p_2 \ldots p_n}_{q_1 q_2 \ldots q_m} \qquad (1.1.1.20)$$

From the transformation law of tensors it follows that if the tensor in some system of coordinates is symmetric (antisymmetric) relative to two indices, than the symmetry (antisymmetry) according to these indices are maintained in any system of coordinates. Really, if

$$A_{\ldots ik \ldots} = A_{\ldots ki \ldots}$$

than

$$A'_{\ldots ik \ldots} = \ldots \frac{\partial x^p}{\partial x'^i} \frac{\partial x^q}{\partial x'^k} \ldots A_{\ldots pq} = \ldots \frac{\partial x^q}{\partial x'^i} \frac{\partial x^p}{\partial x'^k} \ldots A_{\ldots qp \ldots} =$$

$$= \ldots \frac{\partial x^p}{\partial x'^k} \frac{\partial x^q}{\partial x'^i} \ldots A_{\ldots pq} = A'_{\ldots ki \ldots}$$

For antisymmetric tensor $A_{\ldots ik \ldots} = -A_{\ldots ki \ldots}$ has:

$$A'_{...ik....}=...\frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^k}...A_{....pq....}=...\frac{\partial x^q}{\partial x'^i}\frac{\partial x^p}{\partial x'^k}....A_{....qp...}=$$
$$=...\frac{\partial x^p}{\partial x'^k}\frac{\partial x^q}{\partial x'^i}...A_{...pq...}=-A'_{...ki...}$$

A similar circumstance takes place for contra variant tensors.

The components of infinitesimal contravariant vector dx^k are absolutely arbitrary values, but if the increment of coordinates corresponds to increment of some invariant parameter *p,* than from them it is possible to create a final contravariant vector $\dfrac{dx^k}{dp}$.

Let's consider some partial versions of this vector.

If [1])

1) $dx^0=d\underset{1}{x}^0\neq 0,\ dx^1=d\underset{1}{x}^1=0,\ dx^2=d\underset{1}{x}^2=0,\ dx^3=d\underset{1}{x}^3=0;$

2) $dx^0=d\underset{2}{x}^0=0,\ dx^1=d\underset{2}{x}^1\neq 0,\ dx^2=d\underset{2}{x}^2=0,\ dx^3=d\underset{2}{x}^3=0;$

3) $dx^0=d\underset{3}{x}^0=0,\ dx^1=d\underset{3}{x}^1=0,\ dx^2=d\underset{3}{x}^2\neq 0,\ dx^3=d\underset{3}{x}^3=0;$

4) $dx^0=d\underset{4}{x}^0=0,\ dx^1=d\underset{4}{x}^1=0,\ dx^2=d\underset{4}{x}^2=0,\ dx^3=d\underset{4}{x}^3\neq 0;$

The corresponding final vectors have such a form:

1) $\underset{1}{\dot x}^0=d\underset{1}{x}^0\big/dp\neq 0,\ \underset{1}{\dot x}^1=d\underset{1}{x}^1\big/dp=0,\ \underset{1}{\dot x}^2=d\underset{2}{x}^1\big/dp=0,$
$\underset{1}{\dot x}^3=d\underset{1}{x}^3\big/dp=0;$

2) $\underset{2}{\dot x}^0=d\underset{2}{x}^0\big/dp=0,\ \underset{2}{\dot x}^1=d\underset{2}{x}^1\big/dp\neq 0,\ \underset{2}{\dot x}^2=d\underset{2}{x}^2\big/dp=0,$
$\underset{2}{\dot x}^3=d\underset{2}{x}^3\big/dp=0;$ *(1.1.1.21)*

3) $\underset{3}{\dot x}^0=d\underset{3}{x}^0\big/dp=0,\ \underset{3}{\dot x}^1=d\underset{3}{x}^1\big/dp=0,\ \underset{3}{\dot x}^2=d\underset{3}{x}^2\big/dp\neq 0,$
$\underset{3}{\dot x}^3=d\underset{3}{x}^3\big/dp=0;$

4) $\underset{4}{\dot x}^0=d\underset{4}{x}^0\big/dp=0,\ \underset{4}{\dot x}^1=d\underset{4}{x}^1\big/dp\neq 0,\ \underset{4}{\dot x}^2=d\underset{4}{x}^2\big/dp=0,$
$\underset{4}{\dot x}^3=d\underset{3}{x}^3\big/dp\neq 0;$

These vectors are not linearly dependent, so as

$$\begin{vmatrix}\underset{1}{\dot x}^0 & 0 & 0 & 0\\ 0 & \underset{2}{\dot x}^1 & 0 & 0\\ 0 & 0 & \underset{3}{\dot x}^2 & 0\\ 0 & 0 & 0 & \underset{4}{\dot x}^3\end{vmatrix}=\underset{1}{\dot x}^0\cdot\underset{2}{\dot x}^1\cdot\underset{3}{\dot x}^2\cdot\underset{4}{\dot x}^3\neq 0$$

dp is arbitrarily small value and, if in the first line of the system (1.1.1.21) we'll admit that $d\underset{1}{x}^0=dp$, in the second – $d\underset{2}{x}^1=dp$, in the third – $d\underset{3}{x}^2=dp$ and in the last – $d\underset{4}{x}^3=dp$, than we'll obtain the following linearly independent vectors:

[1] Index above the letter shows the number of the version and is not a number of the component of tensor value.

$$\underset{1}{\vec{E}}(1,0,0,0),\quad \underset{2}{\vec{E}}(0,1,0,0),\quad \underset{3}{\vec{E}}(0,0,1,0),\quad \underset{4}{\vec{E}}(0,0,0,1) \qquad (1.1.1.22$$

Thus, in any point of the space it is possible to construct 4 linearly independent vectors (1.1.1.22), so-called independent basic vectors, which are obtained from the established law of addressing points of space (see (1.1.1.21). In other words the basic vectors (1.1.1.22) are chosen not by arbitrary way but are determined by a structure of coordinate system.

Any contravariant vector A^k, which depends on the coordinates of points in considered space, let's imagine in a linear combination of basic vectors (1.1.1.22), i.e.:

$$C\vec{A} + \sum_{k=4}^{4} C_k \underset{k}{\vec{E}} = 0 \qquad (1.1.1.23)$$

This equity really takes place if we admit that:

$$A^k = -\frac{C_k}{C} \qquad (1.1.1.24)$$

On the basis of these latter equities of the values A^k we have received the indicated components of the vector $\vec{A}$ in the basis of (1.1.1.22).

Here it should be done one important note, in the Riemannian geometry while investigating the arbitrary question there are applied exclusively basic vectors (1.1.1.22).

In principle, introduction of basic vectors in the form of (1.1.1.22), depending on law of addressing the space points, is not unique, they might be selected by completely arbitrary way in each space point as well as the coordinates (of the address) of this point. The only limitation, which the basic vectors $\underset{k}{\vec{e}}$ have to satisfy, should meet requirements, is their linear independencies, i.e. the components of these vectors in the basis (1.1.1.22) should meet the requirement:

$$\begin{vmatrix} \underset{1}{e}^0 & \underset{1}{e}^1 & \underset{1}{e}^2 & \underset{1}{e}^3 \\ \underset{2}{e}^0 & \underset{2}{e}^1 & \underset{2}{e}^2 & \underset{2}{e}^3 \\ \underset{3}{e}^0 & \underset{3}{e}^1 & \underset{3}{e}^2 & \underset{3}{e}^3 \\ \underset{4}{e}^0 & \underset{4}{e}^1 & \underset{4}{e}^2 & \underset{4}{e}^3 \end{vmatrix} \neq 0, \qquad (1.1.1.25)$$

where, $\underset{i}{e}^k$ – the components of vector in the basis of (1.1.1.22). They are the functions of the coordinates of separate space points.

It is easy to show, that if $A^k(x^0,x^1,x^2,x^3)$ is any contravariant vector in the given space point, than the system of the vectors $\vec{A},\underset{1}{\vec{e}},\underset{2}{\vec{e}},\underset{3}{\vec{e}},\underset{4}{\vec{e}}$ is linearly dependent, i.e. the following equity takes place:

$$b\vec{A} + \sum_{k=1}^{4} b_k \underset{k}{\vec{e}} = 0 \qquad (1.1.1.26)$$

Really, in force of (1.1.1.25) the system (1.1.1.26) which we'll rewrite in such a form:

$$\sum_{k=1}^{4} \underset{k}{e}^{i}(-\frac{b_k}{b}) = A^i \qquad (1.1.1.27)$$

has the only solution

$$-\frac{b_k}{b} = \overline{A}^k = \alpha_i^k A^i \qquad (1.1.1.28)$$

where,

$$\alpha_i^k = \frac{\Delta_i^k}{\Delta} \qquad (1.1.1.29)$$

Δ - value of a determinant (1.1.1.25), and Δ_i^k – algebraic addition to the element $\underset{i}{e}^{k}$ *in* a determinant Δ. Values of $\overline{A}^k$ determined according to (1.1.1.28), by analogy to the previous one, are called the components of the same $\vec{A}$ vector in the basis $\underset{i}{\vec{e}}$. Obviously, these equities (1.1.1.127) and (1.1.1.28) establish the contact between the components of one and the same vector $\vec{A}$ in various basis in $\underset{i}{\vec{E}}$ and $\underset{i}{\vec{e}}$.

From the above indicated we come to the following conclusion: for a tensor calculus an important value has not only the law of addressing of space points (on the law of addressing depends the functional image of separate tensor components), but the choice of base vectors in separate space points. In this connection, in future, in comparison to the system of coordinates, we shall introduce the notion reference system, under which we'll understand the combination of addresses (coordinates) of points x^k and systems of basis vectors $\underset{k}{\vec{e}}$, chosen in each space point. A reference system we'll denote through the symbol $SR(x^k, \underset{k}{\vec{e}})$. A partial type of the reference system is $SR(x^k, \underset{k}{\vec{E}})$; In this reference system in each space point a basis vector is (1.1.1.22). $SR(x^k, \underset{k}{\vec{E}})$ is the main reference system for Riemannian geometry (in four-dimensional space) and Einstein's general relativity theory.

During transformation of only the system of coordinates $SR(x^k, \underset{k}{\vec{e}})$ will pass to $SR(x'^k, \underset{k}{\vec{e}})$ which differs only by the fact that the system of basis vectors coincides with the old system of basis vectors. During transformation of only the system of basis vectors a new reference system is symbolically written down this way: $SR(x^k, \underset{k}{\vec{e}}')$, and during transformation both of the systems of coordinates, and the basis vectors - $SR(x'^k, \underset{k}{\vec{e}}')$.

If in flat three-dimensional space for investigation of several problems of geometry there are used the Cartesian coordinates and basis vectors being parallel to coordinate axis, than the reference system $SR(x^k, \underset{k}{\vec{E}})$ consists of these coordinates and basis vectors. In the same form is written down the reference system in case of using the spherical coordinates and basis vectors, relative to spherical coordinate lines, passing through a given space point. On the other side, it is clear that for investigation of geometrical problems in a considered space can be used the Cartesian coordinates and basis vectors relative to spherical coordinate lines, passing through the given space point, or vice versa - the spherical coordinates and basis vector, being parallel to the axe of Cartesian system of coordinates, in such cases the reference system is written down as $SR(x^k, \underset{k}{\vec{e}})$ (about it see below):

1.1.2. INTRODUCTION OF A METRIC [1]

Addressing of the space points - is an arbitrary operation and it is not connected with any limitations that is why it can't influence on metric properties of space. The metric properties of space being its inner characteristics are determined by symmetric, nondegenare tensor of second order g_{ij}. In particular, the length of infinitesimal contravariant vector dx^k is determined by equity:

$$ds = \sqrt{e g_{ik} dx^i dx^k} \qquad (1.1.2.1)$$

and the length of some contravariant vectors A^k - by equity

$$A = \sqrt{e g_{ik} A^i A^k} \qquad (1.1.2.2)$$

where,

$$e = \begin{cases} +1, & if \quad g_{ik} A^i A^k > 0, \\ -1, & if \quad g_{ik} A^i A^k < 0. \end{cases}$$

g_{ik} is called to be a metric tensor of the space.

By means of g_{ik} it is possible to form a covariant vector A_i, corresponding to contravariant vector A^k, in particular,

$$A_i = g_{ik} A^k. \qquad (1.1.2.3)$$

A vector character A_i can be easily proved if in the right part of final equity we make a substitution:

$$A^k = \frac{\partial x^k}{\partial x'^p} A'^p, \qquad g_{ik} = \frac{\partial x'^q}{\partial x^i} \frac{\partial x'^r}{\partial x^k} g'_{q^r}$$

we'll receive

$$A_i = \frac{\partial x'^q}{\partial x^i} \frac{\partial x'^r}{\partial x^k} \frac{\partial x^k}{\partial x^p} A'^p g'_{q^r} = \frac{\partial x'^q}{\partial x^i} g'_{pq} A'^p,$$

i.e.

$$A_i = \frac{\partial x'^q}{\partial x^i} A'_q.$$

In connection to the fact that g_{ik} are characteristic parameters for a given space the value of A_i and A^i can be considered as covariant and contravariant components of one and the same vector in a considered reference system $SR(x^k, \underset{k}{\vec{E}})$ relative to metric tensor g_{ik}. Formation of covariant components of the vector by means of contravariant components is called dipping of index. Dipping of indices can be realized as well for tensors of high orders, thus, e.g.:

$$A_{ij} = g_{ip} A_j^p, \qquad A_i^k = g_{ip} A^{pk}, \qquad A_i^{jke} = g_{ip} A^{pjke}, \ldots \qquad (1.1.2.3')$$

From covariant metric tensor g_{ik} can be formed a contravariant metric tensor of second order:

$$g^{ik} = \frac{1}{g} \Delta^{ik} \qquad (1.1.2.4)$$

where, g is a determinant:

$$g = \begin{vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{vmatrix} \qquad (1.1.2.5)$$

and Δ^{ik} – algebraic addition to the element of g_{ik} in a determinant g.

So as $g_{ik} = g_{ki}$, than it is obvious that $\Delta^{ik} = \Delta^{ki}$, that is why g^{ik} is symmetric to $g^{ik} = g^{ki}$.

It is known that

$$g_{ip}\Delta^{pk} = \delta_i^k g,$$

that is why

$$g_{ip} g^{pk} = \delta_i^k \qquad (1.1.2.6)$$

With allowance that g_{ik} and δ_i^k are tensors from this equity comes that g^{ik} is also a contravariant tensor of second order;

On the basis of the law of multiplication the determinants we have:

$$g \cdot \bar{g} = \begin{vmatrix} g_{0p}g^{p0} & g_{0p}g^{p1} & g_{0p}g^{p2} & g_{0p}g^{p3} \\ g_{1p}g^{p0} & g_{1p}g^{p1} & g_{1p}g^{p2} & g_{1p}g^{p3} \\ g_{2p}g^{p0} & g_{2p}g^{p1} & g_{2p}g^{p2} & g_{2p}g^{p3} \\ g_{3p}g^{p0} & g_{3p}g^{p1} & g_{3p}g^{p2} & g_{3p}g^{p3} \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1,$$

where,

$$\bar{g} = \begin{vmatrix} g^{00} & g^{01} & g^{02} & g^{03} \\ g^{01} & g^{11} & g^{12} & g^{13} \\ g^{02} & g^{12} & g^{22} & g^{23} \\ g^{03} & g^{13} & g^{23} & g^{33} \end{vmatrix} \qquad (1.1.2.7)$$

thus,

$$\bar{g} = \frac{1}{g} \qquad (1.1.2.8)$$

During using of tensor g^{ik} it is possible to realize the index of tensors, thus, e.g.:

$$A^i = g^{ip} A_p, \; A_i^k = g^{pk} A_{ip}, \; A^{ik} = g^{ip} A_p^k, \ldots$$

Tensor character of these values comes from the structure of right hand parts of the last equations.

From (1.1.2.2) and (1.1.1.18) it fallows that the length of a vector is covariant.

If in some points of space, this length of a vector is equal to zero $A = 0$ than we'll say that the vector is isotropic in this point, and if this equity is true in some domains than the vector field is isotropic in this domain.

If $x^i(p)$ – is some curve passing through two points $x^i(p_1)$ and $x^i(p_2)$, p_1 and p_2 – fixed values of the parameter p that the length of the curve are between these points is determined according to the equity:

$$s = \int_{p_1}^{p_2} \sqrt{e g_{ik} x'^i x'^k}\, dp\,, \qquad (1.1.2.9)$$

where p is an invariant parameter, and $\dot{x}^i = \frac{dx^i}{dp}$.

Angle between two nonisotropic vectors A^i and B^i is determined according to the following equity:

$$\cos\alpha = \frac{g_{ik} A^i B^k}{\sqrt{e g_{ik} A^i A^k}\sqrt{e g_{ik} B^i B^k}} \qquad (1.1.2.10)$$

or

$$\cos\alpha = \frac{g_{ik} A^i B^k}{AB} = \frac{A^i B_i}{AB} = \frac{A_i B^i}{AB} \qquad (1.1.2.11)$$

where, $A = \sqrt{e g_{ik} A^i A^k}$, $B = \sqrt{e g_{ik} B^i B^k}$ are the lengths of considered vectors relatively.

The condition of orthogonality of nonisotropic vectors is received from (1.1.2.11) with allowance of $\cos\alpha = 0$, in particular,

$$g_{ik} A^i B^k = 0 \qquad (1.1.2.12)$$

From the expression of $\cos\alpha$ it is seen that: 1) it is an invariant value;

2) the inequality is not always valid $|\cos\alpha| \le 1$, in some cases $|\cos\alpha| > 1$. In the first case the (1.1.2.11) determines the material value of an angle α, and in the second α has a complex value. In the latter case the (1.1.2.11) bears a formal character. It is possible to prove, that, if the matrix $\|g_{ik}\|$ is positively determined in some point (domain) of space, than in this point (domain) inequity $|\cos\alpha| \le 1$ is always valid.

Really, in a considered case in a selected point (domain) of space the inequity $g_{ik}\zeta^i\zeta^k \ge 0$ is valid for any values of parameters $\zeta^0, \zeta^1, \zeta^2, \zeta^3$. Let's represent ζ^k in the following way:

$$\zeta^k = \zeta'^k + \lambda\zeta''^k$$

where, λ – some parameter. After substitution we'll receive

$$g_{ik}\zeta'^i\zeta'^k + 2\lambda(g_{ik}\zeta'^i\zeta''^k) + \lambda^2(g_{ik}\zeta''^i\zeta''^k) \ge 0\,.$$

From this inequity it comes out that a quadrate trinomial in any part of this equation relative to arbitrary value λ – is nonnegative, this means that its discriminant is non-positive, i.e.

$$(g_{ik}\zeta'^i\zeta''^k)^2 - (g_{ik}\zeta'^i\zeta'^k)(g_{ik}\zeta''^i\zeta''^k) \le 0$$

Consequently

$$-1 \le \frac{g_{ik}\zeta'^i\zeta''^k}{\sqrt{|g_{ik}\zeta'^i\zeta'^k|}\sqrt{|g_{ik}\zeta''^i\zeta''^k|}} \le 1\,.$$

Substituting here instead of ζ'^i A^i and instead of $\zeta''^i - B^i$, it becomes evident that the condition $|\cos\alpha| \le 1$ is valid. By analogous means it is possible to show the validity of this condition in case, when the matrix $\|g_{ik}\|$ is negatively determined.

Let's rewrite the equity (1.1.2.11) this way:

$$A\cos\alpha = \frac{g_{ik} A^i B^k}{B} \qquad (1.1.2.13)$$

$A\cos\alpha$ is a projection of vector A^k to the vector B^k. Similarly, a projection of vector B^k is calculated on A^k. If B^k is a unit vector $B=1$, than from (1.1.2.13) we have:

$$A\cos\alpha = g_{ik}A^i B^k \qquad \textit{(1.1.2.14)}$$

Using the basis vectors (1.1.1.22) it is possible to determine the projection of vector A^k on coordinate lines, in particular if $\vec{B}=\vec{E}$, than from (1.1.2.13) we shall have:

$$A\cos\alpha_0 = g_{0i}A^i \big/ \sqrt{g_{00}}$$

Let's similarly determine the other projections using the $\vec{B}=\underset{2}{\vec{E}},\ \vec{B}=\underset{3}{\vec{E}},\ \vec{B}=\underset{4}{\vec{E}}$. All these projections can be written down in the form of one equity:

$$A\cos\alpha_k = g_{ki}\,A^i \big/ \sqrt{g_{kk}} \qquad \textit{(1.1.2.15)}$$

From this equation it is evident, that the projections of the vector A^k on the coordinate lines do not coincide with its components, only in one case, when $g_{ii}=1$ and $g_{ik}=0$ $(i\neq k)$ (in case of flat space with Decart system of coordinates).

$$A\cos\alpha_k = A^k \qquad \textit{(1.1.2.16)}$$

Besides, from (1.1.2.15) it follows, that totality of projection of vector A^k on coordinate lines does not constitute a vector.

If the components of vectors $\vec{A}$ and $\vec{B}$ are proportional to

$$B^k = \lambda A^k \qquad \textit{(1.1.2.17)}$$

than, from (1.1.2.10) we have

$$\cos\alpha = \frac{\lambda g_{ik}A^i A^k}{\sqrt{\lambda^2}\left|g_{ik}A^i A^k\right|}.$$

In case, when $\|g_{ik}\|$ is nonnegative definite determined matrix, from here we have:

$$\cos\alpha = \begin{cases} 1, & \lambda > 0 \\ -1, & \lambda < 0 \end{cases}$$

i.e.

$$\alpha = \begin{cases} 0, & \lambda > 0 \\ \pi, & \lambda < 0 \end{cases}.$$

Thus, in case of nonnegatively determined matrix $\|g_{ik}\|$ the $\vec{A}$ and $\vec{B}$ vectors meeting the requirements of (1.1.2.17) – are parallel. In this connection the condition (1.1.2.17) is called the condition of parallelism in any case.

By means of a tensor g_{ik} there can be also formed other invariants, having a great value in appendix. For example, from the equity

$$g'_{ik} = \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^k} g_{pq},$$

it is evident that

$$g' = \frac{1}{I^2} g \qquad \textit{(1.1.2.18)}$$

where,

$$I = \frac{\partial(x'^0, x'^1, x'^2, x'^3)}{\partial(x^0, x^1, x^2, x^3)}$$

is a Iakobean transformation (1.1.1.1). On the other hand, as it is known, the following equation is valid:

$$dx'^0 dx'^1 dx'^2 dx'^3 = I dx^0 dx^1 dx^2 dx^3.$$

If here instead of I we substitute its value form (1.1.2.18), than we'll receive :

$$\sqrt{g'}dx'^0 dx'^1 dx'^2 dx'^3 = \sqrt{g}dx^0 dx^1 dx^2 dx^3 \qquad (1.1.2.19)$$

i.e. $\sqrt{g}dx^0 dx^1 dx^2 dx^3$ - is invariant. The final equity has an important value during integration.

1.1.3. GEODESICAL LINE [1]

As it is known, in flat space between two points there can be traced multiple arcs of various lines, among which a section of the straight line is the shortest in length. The straight line is called a geodesical line of flat space. Let's consider the task for curved space.

According to (1.1.2.9) length of the arc of any curved line, passing through the point $x^i(p_1)$ and $x^i(p_2)$ is equal to:

$$S = \int_{p_1}^{p_2} \sqrt{e g_{ik} \dot{x}^i \dot{x}^k}\, dp \qquad (1.1.3.1)$$

From all-possible curves, passing through the indicated point let's select the subsystem of curves for which e maintains the constant symbol on the section of line between the points $x^i(p_1)$ and $x^i(p_2)$. Let's introduce again the parameter s - the current length of a line, been read from the point $x^i(p_1)$. In future we shall mean, that s - function of the parameter p, i.e. $s = s(p), x^i = x^i[s(p)]$.

The task determined by the geodesical line is reduced to minimization of the functional (1.1.3.1). From this equity it is evident that sub-integral expression

$$L = \sqrt{e g_{ik} \dot{x}^i \dot{x}^k} \qquad (1.1.3.2)$$

is nonnegative value and that is why there exists minimum functional S, which is attained for $x^i(p)$, satisfying the equations of Euler:

$$\frac{d}{dp}\left(\frac{\partial L}{\partial \dot{x}^i}\right) - \frac{\partial L}{\partial x^i} = 0 \qquad (1.1.3.3)$$

From (1.1.3.2) it is evident, that

$$\frac{\partial L}{\partial \dot{x}^i} = e g_{ik} \dot{x}^k \frac{dp}{ds}, \qquad (1.1.3.4)$$

$$\frac{dL}{\partial x^i} = \frac{1}{2} e \frac{\partial g_{jk}}{\partial x^i} x'^j x'^k \frac{dp}{ds} \qquad (1.1.3.5)$$

After substitution of (1.1.3.4) and (1.1.3.5) into (1.1.3.3) we shall receive:

$$g_{ik}\ddot{x}^k + \frac{\partial g_{ij}}{\partial x^k}\dot{x}^j \dot{x}^k - \frac{1}{2}\frac{\partial g_{jk}}{\partial x^i}\dot{x}^j \dot{x}^k - g_{ik}\dot{x}^k \frac{\frac{d^2 s}{dp^2}}{\frac{ds}{dp}} = 0 \qquad (1.1.3.6)$$

With allowance that $g_{ik} = g_{ki}$, the second member of the left part can be represented in the following form:

$$\frac{\partial g_{ij}}{\partial x^k}\dot{x}^j \dot{x}^k = \frac{1}{2}\left(\frac{\partial g_{ij}}{\partial x^k} + \frac{\partial g_{ki}}{\partial x^j}\right)\dot{x}^j \dot{x}^k$$

Then the equation (1.1.3.6) is reduced to the system:

$$g_{ik}\ddot{x}^k + \frac{1}{2}\left(\frac{\partial g_{ij}}{\partial x^k} + \frac{\partial g_{ki}}{\partial x^j} - \frac{\partial g_{kj}}{\partial x^i}\right)\dot{x}^j\dot{x}^k - g_{ik}\dot{x}^k \frac{\frac{d^2s}{dp^2}}{\frac{ds}{dp}} = 0 .$$

From this, after multiplying by g^{ei} and convolution on index i we'll finally obtain:

$$\ddot{x}^l + \Gamma^l_{jk}\dot{x}^j\dot{x}^k - \dot{x}^l \frac{\frac{d^2s}{dp^2}}{\frac{ds}{dp}} = 0 , \qquad (1.1.3.7)$$

where,

$$\Gamma^k_{ij} = \frac{1}{2} g^{kp}\left(\frac{\partial g_{ip}}{\partial x^j} + \frac{\partial g_{jp}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^p}\right). \qquad (1.1.3.8)$$

These values are called the Christoffel symbols of the second type. It is evident, that

$$\Gamma^k_{ij} = \Gamma^k_{ji} . \qquad (1.1.3.9)$$

If the parameter p coincides with s $(s = p)$, which, in considered case, is the current length of the arc of geodesical line, than from (1.1.3.7) we'll receive more than a simple system of the equations of nonisotropic geodesical lines:

$$\ddot{x}^k + \Gamma^k_{ij}\dot{x}^i\dot{x}^j = 0 , \qquad (1.1.3.10)$$

where,

$$\dot{x}^i = \frac{dx^i}{ds}, \quad \ddot{x}^i = \frac{d^2x^i}{ds^2}$$

The isotropic geodesical line is determined also according to the system (1.1.3.10), in which, under s there is meant some invariant parameter.

It should be noted, that the type of geodesical line is unambiguously determined along the initial point x^i_0 and to the direction of $\dot{x}^i_0$, in other words, solution of the system (1.1.3.10) meeting the following initial conditions:

$$x^i = x^i_0, \ \frac{dx^i}{ds} = x'^i_0 \quad \text{at} \quad s = 0 \qquad (1.1.3.11)$$

is the only.

The sought for functions $x^i(s)$ in the vicinity of initial point x^i_0 can be represented in the following form:

$$x^i(s) = x^i_0 + \frac{\dot{x}^i_0}{1!}s + \frac{1}{2!}\left(\frac{d^2x^i}{ds^2}\right)_0 s^2 + ... = \sum_{k=0}^{\infty}\frac{s^k}{k!}\left(\frac{d^kx^i}{ds^k}\right)_0 \qquad (1.1.3.12)$$

In the right hand side x^i_0 and $\dot{x}^i_0$ are known values according to (1.1.3. 11), and the other $\left(\frac{d^kx^i}{ds^k}\right)_0$ $k \geq 2$ are determined from (1.1.3.10) while using (1.1.3.11). Really:

$$\left(\frac{d^2x^i}{ds^2}\right)_0 = -\Gamma^i_{jk}\dot{x}^j\dot{x}^k \big/ s = 0,$$

$$\left(\frac{d^3x^i}{ds^3}\right)_0 = -\frac{d}{ds}\Gamma^i_{jk}\dot{x}^j\dot{x}^k \big/ s = 0, ...$$

The right hand sides of these equities are determined in succession with allowance of (1.1.3.11).

It is easy to show, that one of the first integrals of the system (1.1.3.10) has the following form:

$$g_{kp}\dot{x}^k\dot{x}^p = const \qquad (1.1.3.13)$$

i.e., the value of $I = g_{kp}\dot{x}^k\dot{x}^p$ *(1.1.3.14)*

maintains a constant value along the geodesical line. Indeed, from (1.1.3.14) we have:

$$\frac{dI}{ds} = 2g_{ki}\ddot{x}^k\dot{x}^i + \frac{\partial g_{pi}}{\partial x^j}\dot{x}^p\dot{x}^i\dot{x}^j$$

In the right hand side of this equity instead of $\ddot{x}^k$ let's substitute its value from (1.1.3.10), than we'll receive:

$$\frac{dI}{ds} = \left(\frac{\partial g_{pi}}{\partial x^j} - 2g_{ki}\Gamma^k_{pj}\right)\dot{x}^p\dot{x}^i\dot{x}^j = 0,$$

i.e. along the geodesical line the condition (1.1.3.13) is valid.

1.1.4. COVARIANT DIFFERENTIATION [1]

The question discussed here is closely related to some features of Christoffel symbols, that is why we shall try to bring here some basic of them.

Let's calculate Γ^i_{ik}. From the (1.1.3.8) it is evident that:

$$\Gamma^i_{ik} = \frac{1}{2}g^{ip}\left(\frac{\partial g_{ip}}{\partial x^k} + \frac{\partial g_{kp}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^p}\right).$$

In the last member of the right part let's represent the indices i and p taking into account, that $g^{ip} = g^{pi}$, than for Γ^i_{ik} we'll receive:

$$\Gamma^i_{ik} = \frac{1}{2}g^{ip}\frac{\partial g_{ip}}{\partial x^k} \qquad (1.1.4.1)$$

On the other hand, according to determination of determinants it is evident, that

$$\frac{\partial g}{\partial x^k} = \Delta^{ip}\frac{\partial g_{ip}}{\partial x^k},$$

where, Δ^{ip} is an algebraic addition of the elements of g_{ip} in a determinant g, i.e.

$$\Delta^{ip} = g \cdot g^{ip}.$$

After substitution from (1.1.4.1) we'll receive:

$$\Gamma^i_{ik} = \frac{1}{2}\frac{1}{g}\frac{\partial g}{\partial x^k}$$

or

$$\Gamma^i_{ik} = \frac{\partial \ln\sqrt{|g|}}{\partial x^k} \qquad (1.1.4.2)$$

Transformation law of Christoffel symbols during transformation of the system of coordinates it is possible to determine from the equation (1.1.3.8). With this aim let's differentiate the equity:

$$g'_{ik} = \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^k}g_{pq},$$

by x'^l -

$$\frac{\partial g'_{ik}}{\partial x'^l} = \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^k}\frac{\partial x^r}{\partial x'^l}\frac{\partial g_{pq}}{\partial x^r} + \frac{\partial^2 x^p}{\partial x'^i \partial x'^l}\frac{\partial x^q}{\partial x'^k} g_{pg} + \frac{\partial x^p}{\partial x'^i}\frac{\partial^2 x^q}{\partial x'^k \partial x'^l} g_{pg}$$

From here:

$$\Gamma'^k_{ij} = \frac{1}{2}\frac{\partial x'^k}{\partial x^t}\frac{\partial x'^l}{\partial x^s} g^{ts}\frac{\partial g_{pq}}{\partial x^r}\left(\frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^l}\frac{\partial x^r}{\partial x'^j} + \frac{\partial x^p}{\partial x'^l}\frac{\partial x^q}{\partial x'^j}\frac{\partial x^r}{\partial x'^i} - \right.$$

$$\left. - \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^j}\frac{\partial x^r}{\partial x'^l}\right) + g'^{kl} g_{pq}\frac{\partial x^q}{\partial x'^l}\frac{\partial^2 x^p}{\partial x'^i \partial x'^j}.$$

These equations can be rewritten as follows:

$$\frac{\partial x^k}{\partial x'^l}\Gamma'^l_{ij} = \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^j}\Gamma^k_{pq} + \frac{\partial^2 x^k}{\partial x'^i \partial x'^j} \qquad (1.1.4.3)$$

From this transformation law it is evident that Christoffel symbols do not constitute a tensor value.

Let's differentiate the equity

$$A^i = \frac{\partial x^i}{\partial x'^p} A'^p \qquad (1.1.4.4)$$

by x^j

$$\frac{\partial A^i}{\partial x^j} = \frac{\partial x^i}{\partial x'^p}\frac{\partial x'^q}{\partial x^j}\frac{\partial A'^p}{\partial x'^q} + \frac{\partial^2 x^i}{\partial x'^q \partial x'^p}\frac{\partial x'^q}{\partial x^j} A'^p .$$

In the right hand let's change the second derivative by its values from (1.1.4.3)

$$\frac{\partial^2 x^i}{\partial x'^p \partial x'^q} = \frac{\partial x^i}{\partial x'^l}\Gamma'^l_{pq} - \frac{\partial x^t}{\partial x'^p}\frac{\partial x^s}{\partial x'^q}\Gamma^i_{ts} . \qquad (1.1.4.5)$$

After substitution we'll obtain:

$$\frac{\partial A^i}{\partial x^j} = \frac{\partial x^i}{\partial x'^p}\frac{\partial x'^q}{\partial x^j}\frac{\partial A'^p}{\partial x'^q} + \frac{\partial x^i}{\partial x'^l}\frac{\partial x'^q}{\partial x^j}\Gamma'^l_{pq} A'^p - \frac{\partial x'^q}{\partial x^j}\frac{\partial x^t}{\partial x'^p}\frac{\partial x^s}{\partial x'^q}\Gamma^i_{ts} A'^p$$

From it is clear that:

$$\frac{\partial A^i}{\partial x^j} + \Gamma^i_{pj} A^p = \frac{\partial x^i}{\partial x'^p}\frac{\partial x'^q}{\partial x^j}\left(\frac{\partial A'^p}{\partial x'^q} + \Gamma^p_{\ell q} A'^l\right) \qquad (1.1.4.6)$$

Thus, the tensor value is not a simple derivative of the vector A^i, but the values

$$A,^i_j = \frac{\partial Ai}{\partial x^j} + \Gamma^i_{pj} A^p , \qquad (1.1.4.7)$$

which are called the covariant derivative of a contravariant vector.

In full analogy is determined the surface derivative of covariant vector A_i, in particular

$$A_{i\,,\,j} = \frac{\partial A_i}{\partial x^j} - \Gamma^p_{ij} A_p . \qquad (1.1.4.8)$$

From which it is clear, that:

$$A_{i\,,\,j} - A_{j\,,\,i} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i} \qquad (1.1.4.9)$$

The covariant derivatives of tensors of the second order are determined according to following equities:

$$A_{ij\,,\,k} = \frac{\partial A_{ij}}{\partial x^k} - \Gamma^p_{ik} A_{pj} - \Gamma^p_{jk} A_{ip} , \quad A,^{ij}_k = \frac{\partial A^{ij}}{\partial x^k} +$$

$$+ \Gamma^i_{kp} A^{pj} + \Gamma^j_{kp} A^{ip} , \; A^i_{j\cdot k} = \frac{\partial A^i_j}{\partial x^k} + \Gamma^i_{pk} A^p_j - \Gamma^p_{jk} A^i_p . \qquad (1.1.4.10)$$

These formula are easily generalized for tensors of second order.

1.1.5. CURVATURE TENSOR AND TENSOR OF RICHI [1]

Let's differentiate (1.1.4.5) according to x'^r:

$$\frac{\partial^3 x^i}{\partial x'^p \partial x'^q \partial x'^r} = \frac{\partial x^i}{\partial x'^l}\frac{\partial \Gamma'^l_{pq}}{\partial x'^r} - \frac{\partial x^t}{\partial x'^p}\frac{\partial x^s}{\partial x'^q}\frac{\partial x^k}{\partial x'^r}\frac{\partial \Gamma^i_{st}}{\partial x^k} +$$
$$+\frac{\partial^2 x^i}{\partial x'^l \partial x'^r}\Gamma'^l_{pq} - \frac{\partial^2 x^t}{\partial x'^p \partial x'^r}\frac{\partial x^s}{\partial x'^q}\Gamma^i_{ts} - \frac{\partial^2 x^s}{\partial x'^q \partial x'^r}\frac{\partial x^t}{\partial x'^p}\Gamma^i_{ts}$$

If here we substitute the value of the second derivatives from (1.1.4.5), permutate the indices q and r and exclude the third derivative, finally we'll receive:

$$R'^k_{\ ijl} = \frac{\partial x'^k}{\partial x^p}\frac{\partial x^q}{\partial x'^i}\frac{\partial x^s}{\partial x'^j}\frac{\partial x^t}{\partial x'^l}R^p_{\ qst}\ , \qquad (1.1.5.1)$$

where:

$$R^k_{\ ijl} = \frac{\partial \Gamma^k_{il}}{\partial x^j} - \frac{\partial \Gamma^k_{ij}}{\partial x^l} + \Gamma^p_{il}\Gamma^k_{pj} - \Gamma^p_{ij}\Gamma^k_{pl}\ , \qquad (1.1.5.2)$$

$R'^k_{\ ijl}$ – is expressed through Γ'^k_{ij}.

From (1.1.5.1) it is clear that $R^k_{\ ijl}$ constitutes one-time contravariant and three-times covariant mixed tensor of fourth order. In literature, according to Reimannian geometry it is known as a curvature tensor or tensor of Reimannian. From the structure $R^k_{\ ijl}$ it is easy to notice that the components of curvature tensor are antisymmetric relative to indices j and l, i.e.

$$R^k_{\ ijl} = -R^k_{\ ilj} \qquad (1.1.5.3)$$

Convolution $R^k_{\ ijl}$ according to indices k and l will give a covariant tensor of second order:

$$R_{ij} = R^l_{\ ijl} = \frac{\partial \Gamma^l_{li}}{\partial x^j} - \frac{\partial \Gamma^l_{ij}}{\partial x^l} + \Gamma^p_{il}\Gamma^l_{pj} - \Gamma^p_{ij}\Gamma^l_{pl}\ , \qquad (1.1.5.4)$$

which is called a Tensor of Richi.

By means of Richi tensor it is possible to form a spectrum:

$$R = g^{ij}R_{ij} \qquad (1.1.5.5)$$

known under the name of velocity curvature.

1.2. SOME MAIN ELEMENTS OF GEOMETRY OF NONHOLONOMIC TRANSFORMERS

1.2.1. NONHOLONOMIC TRANSFORMATIONS NONHOLONOMIC TENSOR ALGEBRA [2,3]

In previous paragraph there have been considered some main problems of Riemannian geometry, according to which an inner structure of the space unambiguously is characterized by metric tensor g_{ik}. Readdressing of points, i.e. transition from one state of reference system $SR(x^k, \underset{k}{\vec{E}})$ into another does not change the geometrical features of the space, to be invariant are left such parameters, as: length of vector, angle between two vectors, etc. which can be determined by means of direct measurement. In this respect, all the reference systems $SR(x^k, \underset{k}{\vec{E}})$ are equivalent. Such reference systems in future will be called holonomic.

Let's consider the reference system of more common type of $SR(x^k, \underset{k}{\vec{e}})$ in which $\underset{k}{\vec{e}}$ is the arbitrary linearly independent 4 vectors. According to (1.1.1.28) the components of some contravariant vectors $\underset{e}{A^k}$ in $SR(x^k, \underset{k}{\vec{e}})$ are of following type[1]:

$$\underset{e}{A^k} = \alpha_p^k \underset{E}{A^p}, \qquad (1.2.1.1)$$

where:

$$\alpha_i^k = \frac{\Delta_i^k}{\Delta}, \qquad \Delta = \begin{vmatrix} \underset{1}{e^0} & \underset{1}{e^1} & \underset{1}{e^2} & \underset{1}{e^3} \\ \underset{2}{e^0} & \underset{2}{e^1} & \underset{2}{e^2} & \underset{2}{e^3} \\ \underset{3}{e^0} & \underset{3}{e^1} & \underset{3}{e^2} & \underset{3}{e^3} \\ \underset{4}{e^0} & \underset{4}{e^1} & \underset{4}{e^2} & \underset{4}{e^3} \end{vmatrix} \qquad (1.2.1.2)$$

And Δ_i^k – an algebraic addition to the element e_k^i in a determinant Δ. Completely similarly, in $SR(x^k, \underset{k}{\vec{e}'})$ well have:

$$\underset{e}{A'^i} = \alpha_p'^i \underset{E}{A^p} \qquad (1.2.1.3)$$

where:

$$\alpha_i'^k = \frac{\Delta_i'^k}{\Delta'}, \qquad \Delta' = \begin{vmatrix} \underset{1}{e'^0} & \underset{1}{e'^1} & \underset{1}{e'^2} & \underset{1}{e'^3} \\ \underset{2}{e'^0} & \underset{2}{e'^1} & \underset{2}{e'^2} & \underset{2}{e'^3} \\ \underset{3}{e'^0} & \underset{3}{e'^1} & \underset{3}{e'^2} & \underset{3}{e'^3} \\ \underset{4}{e'^0} & \underset{4}{e'^1} & \underset{4}{e'^2} & \underset{4}{e'^3} \end{vmatrix} \neq 0$$

And $\Delta_i'^k$ – is the algebraic addition of the element $e_k'^i$ in the determinant Δ'. It is obvious, that

$$\underset{p}{e^i} \alpha_k^p = \delta_k^i, \qquad \underset{p}{e'^i} \alpha_k^p = \delta_k^i \qquad (1.2.1.4)$$

According to (1.2.1.4) and (1.2.1.1) we have:

$$\underset{E}{A^p} = \underset{k}{e^p} \underset{e}{A^k} \qquad (1.2.1.5)$$

Substituting this value in to (1.2.1.3), we'll receive:

$$\underset{e}{A'^i} = a_p^i \underset{e}{A^p} \qquad (1.2.1.6)$$

where,

$$a_k^i = \alpha_p'^i \underset{k}{e^p} \qquad (1.2.1.7)$$

These a_k^i coefficients determine the transformation law not only of contravariant vectors, but of any tensors, independently from their character and order.

From (1.2.1.7) with allowance of (1.2.1.4) , we have:

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[1] To differ the values in various reference frames we'll apply temporarily the lower indices "e" and "E". For example $\underset{e}{A^k}$ is a contravariant vector in the system $SR(x^k, \underset{k}{\vec{e}})$ and $\underset{E}{A^k}$ – in the system $SR(x^k, \underset{k}{\vec{E}})$.

$$\underset{p}{e}'^{i}\, a_k^p = \underset{k}{e}^{i} \quad \text{and} \quad \underset{k}{e}'^{i} = \bar{a}_k^p\, \underset{p}{e}^{i} \qquad (1.2.1.8)$$

$$a_p^k \alpha_i^p = \alpha_i'^k \quad \text{and} \quad \alpha_i^k = \bar{a}_p^k \alpha_i'^p \qquad (1.2.1.9)$$

where, $\left\|\bar{a}_i^k\right\|$ – is a reciprocal matrix of the matrix $\left\|a_i^k\right\|$.

dx^i – is an infinitesimal contravariant vector in the system $SR(x^k, \underset{k}{\vec{E}})$. This vector we shall refer to a point with the coordinates x^k. Components of this vector in the reference system $SR(x^k, \underset{k}{\vec{e}})$, according to (1.2.1.1) will be $\alpha_p^k dx^p$. These infinitesimals comprise 16 function $\alpha_i^k(x^0, x^1, x^2, x^3)$, which, to say generally, do not meet the requirements of differentiability of considered infinitesimal components, i.e.:

$$\frac{\partial \alpha_i^k}{\partial x^l} \neq \frac{\partial \alpha_l^k}{\partial x^i} \qquad (1.2.1.10)$$

That is why, in such cases, the values of $\alpha_p^k dx^p$ are not the differentials of some functions.

Thus to the infinitesimal contravariant vector, dx^k the components of which are exact differentials in $SR(x^k, \underset{k}{\vec{E}})$, in $SR(x^k, \underset{k}{\vec{e}})$ – correspond as well to the infinitesimal components:

$$d\underset{e}{x}^k = \alpha_p^k dx^p \qquad (1.2.1.11)$$

of which in force of (1.2.1.10) are not exact differentials of some functions [4].

In full analogy, the components of considered vector in $SR(x^k, \underset{k}{\vec{e}}')$ are determined by the equities:

$$d\underset{e}{x}'^k = \alpha_p'^k dx^p \qquad (1.2.1.12)$$

which are infinitesimal, but not the exact differentials.

From (1.2.1.11) and (1.2.1.12), with allowance of previous equities, we have:

$$d\underset{e}{x}'^k = a_p^k\, d\underset{e}{x}^p \qquad (1.2.1.13)$$

i.e. $d\underset{e}{x}^k$ and $d\underset{e}{x}'^k$ are the components of one contravariant vector in $SR(x^k, \underset{k}{\vec{e}})$ and $SR(x^k, \underset{k}{\vec{e}}')$ correspondingly.

It should be noted that the coefficients of transformations a_i^k, according to (1.2.1.7), to say generally, meet the following equations:

$$\frac{\partial a_i^k}{\partial x^j} \neq \frac{\partial a_j^k}{\partial x^i} \qquad (1.2.1.14)$$

In this connection (1.2.1.6) (or 1.2.1.13) in future we'll call nonholonomic transformation. The holonomoic transformation (1.1.1.1), which is used in Reimannian geometry, may be as well written in infinitesimal parameters:

$$dx'^k = \frac{\partial x'^k}{\partial x^p} dx^p \qquad (1.2.1.15)$$

The equity (1.2.1.8) establishes the contact between the basic vectors $\underset{k}{\vec{e}}$ and $\underset{k}{\vec{e}}'$, that is why the nonholonomic transformation should not be explained as the transformation of coordinates (thy, at the same time can remain unchanged) but as a transformation of basic vectors.

Here, it should be mentioned one important circumstance.

According to (1.2.1.7) a_i^k are the composition of α_i^k and $\alpha_i'^k$ (or $\underset{i}{e}^k$ and $\underset{i}{e}'^k$) functions, meeting the requirements of (1.2.1.10) that is why in some cases α_i^k functions can meet the requirements of integrity (holonomicity)

$$\frac{\partial a_i^k}{\partial x^j} = \frac{\partial a_j^k}{\partial x^i} \tag{1.2.1.16}$$

For example, if α_i^k (or $\underset{k}{e}^k$) are the prescribed functions than it is always possible to select such $\alpha_i'^k$ (or $\underset{k}{e}'^k$) functions, for which the conditions of (1.2.1.16) for a_i^k are fulfilled. Really, selecting in advance such a_i^k functions, which during all fixed values of the index "k" meet the requirements of (1.2.1.16), than the corresponding values of the function $\alpha_i'^k$ are determined from (1.2.1.9).

In such cases any contravariant vectors including the infinitesimal contravariant vectors (infinitesimal of dx^k and dx'^k, are not the exact differentials), are connected between each other by means of holonomic transformations.

Let's admit that $u(x^0, x^1, x^2, x^3)$ is a scalar. During transition from the point x^k into point $x^k + dx^k$ it receives the increment

$$du = \frac{\partial u}{\partial x^k} dx^k \tag{1.2.1.17}$$

It is evident, that du is also a scalar, that is why its value in $SR(x^k, \underset{k}{\vec{e}})$ is not changed. Similarly to partial derivatives, valid for $SR(x^k, \underset{k}{\vec{E}})$, let's introduce the partial derivatives in $SR(x^k, \underset{k}{\vec{e}})$, as relation of infinitesimal du to $d\underset{e}{x}^k$, when the whole $d\underset{e}{x}^j = 0$, when $j \neq k$ i.e.:[1]

$$\frac{\partial u}{\partial \underset{e}{x}^k} = \frac{du}{d\underset{e}{x}^k} \qquad \text{when} \qquad d\underset{e}{x}^j = 0, \qquad j \neq k$$

If, here instead of du we'll substitute its value from (1.2.1.17) we'll receive:

$$\frac{\partial u}{\partial \underset{e}{x}^k} = \frac{\partial u}{\partial x^k} \frac{dx^p}{d\underset{e}{x}^k} \tag{1.2.1.18}$$

For further transformation of this equity, let's use the (1.2.1.11), from which

$$dx^p = \underset{j}{e}^p \, d\underset{e}{x}^j$$

From here $d\underset{e}{x}^k \neq 0$ and $d\underset{e}{x}^j = 0$ $(j \neq k)$ is clear, that:

$$\frac{dx^p}{d\underset{e}{x}^k} = \underset{k}{e}^p \tag{1.2.1.19}$$

Than, from (1.2.1.18) we have:

$$\frac{\partial u}{\partial \underset{e}{x}^k} = \underset{k}{e}^p \frac{\partial u}{\partial x^p} \tag{1.2.1.20}$$

In full analogy, $SR(x^k, \underset{k}{\vec{e}}')$ we have:

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[1] For generalized partial derivatives, in the sense indicated here, in future we'll apply the symbol "$\partial / \partial \underset{e}{x}^k$".

$$\frac{\partial u}{\partial \underset{e}{x}'^{k}} = \underset{k}{e}'^{p} \frac{\partial u}{\partial x^{p}} \qquad (1.2.1.21)$$

From (1.2.1.20) and (1.2.1.21) it is evident, that:

$$\frac{\partial u}{\partial x'^{k}} = \underset{k}{e}'^{p} \alpha_{p}^{q} \frac{\partial u}{\partial \underset{e}{x}^{q}} \qquad (1.2.1.22)$$

Which, according to (1.2.1.8) is possible to rewrite thus:

$$\frac{\partial u'}{\partial \underset{e}{x}'^{k}} = \bar{a}_{k}^{p} \frac{\partial u}{\partial \underset{e}{x}^{p}} \qquad (1.2.1.23)$$

This latter equity determines the transformation law of generalized partial derivatives of scalar function during the transition from $SR(x^k, \underset{k}{\vec{e}})$ into $SR(x^k, \underset{k}{\vec{e}}')$.

In this connection, by analogy to the previous, the values of $\underset{e}{A_i}$ constitute a covariant vector, if its components $\underset{e}{A_i'}$ in the reference system $SR(x^k, \underset{k}{\vec{e}}')$ are calculated according to the formula:

$$\underset{e}{A}'_{i} = \bar{a}_{i}^{p} \underset{e}{A}_{p} \qquad (1.2.1.24)$$

From (1.2.1.6) and (1.2.1.24) it is seen, that in case of nonholonomic transformations the vector values are transformed completely analogously to the case of holonomic transformations. These transformations differ from each other only by coefficients; In holonomic transformations there are used $\frac{\partial x'^{k}}{\partial x^{i}}$ and $\frac{\partial x^{k}}{\partial x'^{i}}$, and in nonholonomic ones a_i^k and $\bar{a}_i^k$.

Tensors of nonholonomic transformations are determined in full similarity, in particular, if

$$\begin{aligned} \underset{e}{A}'^{k_1 k_2 \ldots k_n} &= a_{p_1}^{k_1} a_{p_2}^{k_2} \ldots a_{p_n}^{k_n} \underset{e}{A}^{p_1 p_2 \ldots p_n}, \\ \underset{e}{A}'_{i_1 i_2 \ldots i_n} &= \bar{a}_{i_1}^{p_1} \bar{a}_{i_2}^{p_2} \ldots \bar{a}_{i_n}^{p_n} \underset{e}{A}_{p_1 p_2 \ldots p_n}, \\ \underset{e}{A}'^{k_1 k_2 \ldots k_n}_{i_1 i_2 \ldots i_m} &= a_{p_1}^{k_1} a_{p_2}^{k_2} \ldots a_{p_n}^{k_n} \bar{a}_{i_1}^{q_1} \bar{a}_{i_2}^{q_2} \ldots \bar{a}_{i_m}^{q_m} \underset{e}{A}^{p_1 p_2 \ldots p_n}_{q_1 q_2 \ldots q_m}, \end{aligned} \qquad (1.2.1.25)$$

than $\underset{e}{A}^{k_1 k_2 \ldots k_n}$ – is a contravariant tensor of n order,

$\underset{e}{A}_{i_1 i_2 \ldots i_m}$ – covariant tensor of n order, and

$\underset{e}{A}^{k_1 k_2 \ldots k_n}_{i_1 i_2 \ldots i_m}$ – n-times contravariant and m-times covariant (mixed) tensor.

As it was noted above, the length of infinitesimal of the curve ds is invariant value in all reference systems $SR(x^k, \underset{k}{\vec{E}})$. Requirements to invariance ds in all reference systems $SR(x^k, \underset{k}{\vec{e}})$ leads to the equities:

$$\underset{e}{g}_{ke} = \underset{k}{e}^{p} \underset{e}{e}^{q} \underset{E}{g}_{pq}, \qquad \underset{e}{g}'_{ke} = \underset{k}{e}'^{p} \underset{e}{e}'^{q} \underset{E}{g}_{pq} \qquad (1.2.1.26)$$

From here, after exclusion a tensor $\underset{E}{g}_{pq}$, we'll receive

$$\underset{e}{g}'_{ke} = \bar{a}_{k}^{p} \bar{a}_{e}^{q} \underset{e}{g}_{pq} \qquad (1.2.1.27)$$

i.e. $\underset{e}{g}_{ke}$ – represents a covariant tensor relative to nonholonomomic transformations.

Length of a vector $\underset{e}{A}^i$ and cosine of an angle between two vectors $\underset{e}{A}^i$ and $\underset{e}{B}^k$ are determined according to formula:

$$\underset{e}{A}^2 = e\underset{e}{g}_{ik}\underset{e}{A}^i\underset{e}{A}^k$$

$$\cos\alpha = \frac{\underset{e}{g}_{ik}\underset{e}{A}^i\underset{e}{B}^k}{\sqrt{e\underset{e}{g}_{ik}\underset{e}{A}^i\underset{e}{A}^k}\sqrt{e\underset{e}{g}_{ik}\underset{e}{B}^i\underset{e}{B}^k}} \qquad (1.2.1.28)$$

These values are invariant.

From (1.2.1.27) we have:

$$\underset{e}{g}' = \frac{1}{a^2}\underset{e}{g} \qquad (1.2.1.29)$$

where, a is a determinant, composed of elements a_i^k. On the other hand, from (1.2.1.13) we have:

$$d\underset{e}{x}'^0\, d\underset{e}{x}'^1\, d\underset{e}{x}'^2\, d\underset{e}{x}'^3 = a\,d\underset{e}{x}^0\, d\underset{e}{x}^1\, d\underset{e}{x}^2\, d\underset{e}{x}^3 \qquad (1.2.1.30)$$

Comparing (1.2.1.29) and (1.2.1.30) we'll receive:

$$\sqrt{|\underset{e}{g}'|}\, d\underset{e}{x}'^0\, d\underset{e}{x}'^1\, d\underset{e}{x}'^2\, d\underset{e}{x}'^3 = \sqrt{|\underset{e}{g}|}\, d\underset{e}{x}^0\, d\underset{e}{x}^1\, d\underset{e}{x}^2\, d\underset{e}{x}^3 \qquad (1.2.1.30)$$

i.e.

$$\sqrt{|\underset{e}{g}|}\, d\underset{e}{x}^0\, d\underset{e}{x}^1\, d\underset{e}{x}^2\, d\underset{e}{x}^3 \qquad (1.2.1.31)$$

invariant. It has an important application in calculations of integrals.

1.2.2. ELEMENTS OF NONHOLONOMIC TENSOR ANALYSIS [3,5]

The main parameters of tensor analysis of Riemannian geometry, as it was shown above, are the symbols of Christoffel, that is why this paragraph we shall start with generalization of these symbols in case of nonholonomic transformations.

In $SR(x^k, \underset{k}{\vec{e}})$ the Christoffel symbols let's formally determine completely analogous to the reference system $SR(x^k, \underset{k}{\vec{E}})$:

$$\underset{e}{\Gamma}{}^k_{ij} = \frac{1}{2}g^{kp}\left(\frac{\partial \underset{e}{g}_{ip}}{\partial \underset{e}{x}^j} + \frac{\partial \underset{e}{g}_{jp}}{\partial \underset{e}{x}^i} - \frac{\partial \underset{e}{g}_{ij}}{\partial \underset{e}{x}}\right) \qquad (1.2.2.1)$$

If we apply (1.2.1.27) for $\underset{e}{\Gamma}{}'^k_{ij}$ we'll receive:

$$\underset{e}{\Gamma}{}'^k_{ij} = \frac{1}{2}a^k_m a^l_n \underset{e}{g}^{mn}\left(\overline{a}_i^p\overline{a}_e^q\overline{a}_i^r + \overline{a}_e^p\overline{a}_j^q\overline{a}_i^r - \overline{a}_i^p\overline{a}_j^q\overline{a}_\ell^r\right)\frac{\partial \underset{e}{g}_{pq}}{\partial \underset{e}{x}'^r} +$$

$$+\frac{1}{2}a^k_m a^l_n \underset{e}{g}^{mn}\left(\overline{a}_l^q\frac{\partial \overline{a}_i^p}{\partial \underset{e}{x}'^j} + \overline{a}_j^q\frac{\partial \overline{a}_l^p}{\partial \underset{e}{x}'^i} - \overline{a}_j^q\frac{\partial \overline{a}_i^p}{\partial \underset{e}{x}'^l}\right)\underset{e}{g}_{pq} +$$

$$+\frac{1}{2}a_m^k a_n^l \underset{e}{g}^{mn}\left(\bar{a}_i^p \frac{\partial \bar{a}_l^q}{\partial \underset{e}{x}'^j}+\bar{a}_l^p \frac{\partial a_j^q}{\partial \underset{e}{x}'^i}-a_i^p \frac{\partial \bar{a}_j^q}{\partial \underset{e}{x}'^l}\right)\underset{e}{g}_{pq}\ .$$

With allowance, that $a_i^p \bar{a}_p^k=\delta_i^k$, than from this it follows:

$$2\left(\underset{e}{\Gamma}'^k_{ij}-a_m^k \bar{a}_i^p \bar{a}_j^q \underset{e}{\Gamma}^m_{pq}\right)=a_m^k\left(\frac{\partial \bar{a}_i^m}{\partial \underset{e}{x}'^j}+\frac{\partial \bar{a}_j^m}{\partial \underset{e}{x}'^i}\right)+$$
$$+a_m^k a_n^l \underset{e}{g}^{mn}\underset{e}{g}_{pq}\left[\bar{a}_i^p\left(\frac{\partial \bar{a}_l^q}{\partial \underset{e}{x}'^j}-\frac{\partial \bar{a}_j^q}{\partial \underset{e}{x}'^l}\right)+\bar{a}_j^q\left(\frac{\partial \bar{a}_l^p}{\partial \underset{e}{x}'^i}-\frac{\partial \bar{a}_i^p}{\partial \underset{e}{x}'^l}\right)\right] \qquad (1.2.2.2)$$

This equity determines the transformation law of symbols of $\underset{e}{\Gamma}^k_{ij}$ obtained according to (1.2.1.1) relative to nonholonomic transformers, i.e. while transition from $SR(x^k,\underset{k}{\vec{e}})$ into $SR(x^k,\underset{k}{\vec{e}}')$.

Let's introduce the notation

$$\eta'^k_{ij}=\eta'^k_{ji}=\frac{\partial \bar{a}_i^k}{\partial \underset{e}{x}'^j}+\frac{\partial \vec{a}_j^k}{\partial \underset{e}{x}'^i},$$
$$\omega'^k_{ij}=-\omega'^k_{ji}=\frac{\partial \bar{a}_i^k}{\partial \underset{e}{x}'^j}-\frac{\partial \bar{a}_j^k}{\partial \underset{e}{x}'^i}$$

Than from (1.2.2.2) we'll receive that:

$$\eta'^k_{ij}=-a_n^l \underset{e}{g}^{kn}\underset{e}{g}_{pq}\left(\bar{a}_i^p \omega'^q_{lj}+\bar{a}_j^q \omega'^p_{li}\right)+$$
$$+2\left(\bar{a}_m^k \underset{e}{\Gamma}'^m_{ij}-\bar{a}_i^p \bar{a}_j^q \underset{e}{\Gamma}^k_{pq}\right), \qquad (1.2.2.3)$$

as well

$$\frac{\partial \bar{a}_i^k}{\partial \underset{e}{x}'^j}=\frac{1}{2}\left(\eta'^k_{ij}+\omega'^k_{ij}\right) \qquad (1.2.2.4)$$

System of (1.2.2.3) consists of $\frac{1}{2}n^2(n+1)$ of the equations, which comprise

$$\frac{1}{2}n^2(n+1)+\frac{1}{2}n^2(n-1)=n^3$$

being unknown η'^k_{ij} and ω'^k_{ij}, that is why the parameters ω'^k_{ij}, number of which equals to $\frac{1}{2}n^2(n-1)$, can satisfy the definite additional conditions of calibration. Here, we shall consider one of the possible terms of calibration, which from all-possible reference systems $SR(x^k,\underset{k}{\vec{e}})$, connected between each other by nonholonomic transformations, distinguishes one of special subclasses. Group properties corresponding to nonholonomic transformations will be discussed in the following paragraph.

Let's admit that $\underset{e}{\varphi}_i$ and $\underset{e}{\psi}^i$ are two prescribed vectors relative to nonholonomic transformations, meeting the following term:

$$\underset{e}{\varphi}_i \underset{e}{\psi}^i = \beta \tag{1.2.2.5}$$

where, β – is some invariant, constant value. From all-possible reference system $SR(x^k, \underset{k}{\vec{e}})$ we'll distinguish only those, which are connected with each other by nonholomonic transformations, meeting the following terms of calibration:

$$\frac{\partial a_i^k}{\partial \underset{e}{x}^j} - \frac{\partial a_j^k}{\partial \underset{e}{x}^i} = \frac{1}{\eta} a_p^k \underset{e}{\psi}^p \left(\frac{\partial \underset{e}{\varphi}_i}{\partial \underset{e}{x}^j} - \frac{\partial \underset{e}{\varphi}_j}{\partial \underset{p}{x}^i} \right), \tag{1.2.2.6}$$

where, η – some constant value in reference system $SR(x^k, \underset{k}{\vec{e}})$.

This term can be written in a short form:

$$\omega_{ij}^k = \frac{1}{\eta} a_p^k \underset{e}{\psi}^p \underset{e}{F}_{ij}, \tag{1.2.2.7}$$

where,

$$\underset{e}{F}_{ij} = \frac{\partial \underset{e}{\varphi}_i}{\partial \underset{e}{x}^j} - \frac{\partial \underset{e}{\varphi}_j}{\partial \underset{e}{x}^i} \tag{1.2.2.8}$$

It is evident, that $\underset{e}{F}_{ij} = -\underset{e}{F}_{ji}$.

Taking into account the vector character $\underset{e}{\varphi}_i$ and $\underset{e}{\psi}^i$, it is easy to establish the transformation law of the values F_{ij}, in particular:

$$\underset{e}{F}_{ij} = a_i^p \frac{\partial \underset{e}{\varphi}'_p}{\partial \underset{e}{x}^j} - a_j^q \frac{\partial \underset{e}{\varphi}'_q}{\partial \underset{e}{x}^i} + \varphi'_p \left(\frac{\partial a_i^p}{\partial \underset{e}{x}^j} - \frac{\partial a_j^p}{\partial \underset{e}{x}^i} \right) =$$

$$= a_i^p a_j^q \left(\frac{\partial \underset{e}{\varphi}'_p}{\partial \underset{e}{x}'^q} - \frac{\partial \underset{e}{\varphi}'_q}{\partial \underset{e}{x}'^p} \right) + \frac{1}{\eta} \underset{e}{\varphi}'_p \underset{e}{\psi}'^p \underset{e}{F}_{ij}.$$

Taking into account the (1.2.2.5) we have

$$\underset{e}{F}_{ij} = \frac{\eta}{\eta - \beta} a_i^p a_j^q \underset{e}{F}'_{pq} \tag{1.2.2.9}$$

i.e. $\underset{e}{F}_{ij}$ – is a quasitensor relative to nonholonomic transformations of considered type.

With the aim to establish the terms of calibration in reference system $SR(x^k, \underset{k}{\vec{e}}')$, let's multiply the (1.2.2.6) by $\bar{a}_k^e \bar{a}_n^i \bar{a}_m^j$ and summarize according to indices k, i and j, we'll receive:

$$-\bar{a}_n^i \bar{a}_m^j \left(a_i^k \frac{\partial \bar{a}_k^l}{\partial \underset{e}{x}^j} - a_j^k \frac{\partial \bar{a}_k^l}{\partial \underset{e}{x}^i} \right) = \frac{1}{\eta} \bar{a}_n^i \bar{a}_m^j \underset{e}{\psi}^l \left(\frac{\partial \underset{e}{\varphi}_i}{\partial \underset{e}{x}^j} - \frac{\partial \underset{e}{\varphi}_j}{\partial \underset{e}{x}^i} \right).$$

from it, according to (1.2.1.23) and (1.2.2.9) we'll have:

$$\frac{\partial \bar{a}_m^l}{\partial \underset{e}{x}'^n} - \frac{\partial \bar{a}_n^l}{\partial \underset{e}{x}'^m} = \frac{1}{\eta - \beta} \bar{a}_n^i \bar{a}_m^j \underset{e}{\psi}^l a_i^p a_j^q \underset{e}{F}'_{pq},$$

or

$$\frac{\partial \bar{a}_m^l}{\partial \underset{e}{x}'^n} - \frac{\partial \bar{a}_n^l}{\partial \underset{e}{x}'^m} = \frac{1}{\eta - \beta} \bar{a}_p^l \underset{e}{\psi}'^p \underset{e}{F}'_{nm} = \frac{1}{\eta'} \bar{a}_p^l \underset{e}{\psi}'^p \underset{e}{F}'_{mn}, \tag{1.2.2.10}$$

where:

$$\eta' = \eta - \beta$$

A new constant value in the reference system is $SR(x^k, \underset{k}{\vec{e}})$. From it, and from (1.2.2.9) it is evident that $\frac{1}{\eta}\underset{e}{F}_{ij}$ is a tensor.

In case when $\eta \to \infty$ the (1.2.2.6) becomes a holonomic transformation and from (1.2.2.9) we'll receive $\underset{e}{F}_{ij} = a_i^p a_j^q \underset{e}{F}'_{pq}$, i.e. F_{ij} tensor relative to holonomic transformations. As well, during $\beta = 0$ $\underset{e}{F}_{ij}$ tensor relative to considering nonholonomic transformations.

From (1.2.2.3), (1.2.2.4) and (1.2.2.8) by means of simple computations it is possible to show the validity of the following equity:

$$\frac{\delta \bar{a}_i^k}{\partial \underset{e}{x}'^j} = \bar{a}_p^k H'^p_{ij} - \bar{a}_i^p \bar{a}_j^q H^k_{pq}, \qquad (1.2.2.11)$$

where,

$$H^k_{ij} = \underset{e}{\Gamma}{}^k_{ij} + \frac{1}{4\eta}\underset{e}{\psi}{}^k \underset{e}{F}_{ij} + \frac{1}{4\eta}\left(\underset{e}{\psi}_i \underset{e}{F}_j{}^k + \underset{e}{\psi}_j \underset{e}{F}_i{}^k\right),$$
$$\underset{e}{F}_i{}^k = \underset{e}{g}{}^{pk} \underset{e}{F}_{ip}, \quad \underset{e}{\psi}_i = \underset{e}{g}_{ip} \underset{e}{\psi}{}^p, \quad H^k_{ij} \neq H^k_{ji}, \qquad (1.2.2.12)$$

and H'^k_{ij} is the same expression in $SR(x^k, \underset{k}{\vec{e}}')$.

Let $\underset{e}{A}{}^i$ – is a contravariant vector relative to nonholonomic transformations of considered partial type, than

$$\underset{e}{A}{}^i = \bar{a}_p^i \underset{e}{A}{}'^p$$

Hence:

$$\frac{\partial \underset{e}{A}{}^i}{\partial \underset{e}{x}{}^j} = \bar{a}_p^i a_j^q \frac{\partial \underset{e}{A}{}'^p}{\partial \underset{e}{x}{}'^q} + a_j^q \underset{e}{A}{}'^p \frac{\partial \bar{a}_p^i}{\partial \underset{e}{x}{}'^q}.$$

In the first part of this equity $\frac{\partial \bar{a}_p^i}{\partial \underset{e}{x}{}'^q}$ let's change it by its value from (1.2.2.11):

$$\frac{\partial \underset{e}{A}{}^i}{\partial \underset{e}{x}{}^j} = \bar{a}_p^i a_j^q \frac{\partial \underset{e}{A}{}'^p}{\partial \underset{e}{x}{}'^q} + \bar{a}_\ell^i a_j^q H'^\ell_{pq} \underset{e}{A}{}'^p - a_j^q \bar{a}_p^r \bar{a}_q^\ell H^i_{r\ell} \underset{e}{A}{}'^p.$$

Hence, it is evident, that

$$\frac{\partial \underset{e}{A}{}^i}{\partial \underset{e}{x}{}^j} + H^i_{pj} \underset{e}{A}{}^p = \bar{a}_p^i a_j^q \left(\frac{\partial \underset{e}{A}{}'^p}{\partial \underset{e}{x}{}'^q} + H^p_{\ell q} \underset{e}{A}{}'^\ell \right) \qquad (1.2.2.13)$$

This equity shows that the values

$$\underset{e}{A}{}^i_{\ ;j} = \frac{\partial \underset{e}{A}{}^i}{\partial \underset{e}{x}{}^j} + H^i_{pj} \underset{e}{A}{}^p \qquad (1.2.2.14)$$

relative to nonholonomic transformations are transformed as components of mixed tensor of second order, i.e. they constitute a mixed tensor of second order. In this connection $\underset{e}{A}{}^i_{\ ;j}$ are the value, in future will be called covariant derivatives (in generalized sense) of contravariant vector.

In full analogy are determined the covariant derivative tensors of various orders, in particular:

$$\underset{e}{A}_{i,j}=\frac{\partial \underset{e}{A}_i}{\partial \underset{e}{x}^j}-H^p_{ij}\underset{e}{A}_p,$$

$$\underset{e}{A}^{ik}_{\ ,j}=\frac{\partial \underset{e}{A}^{ik}}{\partial \underset{e}{x}^j}+H^i_{pj}\underset{e}{A}^{pk}+H^k_{pj}\underset{e}{A}^{ip},$$

$$\underset{e}{A}_{ik,j}=\frac{\partial \underset{e}{A}_{ik}}{\partial \underset{e}{x}^j}-H^p_{ij}\underset{e}{A}_{pk}-H^p_{kj}\underset{e}{A}_{ip},$$

$$\underset{e}{A}^k_{i\ ,j}=\frac{\partial \underset{e}{A}^k_i}{\partial \underset{e}{x}^j}+H^k_{pj}\underset{e}{A}^p_i-H^p_{ij}\underset{e}{A}^k_p,\ \ldots \qquad (1.2.2.15)$$

From these expressions it is evident that the symbols of H^k_{ij} in nonholonomic tensor analysis play such a role, which are played by the symbols of Christoffel in holonomic tensor analysis.

Above it was shown, that $\frac{1}{\eta}F_{ij}$ is a covariant tensor, that is why the summand $\frac{1}{4\eta}\left(\underset{e}{\psi}\,{}^k_e F_{ij}+\underset{e}{\psi}_i\underset{e}{F}^k_{\ j}+\underset{e}{\psi}_j\underset{e}{F}^k_{\ i}\right)$ – in the right hand side of the equity (1.2.2.12) – is a mixed tensor of third order. Taking into account this fact from equations (1.2.2.14) and (1.2.2.15) it is evident, that if in the right parts of these equities H^k_{ij} is changeable through $\underset{e}{\Gamma}^k_{ij}$ than the obtained expressions, according to the physics coincide by form with the classical covariant derivatives, as well they are tensors.

From (1.2.2.14) and from the first equity of the system (1.2.2.15) it is evident that

$$\left(\underset{e}{A}^i\underset{e}{B}_i\right)_{,j}=\underset{e}{A}^i_{\ ,j}\underset{e}{B}_i+\underset{e}{A}^i\underset{e}{B}_{i,j}=\frac{\partial\left(\underset{e}{A}^i\underset{e}{B}_i\right)}{\partial \underset{e}{x}^j}+$$
$$H^i_{pj}\underset{e}{A}^p\underset{e}{B}_i-H^p_{ij}\underset{e}{A}^i\underset{e}{B}_p=\frac{\partial\left(\underset{e}{A}^i\underset{e}{B}_i\right)}{\partial \underset{e}{x}^j} \qquad (1.2.2.16)$$

Using the third equity of the system (1.2.2.15) relative to covariant metric tensor $\underset{e}{g}_{ik}$, we obtain:

$$\underset{e}{g}_{ik,j}=\frac{\partial \underset{e}{g}_{ik}}{\partial \underset{e}{x}^j}-\frac{1}{2}\left(\frac{\partial \underset{e}{g}_{ik}}{\partial \underset{e}{x}^j}+\frac{\partial \underset{e}{g}_{jk}}{\partial \underset{e}{x}^i}-\frac{\partial \underset{e}{g}_{ij}}{\partial \underset{e}{x}^k}\right)-$$
$$-\frac{1}{2}\left(\frac{\partial \underset{e}{g}_{ij}}{\partial \underset{e}{x}^k}+\frac{\partial \underset{e}{g}_{ik}}{\partial \underset{e}{x}^j}-\frac{\partial \underset{e}{g}_{kj}}{\partial \underset{e}{x}^i}\right)-\frac{1}{4\eta}\left(\underset{e}{\psi}_k\underset{e}{F}_{ij}+\right.$$
$$\left.\underset{e}{\psi}_i\underset{e}{F}_{jk}+\underset{e}{\psi}_j\underset{e}{F}_{ik}+\underset{e}{\psi}_i\underset{e}{F}_{kj}+\underset{e}{\psi}_k\underset{e}{F}_{ji}+\underset{e}{\psi}_j\underset{e}{F}_{ki}\right)=0.$$

In full analogy it is possible to show, that $\underset{e}{g}^{ik}_{\ ,j}=0$.

Thus,

$$\underset{e}{g}_{ik,j} = \underset{e}{g}^{ik}_{,j} = 0 \qquad (1.2.2.17)$$

As it is known [1] always is possible to select such a $SR(x^k, \underset{k}{\vec{E}})$, that in a given point of the considered space all the Christoffel symbols are equal to zero. The similar circumstance takes place for the symbols H^k_{ij}.

If $SR(x^k, \underset{k}{\vec{E}})$ and $SR(x'^k, \underset{k}{\vec{E}})$ are two reference systems provided that

$$x^k = x^k_0 + x'^k + C^k_{pq} x'^p x'^q + C^k_{pqr} x'^p x'^q x'^r + ...$$

where, x^k_0 – are the coordinates of fixed point in $SR(x^k, \underset{k}{\vec{E}})$, and $C^k_{pq}, C^k_{pqr}...$ are the constant values. It is evident, that in $SR(x'^k, \underset{k}{\vec{E}})$ for coordinates of the point x^k_0 will be $x'^k = 0$.

In this point:

$$\left(\frac{\partial x^k}{\partial x'^i}\right)_{x'^i=0} = \delta^k_i, \left(\frac{\partial x'^k}{\partial x^i}\right)_{x^i=x^i_0} = \delta^k_i, \left(\frac{\partial^2 x^k}{\partial x'^i \partial x'^j}\right)_{x'^i=0} = C^k_{ij}.$$

The considered transformation is holonomic that is why the coefficients of transformation a^k_i and $\overline{a}^k_i$ have such a form:

$$a^k_i = \frac{\partial x'^k}{\partial x^i}, \ \overline{a}^k_i = \frac{\partial x^k}{\partial x'^i}$$

With allowance of this from (1.2.2.11) for H'^k_{ij} we'll obtain the following transformation law

$$H'^k_{ij} = \frac{\partial x^p}{\partial x'^i}\frac{\partial x^q}{\partial x'^j}\frac{\partial x'^k}{\partial x^r} H^r_{pq} + \frac{\partial x'^k}{\partial x^p}\frac{\partial^2 x^p}{\partial x'^i \partial x'^j}$$

These equities in the point $x^k = x^k_0$ (or $x'^k = 0$) is reduced to the following equities:

$$H'^k_{ij} \Big/_{x'^i=0} = H^k_{ij} \Big/_{x^i=x^i_0} + C^k_{ij}$$

and, if

$$C^k_{ij} = -H^k_{ij} \Big/_{x^i=x^i_0}$$

Than

$$H'^k_{ij} \Big/_{x'^i=0} = 0$$

i.e. in $SR(x'^k, \underset{k}{\vec{E}})$ in point of $x'^i = 0$ the symbols H'^k_{ij} are equal to zero.

After the above-indicated generalization of covariant determinants we'll use the generalization of curvature tensor of fourth order. With this aim we'll rewrite the main equation of the whole nonholonomic tensor analysis (1.2.2.11), with allowance of (1.2.1.21), we'll in the following form:

$$\underset{\ell}{e}'^p \frac{\partial \overline{a}^k_i}{\partial x^p} = \overline{a}^k_p H'^p_{i\ell} - \overline{a}^p_i \overline{a}^q_\ell H^k_{pq}$$

From this it is evident, that

$$\frac{\partial \overline{a}^k_i}{\partial x^j} = \alpha'^t_j \overline{a}^k_p H'^p_{it} - \alpha'^t_j \overline{a}^p_i \overline{a}^q_t H^k_{pq}$$

or, according to (1.2.1.9) we have:

$$\frac{\partial \overline{a}^k_i}{\partial x^j} = \alpha'^t_j \overline{a}^k_p H'^p_{it} - \alpha^t_j \overline{a}^p_i H^k_{pt} \qquad (1.2.2.18)$$

Let's differentiate this equity (in general sense) according to x^f, and the equity

$$\frac{\partial \overline{a}_i^k}{\partial x^f} = \alpha'^t_f \overline{a}_p^k H'^p_{it} - \alpha_f^t \overline{a}_i^p H_{pt}^k$$

which is obtained from (1.2.2.18), if we substitute the index j by the index f, differentiate by x^j, and then from them exclude the second derivative

$$\frac{\partial^2 \overline{a}_i^k}{\partial x^j \partial x^f}.$$

After some transformations, with allowance of above obtained main equities, we'll finally receive:

$$\underset{e}{R}'^k_{ijt} = a_\ell^k \overline{a}_i^p \overline{a}_j^q \overline{a}_f^r \underset{e}{R}^\ell_{pqr} \qquad (1.2.2.19)$$

where,

$$\underset{e}{R}^k_{ijf} = \frac{\partial H_{if}^k}{\partial \underset{e}{x}^j} - \frac{\partial H_{ij}^k}{\partial \underset{e}{x}^f} + H_{if}^n H_{nj}^k - H_{ij}^n H_{nf}^k + \\ + H_{it}^k \left(e_f^s \frac{\partial \alpha_s^t}{\partial \underset{e}{x}^j} - e_j^s \frac{\partial \alpha_s^t}{\partial \underset{e}{x}^f} \right) \qquad (1.2.2.20)$$

and $\underset{e}{R}'^k_{ijf}$ – is the same expression in the reference system $SR(x^k, \underset{k}{\vec{e}}')$.

From (1.2.2.19) it is clear, that $\underset{e}{R}^k_{ijf}$ are the components of a tensor of fourth order relative of nonholonomic transformations of considered type. In future $\underset{e}{R}^k_{ijf}$ will show by nonholonomic curvature tensor of the fourth order.

From (1.2.2.20) it is clear, that $\underset{e}{R}^k_{ijf} = -\underset{e}{R}^k_{ifj}$ i.e. nonholonomic curvature tensor, analogous to the previous one, is antisymmetric relative to the index of j and f.

Here we shall not study the main properties of this tensor, so as for final aims of this works sufficient are only several.

By means of mixed curvature tensor of the fourth order it is possible to form various tensor values, in particular:

Nonholonomic covariant curvature tensor of the fourth order:

$$\underset{e}{R}_{kijf} = \underset{e}{g}_{kp} \underset{e}{R}^p_{ijf} \qquad (1.2.2.21)$$

Nonholonomic covariant curvature tensor of the second order:

$$\underset{e}{R}_{ij} = \underset{e}{R}^k_{ijk} = \frac{\partial H_{im}^m}{\partial \underset{e}{x}^j} - \frac{\partial H_{ij}^m}{\partial \underset{e}{x}^m} + H_{im}^n H_{nj}^m - H_{ij}^n H_{nm}^m - E_{jn}^m H_{im}^n \qquad (1.2.2.22)$$

where

$$E_{pq}^k = \underset{p}{e}^r \frac{\partial \alpha_r^k}{\partial \underset{e}{x}^q} - \underset{q}{e}^r \frac{\partial \alpha_r^k}{\partial \underset{e}{x}^p} \qquad (1.2.2.23)$$

From (1.2.2.22) it is evident that $\underset{e}{R}_{ij} \neq \underset{e}{R}_{ji}$.

Nonholonomic scalar curvature:

$$\underset{e}{R} = \underset{e}{g}^{pq} \underset{e}{R}_{pq} \qquad (1.2.2.24)$$

Let's suppose that $SR(x^k, \underset{k}{\vec{E}})$ is such, that in the point of x_0^k the condition $H_{ij}^k = 0$ is valid. In this point $\underset{e}{R}_{kije}$ has such a form:

$$\underset{e}{R}_{kijl}=\underset{e}{g}_{kp}\underset{e}{R}{}^{p}_{ijl}=\frac{1}{2}\left(\frac{\partial^2 \underset{e}{g}_{lk}}{\partial \underset{e}{x}^i\,\partial \underset{e}{x}^j}+\frac{\partial^2 \underset{e}{g}_{ij}}{\partial \underset{e}{x}^k\,\partial \underset{e}{x}^l}-\frac{\partial^2 \underset{e}{g}_{il}}{\partial \underset{e}{x}^k\,\partial \underset{e}{x}^j}-\right.$$

$$\left.-\frac{\partial^2 \underset{e}{g}_{jk}}{\partial \underset{e}{x}^i\,\partial \underset{e}{x}^l}\right)+\frac{1}{4}\left[\frac{\partial}{\partial \underset{e}{x}^j}\left(\underset{e}{\psi}_k\underset{e}{F}_{il}+\underset{e}{\psi}_i\underset{e}{F}_{lk}+\underset{e}{\psi}_l\underset{e}{F}_{ik}\right)-\right. \qquad (1.2.2.25)$$

$$\left.-\frac{\partial}{\partial \underset{e}{x}^l}\left(\underset{e}{\psi}_k\underset{e}{F}_{ij}+\underset{e}{\psi}_i\underset{e}{F}_{jk}+\underset{e}{\psi}_j\underset{e}{F}_{ik}\right)\right].$$

From here, it is clear, that

$$\underset{e}{R}_{ikjl}=-\underset{e}{R}_{kijl}\,,\ \ \underset{e}{R}_{kilj}=-\underset{e}{R}_{kijl}\,,\ \underset{e}{R}_{kijl}\neq-\underset{e}{R}_{jlki} \qquad (1.2.2.26)$$

i.e. $\underset{e}{R}_{ikjl}$ – is an antisymmetric tensor relative to *i*, *k* and *j*, *l*.

According to the results of the paragraph 1.1.1, the features mentioned here of antisymmetry of a tensor $\underset{e}{R}_{kije}$, valid in special reference system, take place in any reference system.

If $SR(x^k,\underset{k}{\vec{E}})$ is such that $H^k_{ij}=0$ in the point x^k_0, than

$$\underset{e}{R}{}^{k}_{ijl,r}=\frac{\partial^2 H^k_{il}}{\partial x^j\partial x^r}-\frac{\partial^2 H^k_{ij}}{\partial x^l\partial x^r}\,,$$
$$\underset{e}{R}{}^{k}_{irj,l}=\frac{\partial^2 H^k_{ij}}{\partial x^j\partial x^l}-\frac{\partial^2 H^k_{ir}}{\partial x^j\partial x^l}\,, \qquad (1.2.2.27)$$
$$\underset{e}{R}{}^{k}_{ilr,j}=\frac{\partial^2 H^k_{ir}}{\partial x^l\partial x^j}-\frac{\partial^2 H^k_{il}}{\partial x^r\partial x^j}$$

From this it's evident, that

$$\underset{e}{R}{}^{k}_{ijl,r}+\underset{e}{R}{}^{k}_{irj,l}+\underset{e}{R}{}^{k}_{ilr,j}=0 \qquad (1.2.2.28)$$

This equity comprises exceptionally the tensor values and, that is why, is valid in any $SR(x^k,\underset{k}{\vec{E}})$.

The (1.2.2.28) is a generalized identity of Bianka. From it can he received other, more elemental, identity. With this aim let's multiply by (1.2.2.28) $\underset{e}{g}^{ik}\,\delta^l_k$. With allowance of the fact, that $\underset{e}{g}{}^{ij}_{,r}=0$ we'll receive:

$$\underset{e}{g}^{ij}\,\delta^l_k R^k_{ijl,r}=\underset{e}{g}^{ij}\,R_{ij,r}=\left(R\delta^j_r\right)_{,j}\ ,$$

$$\underset{e}{g}^{ij}\,\delta^l_k\,\underset{e}{R}{}^{k}_{irj,l}=-\left(\underset{e}{g}^{ij}\,\underset{e}{g}^{kp}\,\underset{e}{R}_{pijr}\right)_{,k}=-\left(\underset{e}{g}^{pj}\,\underset{e}{g}^{ik}\,\underset{e}{R}_{pijr}\right)_{,k}=$$
$$=\left(\underset{e}{g}^{pk}\,\underset{e}{g}^{ij}\,\underset{e}{R}_{pikr}\right)_{,j}=-\left(\underset{e}{g}^{ij}\,\underset{e}{R}{}^{k}{}_{irk}\right)_{,j}=-\left(\underset{e}{g}^{ij}\,\underset{e}{R}_{ir}\right)_{,j}=-\underset{e}{R}{}^{j}_{r,j}\ ,$$

$$\underset{e}{g}^{ij}\,\delta^\ell_k\,\underset{e}{R}{}^{k}_{ier,j}=\left(\underset{e}{g}^{ij}\,\underset{e}{R}{}^{k}{}_{ikr}\right)_{,j}=-\underset{e}{R}{}^{j}_{r,j}\ ,$$

where,

$$\underset{e}{R}{}^{j}_{r}=g^{ij}\,\underset{e}{R}_{ir} \qquad (1.2.2.29)$$

So as $\underset{e}{R}_{ij}\neq\underset{e}{R}_{ji}$, than $\underset{e}{R}{}^{j}{}_{r}\neq\underset{e}{R}{}_{r}{}^{j}$, where $\underset{e}{R}{}_{r}{}^{j}=g^{ij}\,\underset{e}{R}_{ri}$.

After substitution into (1.2.2.28) we have

$$\underset{e}{G}{}^{k}{}_{i,k}=0\,, \qquad (1.2.2.30)$$

where,

$$\underset{e}{G}{}^{k}{}_{i} = \underset{e}{R}{}^{k}{}_{i} - \frac{1}{2}\delta_i^k \underset{e}{R} \qquad (1.2.2.31)$$

(1.2.2.30) is a generalization of known identity of general theory of relativity [6]. In contravariant components the (1.2.2.30) has such a form.

$$\underset{e}{G}{}^{ki}{}_{,k} = 0, \qquad (1.2.2.32)$$

where,

$$\underset{e}{G}{}^{ki} = \underset{e}{g}{}^{ip} \underset{e}{G}{}^{k}{}_{p} = \underset{e}{R}{}^{ki} - \frac{1}{2}\underset{e}{g}{}^{ki} \underset{e}{R}$$
$$\underset{e}{R}{}^{ki} = \underset{e}{g}{}^{kp} \underset{e}{g}{}^{iq} \underset{e}{R}{}_{pq}, \quad \underset{e}{R}{}^{ki} \neq \underset{e}{R}{}^{ik} \qquad (1.2.2.33)$$

provided $\underset{e}{G}{}^{ki} \neq \underset{e}{G}{}^{ik}$, when $\underset{e}{R}{}^{ki} \neq \underset{e}{R}{}^{ik}$.

According to (1.2.2.33) $\underset{e}{G}{}^{ki}$ coincides by form with the tensor of Einstein, applied in relativistic theory of gravitational field, however with allowance of the fact, the $\underset{e}{R}{}_{ik}$ applied in present work, is generalization of the tensor of Richi R_{ik}, than $\underset{e}{G}{}^{ki}$ should be considered as generalization of Einstein tensor.

1.2.3. *EH* GROUP OF NONHOLONOMIC TRANSFORMATIONS [3,5]

The above considered elements of tensor analysis are covariant relative to nonholonomic transformations, meeting the requirements of calibration of (1.2.1.6) (or (1.2.2.7), (1.2.2.19), corresponding to fixed functions φ_i and ψ^i, i.e. all reference systems $SR(x^k, \underset{k}{\vec{e}})$, which are connected between each other by means of nonholonomic transformations of considered type, are equivalent. It is possible to show, that the considered nonholonomic transformations at prescribed φ_i and ψ^i constitute a group. With this aim let's consider two nonolonomic transformations having the similar functions φ_i and ψ^i and various constants η' and η'':

$$d\underset{e}{x}{}'^{i} = a'^{i}_{p} d\underset{e}{x}{}^{p}, \quad d\underset{e}{x}{}''^{i} = a''^{i}_{p} d\underset{e}{x}{}^{p} \qquad (1.2.3.1)$$

in which the function a'^{i}_{p} and a''^{i}_{p} satisfy the conditions of:

$$\frac{\partial a'^{k}_{i}}{\partial \underset{e}{x}{}^{j}} - \frac{\partial a'^{k}_{j}}{\partial \underset{e}{x}{}^{i}} = \frac{1}{\eta'} a'^{k}_{l} \underset{e}{\psi}{}^{l} \underset{e}{F}{}_{ij},$$
$$\frac{\partial a''^{k}_{i}}{\partial \underset{e}{x}{}^{j}} - \frac{\partial a''^{k}_{j}}{\partial \underset{e}{x}{}^{i}} = \frac{1}{\eta''} a''^{k}_{l} \underset{e}{\psi}{}^{l} \underset{e}{F}{}_{ij}. \qquad (1.2.3.2)$$

Combination of these transformations give one transformation

$$d\underset{e}{x}{}''^{i} = a^{i}_{p} d\underset{e}{x}{}'^{p} \qquad (1.2.3.3)$$

where,

$$a^{k}_{i} = a''^{k}_{p} \bar{a}'^{p}_{i} \qquad (1.2.3.4)$$

which connects the reference system $SR(x^k, \underset{k}{\vec{e}}')$ with the system $SR(x^k, \underset{k}{e}'')$.

Let's calculate

$$\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{i}} .$$

From (1.2.3.4) we have:

$$\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{i}} = a''^k_p \left(\frac{\partial \bar{a}'^p_i}{\partial \underset{e}{x}'^{j}} - \frac{\partial \bar{a}'^p_j}{\partial \underset{e}{x}'^{i}} \right) + \bar{a}'^p_i \frac{\partial a''^k_p}{\partial \underset{e}{x}'^{j}} - \bar{a}'^p_j \frac{\partial a''^k_p}{\partial \underset{e}{x}'^{i}} .$$

Let's multiply this equity by $a'^i_m \; a'^j_n$ and sum it according to indices i and j:

$$a'^i_m a'^j_n \left(\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{j}} \right) = a'^i_m a'^j_n a''^k_p \left(\frac{\partial \bar{a}'^p_i}{\partial \underset{e}{x}'^{j}} - \frac{\partial \bar{a}'^p_j}{\partial \underset{e}{x}'^{i}} \right) + \frac{\partial a''^k_m}{\partial \underset{e}{x}^{n}} - \frac{\partial a''^k_n}{\partial \underset{e}{x}^{m}} .$$

Taking into account (1.2.3.2) and (1.2.2.10), we have:

$$a'^i_m a'^j_n \left(\frac{\partial a_i^k}{\partial \underset{e}{x}^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}^{i}} \right) = \frac{1}{\beta - \eta'} a'^i_m a'^j_n a''^k_p \bar{a}'^p_q \underset{e}{\psi}'^q \underset{e}{F}'_{ij} + \frac{1}{\eta''} a''^k_p \underset{e}{\psi}^p \underset{e}{F}_{mn}$$

According to (1.2.2.9)

$$\underset{e}{F}_{mn} = \frac{\eta'}{\eta' - \beta} a'^i_m a'^j_n \underset{e}{F}'_{ij}$$

Substituting this meaning $\underset{e}{F}_{mn}$ into the second summon of the right hand side of the last equity and replacing $\underset{e}{\psi}^p$ through $\bar{a}'^p_q \psi^q$, we'll get:

$$a'^i_m a'^j_n \left(\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{i}} \right) = a'^i_m a'^j_n \left(\frac{1}{\beta - \eta'} + \frac{\eta'}{\eta' - \beta} \frac{1}{\eta''} \right) a''^k_p \bar{a}'^p_q \underset{e}{\psi}'^q \underset{e}{F}'_{ij}$$

From here, with allowance of (1.2.3.4) we finally have:

$$\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{i}} = \frac{1}{\eta} a_p^k \underset{e}{\psi}'^p \underset{e}{F}'_{ij} \qquad (1.2.3.5)$$

where,

$$\eta = \eta'' \frac{\beta - \eta'}{\eta'' - \eta'} . \qquad (1.2.3.6)$$

In full analogy, it is possible to show that:

$$\frac{\partial \bar{a}_i^k}{\partial \underset{e}{x}''^{j}} - \frac{\partial \bar{a}_j^k}{\partial \underset{e}{x}''^{i}} = \frac{1}{\bar{\eta}} \bar{a}_p^k \underset{e}{\psi}''^p \underset{e}{F}''_{ij} ,$$

where: $\quad \bar{\eta} = \eta' \dfrac{\beta - \eta''}{\eta' - \eta''}$, $\quad$ i.e. $\quad \bar{\eta} = \beta - \eta$.

Thus, combination of two nonholonomic transformations satisfying the condition of calibration of (1.2.2.6), constitutes as well the nonholonomic transformation of the same type with the constant η, defined according to (1.2.3.6).

Besides, the transformation of the considered type comprises the identity transformation. Really, the transformation of $d\underset{e}{x}'^{i} = d\underset{e}{x}^{i}$ corresponds to the case $\frac{1}{\eta} = 0$, and thus belongs to the transformation under consideration.

There is available the inverse transformation. This property is a simple consequence of requirement of $\det(a_i^k) \neq 0$. From it, it is clear, that the nonholonomic transformation of considered type constitutes a group, which in future we'll call the *EH* group of nonholonomic transformations.

At specified β and η' it is always possible to find such a value of η'', for which $\frac{1}{\eta} = 0$. Really, from (1.2.3.6) this takes place at

$$\eta'' = \eta' \qquad (1.2.3.7)$$

In compliance to (1.2.3.5) to such a value η'' corresponds a transformation, meeting the requirements of

$$\frac{\partial a_i^k}{\partial \underset{e}{x}'^{j}} - \frac{\partial a_j^k}{\partial \underset{e}{x}'^{i}} = 0 \qquad (1.2.3.8)$$

and if the reference frame $SR(x^k, \underset{k}{\vec{e}}')$ coincides with $SR(x^k, \underset{k}{\vec{E}})$, than the transformation with the coefficient a_i^k is holonomic. Provided $\underset{k}{e}^i = \delta_k^i$ and, according to (1.2.1.20) condition (1.2.3.8) is reduced to the conditions of holonomicity

$$\frac{\partial a_i^k}{\partial x'^{j}} - \frac{\partial a_j^k}{\partial x'^{i}} = 0 .$$

Thus, combination of two transformations of considered character, corresponding to one and the same value η, constitutes a holonomic transformation.

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CHAPTER II

ELEMENTS OF RELATIVISTIC KINEMATICS

Any physical phenomenon runs in four-dimensional space signified as $+---$, i.e. in four-dimensional space-time variety, and consequently only such spaces will be further considered. The subject may investigate the physical phenomena based on application of rational - for a subject - notions, as: time, distance, direction, velocity and movement acceleration, etc. Coordinates used for point addressing, in general case, will not reflect the essence of these rational notions (these are dimensionless numbers), that is why it becomes necessary to draw those main equities, that allow to define the mentioned parameters, depending on metric properties of four-dimensional space-time variety.

The authors will try to define in this Chapter the most common expressions for calculation of kinematics parameters, justified for any four-dimensional space-time variety (both plane and bend), as well as for any coordinate systems describing the investigated four-dimensional space.

Expressions yielded below for calculation of time and distance are valid and then, when none of coordinate lines is timelike, and $g<0$. In that particular case, when one coordinate line is timelike, the common expressions coincide with known expressions [1,2,3].

Such generalization should be recognized as natural, so as from geometric viewpoint it is not important what coordinate lines will be used by us while describing four-dimensional space-time variety, important is that the four coordinate lines would not be laying simultaneously in subspace of n-measurement, where $n<4$.

Investigation of major kinematics issues have shown that for any study of issues related with calculation of metric characteristics of four-dimensional space is reduced to study of resolution of differential equations in partial derivatives of first order. In this connection, in order to present the subject in full, this Chapter will have additional paragraph dealing with elements theory of differential equations in partial derivatives of first order. To put it briefly, the method of characteristics will be reduced to discussion, excluding its geometrical interpretation. These issues are discussed in detail in [4,5,6].

2.1 SOME ASPECTS OF DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES OF FIRST ORDER

2.1.1 LINEAR DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES OF FIRST ORDER

The differential equation

$$\sum_{k=1}^{n} a_k \frac{\partial u}{\partial x^k} = a_0 , \qquad (2.1.1.1)$$

where, $a_k(x^1,x^2,...,x^n,u)$, $a_0(x^1,x^2,...,x^n,u)$ - are prescribed functions of their arguments, and $u(x^1,x^2,...,x^n)$ - the sought function is called quasi-linear differential equation in partial derivatives of first order.

In the $n+1$ - dimensional space of variables x^k, u the values $\frac{\partial u}{\partial x^k}$-1 constitute a normal vector to surface $u(x^1,x^2,...,x^n)-u=0$, that is why, as per (2.1.1.1), the coefficients a_k, a_0 represent the components of tangent vector of the same surface. According to this prescribed vector power lines $x^k(s)$, $u(s)$ can be constructed, belonging to indicated surfaces

$$\frac{dx^k}{ds} = a_k, \quad \frac{\partial u}{\partial s} = a_0 \tag{2.1.1.2}$$

This system of ordinary differential equations is called a characteristic system for (2.1.1.1).

If $a_0 \equiv 0$, and coefficients a_k do not depend on u, then from (2.1.1.1) is yielded:

$$\sum a_k \frac{\partial u}{\partial x^k} = 0 \tag{2.1.1.3}$$

This equation is called a linear differential equation in partial derivatives of first order. Characteristic system of equation (2.1.1.3) is as follows

$$\frac{dx^k}{ds} = a_k, \qquad \frac{du}{ds} = 0 \tag{2.1.1.4}$$

Since a_k doesn't depend on u, a it is reduced to system:

$$\frac{dx^k}{ds} = a_k, \tag{2.1.1.5}$$

which constitutes a complete system of ordinary differential equations in relation to x^k. Any function of $\varphi(x^1, x^2, ..., x^n)$ is called an integral of this system, if

$$\varphi\left[x^1(s), x^2(s), ..., x^n(s)\right] = const, \tag{2.1.1.6}$$

where, $x^k(x)$ – is a solution of system (2.1.1.5). Integral of system (2.1.1.5) can be formed as follows: assuming that one of a_k coefficients differs from zero, i.e. condition of

$$\sum_{k=1}^{n} a_k^2 \neq 0 \tag{2.1.1.7}$$

is valid.

Assuming that such coefficient is a_n, we'll rewrite (2.1.1.5) in the following way:

$$\frac{dx^k}{dx^n} = \frac{a_k}{a_n} \qquad k = 1, 2, ..., n-1 \tag{2.1.1.8}$$

In accord to the theory of ordinary differential equations [5], this system has common solution at rather general conditions relative to coefficients:

$$x^k = \psi^k(x^n, C_1, C_2, ...C_{n-1}) \quad k = 1, 2, ..., n-1 \tag{2.1.1.9}$$

where, $C_1, C_2, ..., C_{n-1}$ – are arbitrary constant integrations. ψ^k functions admit solubility of this system in relation to $C_1, C_2, ..., C_{n-1}$ constants:

$$\varphi_k(x^1, x^2, ..., x^n) = C_k \qquad k = 1, 2, ..., n-1 \tag{2.1.1.10}$$

$n-1$ functions of $\varphi_k(x^1, x^2, ..., x^n)$, determined as per procedure mentioned here, represent themselves as functionally independent integrals of systems (2.1.1.8) or (2.1.1.5). Functional independence of φ_k function is the result of method of their determination, in particular, whereas (2.1.1.9) and (2.1.1.10) – are identity systems, than (2.1.1.10) is resolved in relation to $x^1, x^2, ...x^{n-1}$ variables and that is why rank of matrix

$$\left\| \begin{matrix} \frac{\partial \varphi_1}{\partial x^1} & \frac{\partial \varphi_1}{\partial x^2} & \cdots & \frac{\partial \varphi_1}{\partial x^n} \\ \frac{\partial \varphi_2}{\partial x^1} & \frac{\partial \varphi_2}{\partial x^2} & \cdots & \frac{\partial \varphi_2}{\partial x^n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \varphi_{n-1}}{\partial x^1} & \frac{\partial \varphi_{n-1}}{\partial x^2} & \cdots & \frac{\partial \varphi_{n-1}}{\partial x^n} \end{matrix} \right\| \tag{2.1.1.11}$$

is equal to $n-1$.

Any function of $F(\varphi_1, \varphi_2, ... \varphi_{n-1})$ is also an integral (2.1.1.5), however it is in functional dependance on preceding functions. It is easy to show that system (2.1.1.5) has no other integral independently from $\varphi_1, \varphi_2, ... \varphi_{n-1}$. Actually, if we assume that there is integral $\varphi_n(x^1, x^2, ..., x^n)$, independet from $\varphi_1, \varphi_2, ... \varphi_{n-1}$, than we'll get that

$$\det\left(\frac{\partial \varphi_i}{\partial x^k}\right) \neq 0$$

Than from evident equaties

$$\frac{\partial \varphi_i}{\partial s} = \sum_{k=1}^{n} \frac{\partial \varphi_i}{\partial x^k} \frac{dx^k}{ds} = \sum_{k=1}^{n} a_k \frac{\partial \varphi_i}{\partial x^k} = 0 \qquad i = 1, 2, ..., n \qquad (2.1.1.12)$$

representing the linear algebraic (homogeneous) equations relative to a_k, it follows that $a_k = 0 \ \ k = 1, 2, ..., n$, and this contradicts to condition (2.1.1.7). In accord to (2.1.1.7) the condition of $\det\left(\frac{\partial \varphi_i}{\partial x^k}\right) = 0$ should take place, from which it follows that φ_n function depends on $\varphi_1, \varphi_2, ..., \varphi_{n-1}$. Besides, it can be obtained from (2.1.1.12) that any integral of system (2.1.1.5) satisfies the linear equation (2.1.1.3). Reverse proposal is correct: any solution of equation (2.1.1.3) is an integral of system (2.1.1.5). Indeed, if we in (2.1.1.3) changes x^k by $x^k(s)$ solution of system (2.1.1.5), and a_k is changed through $\frac{dx^k}{ds}$, then we'll get $\frac{du}{ds} = 0$, i.e. $u = const$ at $x^k = x^k(s)$, consequently $u(x^1, x^2, ..., x^n)$ is integral of system (2.1.1.5).

Thus, equation (2.1.1.3) has $n-1$ functionally independent solutions. These solutions of equations of (2.1.1.3) give all independent integrals of system (2.1.1.5); inverse independent integrals of system (2.1.1.5) determine the independent solution of differential equation (2.1.1.3).

Quasi-linear (2.1.1.1) may be reduced to linear, if dependence between sought and independent variables is represented as implicit function

$$F(u, x^1, x^2, ..., x^n) = c_0 \qquad (2.1.1.13)$$

where, c_0 – is some constant. Hence, with allowance that u is a function of $x^1, x^2, ..., x^n$, we'll get:

$$\frac{\partial u}{\partial x^k} = -\frac{\frac{\partial F}{\partial x^k}}{\frac{\partial F}{\partial u}} \qquad (2.1.1.14)$$

After substituting we'll get:

$$\sum_{k=0}^{n} a_k \frac{\partial F}{\partial x^k} = 0 \qquad (2.1.1.15)$$

where, u function is denoted through x^0. Such differential equation will take place not for any $x^0, x^1, ..., x^n$, but only for those which meet the conditions of (2.1.1.13), and due to being set with (2.1.1.13) are not linear. However, considering independently (2.1.1.15) as a linear equation for any $x^0, x^1, ..., x^n$, we'll define some function of $F(x^0, x^1, ..., x^n)$, after that the variables $x^0, x^1, ..., x^n$ should be limited as per equity (2.1.1.13). Function u (in inplicit form) defined this way, will satisfy the equation (2.1.1.1), so as according to (2.1.1.14) being sequence of (2.1.1.13), the equations (2.1.1.1) and (2.1.1.15) are identical.

2.1.2 CAUCHI PROBLEM FOR QUASI-LINEAR DIFFERENTIAL EQUATION IN PARTIAL DERIVATIVES OF FIRST ORDER

In space of $n+1$ dimensional variables $x^0, x^1, ..., x^n, u$ we shall determine some variety of $n-1$ dimension:

$$\begin{aligned} &x^k = x^k(q_1, q_2, ..., q_{n-1}) \qquad k = 1, 2, ..., n, \\ &u = u(q_1, q_2, ..., q_{n-1}), \end{aligned} \qquad (2.1.2.1)$$

where, $\frac{\partial x^k}{\partial q_i}, \frac{\partial u}{\partial q_i}$ $(k = 1, 2, ..., n,\ i = 1, 2, ..., n-1)$ - are continuous functions of their arguments, and rank of matrix $\left\| \frac{\partial x^k}{\partial q_i} \right\|$ is equal to $n-1$.

The essence of Cauchi problem for quasi-linear differential equation (2.1.1.1) concludes in the following: to find such a solution of equation (2.1.1.1) $u(x^1, x^q, ..., x^n)$, which at $x^k = x^k(q_1, q_2 ... q_{n-1})$ coincides with function $u(q_1, q_2, ..., q_{n-1})$, determined by (2.1.2.1). This problem can be formulated also in this way: to find solution of differential equation (2.1.1.1) that is running through variety (2.1.2.1).

Solution of the Cauchi problem can be implemented by application of characteristic system (2.1.1.2), in particular, let

$$\begin{aligned} &x^k = \psi^k(s, c_0, c_1, ... c_n) \qquad k = 1, 2, ..., n, \\ &u = \psi^0(s, c_0, c_1, ... c_n) - \end{aligned} \qquad (2.1.2.2)$$

is general solution of characteristic system (2.1.1.2), where $c_0, c_1, ... c_n$ - are arbitrary constants of integration. Let's select them as functions $q_1, q_2, ..., q_{n-1}$ of parameters in such a way that at $s = 0$ x^k coincide with $x^k(q_1, q_2, ..., q_{n-1})$, and $u - u(q_1, q_2, ..., q_{n-1})$,

$$\begin{aligned} &\psi^k(0, c_0, c_1, ..., c_n) = x^k(q_1', q_2, ..., q_{n-1}) \ \ k = 1, 2, ..., n, \\ &\psi^0(0, c_0, c_1, ..., c_n) = u(q_1, q_2, ..., q_{n-1}) \end{aligned} \qquad (2.1.2.3)$$

In accord to theory of ordinary differential equations [5], functions $\psi^k(s, c_0, c_1, ..., c_n)$ $(k = 0, 1, 2, ..., n)$ are differentiated not only according to parameter s, but in accord to all $c_k (k = 0, 1, 2, ..., n)$, and system (2.1.2.3) has the only one solution in relation to c_k parameter.

If solutions of system (2.1.2.3) $c_k(q_1, q_2, ..., q_{n-1})$ is substituted into (2.1.2.2) than we'll get:

$$\begin{aligned} &x^k = \varphi^k(s, q_1, q_2, ..., q_{n-1}) \qquad k = 1, 2, ..., n, \\ &u = \varphi^0(s, q_1, q_2, ..., q_{n-1}) \end{aligned} \qquad (2.1.2.4)$$

This system is to determine solution of the Cauchi problem in parameter form, provided that the condition

$$\det\left(\frac{\partial \varphi^k}{\partial q_i}\right) = \begin{vmatrix} a_1 & a_2 & ... & a_n \\ \frac{\partial x^1}{\partial q_1} & \frac{\partial x^2}{\partial q_1} & ... & \frac{\partial x^n}{\partial q_1} \\ ... & ... & ... & ... \\ \frac{\partial x^1}{\partial q_{n-1}} & \frac{\partial x^2}{\partial q_{n-1}} & ... & \frac{\partial x^n}{\partial q_{n-1}} \end{vmatrix} \neq 0 \qquad (2.1.2.5)$$

is valid only in points of variety (2.1.2.1) (i.e. at $s = 0$).

By force of continuity of all elements of determinant, this inequity is in force in certain vicinity of variety (2.1.2.1). Than in this vicinity of equity $x^k = \varphi^k(s, q_1, q_2, ..., q_{n-1})$, reversed in relation to variables $s, q_1, q_2, ..., q_{n-1}$, i.e.

$$s = s(x^1, x^2, ..., x^n), \quad q_i = q_i(x^1, x^2, ... x^n) \quad i = 1, 2, ..., n-1,$$

which upon substitution into the latter the equity of system (2.1.2.4) determines solution of the Cauchi problem $u = u(x^1, x^2, ..., x^n)$. Actually from the obvious equity

$$\frac{du}{ds} = \sum_{k=1}^{n} \frac{\partial u}{\partial x^k} \frac{dx^k}{ds}$$

in accord to (2.1.1.2), we'll have

$$\sum_{k=1}^{n} a_k \frac{\partial u}{\partial x^k} = a_0$$

It follows from (2.1.2.5) that system (2.1.2.4) is in the only one way solved relative to variables $s, q_1, q_2, ..., q_{n-1}$; It means that in the vicinity of variety (2.1.2.1) in which (2.1.2.5) is valid, the solution of the Cauchi problem is unique.

In case when condition of (2.1.2.5) does not take place, i.e. when condition of

$$\begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ \frac{\partial x^1}{\partial q_1} & \frac{\partial x^2}{\partial q_1} & \cdots & \frac{\partial x^n}{\partial q_1} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x^1}{\partial q_{n-1}} & \frac{\partial x^2}{\partial q_{n-1}} & \cdots & \frac{\partial x^n}{\partial q_{n-1}} \end{vmatrix} = 0 \qquad (2.1.2.6)$$

is valid variety (2.1.2.1) (i.e. at $s = 0$), it means that there are such multitudes $\lambda_i(q_1, q_2, ..., q_{n-1})$ $i = 1, 2, ..., n-1$, for which the following conditions are valid:

$$a_k = \sum_{i=1}^{n-1} \lambda_i \frac{\partial \varphi^k}{\partial q_i} \quad k = 1, 2, ..., n \qquad (2.1.2.7)$$

If, along with that, condition

$$a_0 = \sum_{i=1}^{n-1} \lambda_i \frac{\partial u}{\partial q_i}, \qquad (2.1.2.8)$$

is valid as well, than the variety (2.1.2.1) is called a characteristic variety.

It is proved, that [4,10] in case when initial variety (2.1.2.1) meets condition of (2.1.2.6), than for solubility of Cauchi problem it is required and sufficient that (2.1.2.1) was characteristic variety. In this case the Cauchi problem is not the only.

Thus: a) The Cauchi problem for quasi-linear differential equation (2.1.1.1) has the only solution provided there is such variety of (2.1.2.1) which is valid the condition (2.1.2.5);

b) If variety (2.1.2.1) satisfies condition (2.1.2.6) than to solve the Cauchi problem it is necessary and sufficient that variety (2.1.2.1) was a characteristic one. In such case the Cauchi problem has infinitively many solutions.

2.1.3 THE CAUCHI PROBLEM FOR NON-LINEAR DIFFERENTIAL EQUATION IN PARTIAL DERIVATIVES OF FIRST ORDER

The non-linear differential equation in partial derivatives of first order has the following form:

$$F\left(x^1, x^2, ..., x^n, u, \frac{\partial u}{\partial x^1}, \frac{\partial u}{\partial x^2}, ..., \frac{\partial u}{\partial x^n}\right) = 0 \qquad (2.1.3.1)$$

where F is some non-linear function continuously differential relative to its arguments.

Let's consider the following system of ordinary differential equations relative to $2n+1$ of variables $x^k(s), u(s)$ and $p_k(s)$

$$\frac{dx^k}{ds} = \frac{\partial F}{\partial p_k}, \quad \frac{du}{ds} = \sum_{k=1}^{n} p_k \frac{\partial F}{\partial p_k}$$
$$\frac{dp^k}{ds} = -\frac{\partial F}{\partial x_k} - p_k \frac{\partial F}{\partial u} \qquad (2.1.3.2)$$

where $p_k(s)$ $k = 1, 2, ..., n$ – are some functions of parameter s, and F – is a function of arguments x^k, u, p_k, determined in accord to (2.1.3.1), in particular,

$$F = F(x^1, x^2, ..., x^n, u, p_1, p_2, ..., p_n).$$

(2.1.3.2) is called a characteristic system for equation (2.1.3.1).

It may be shown that if $x^k(s), u(s), p_k(s)$ - is solution of a system (2.1.3.2), than

$$F\left[x^1(s), x^2(s), ..., x^n(s), u(s), p_1(s), p_2(s), ..., p_n(s)\right] = const \qquad (2.1.3.3)$$

indeed, from obvious equity of

$$\frac{dF}{ds} = \sum_{k=1}^{n} \frac{\partial F}{\partial x^k}\frac{dx^k}{ds} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial s} + \sum_{k=1}^{n} \frac{\partial F}{\partial p^k}\frac{dp_k}{ds},$$

if we substitute here the values $\frac{dx^k}{ds}, \frac{du}{ds}$ and $\frac{dp_k}{ds}$ from (2.1.3.2), we'll get:

$$\frac{dF}{ds} = \sum_{k=1}^{n} \frac{\partial F}{\partial x^k}\frac{\partial F}{\partial p_k} + \frac{\partial F}{\partial u}\sum_{k=1}^{n} p_k \frac{\partial F}{\partial p^k} -$$
$$-\sum_{k=1}^{n} \frac{\partial F}{\partial p_k}\left(\frac{\partial F}{\partial x^k} + p_k \frac{\partial F}{\partial u}\right) = 0, \qquad (2.1.3.4)$$

i.e. (2.1.3.3) is valid.

Let

$$x^k(s, q_1, q_2, ..., q_{n-1}) \quad k = 1, 2, ..., n, \quad u(s, q_1, q_2, ..., q_{n-1}) \qquad (2.1.3.5)$$

$n-1$ - dimensional variety in $n+1$ dimensional space of variables x^k, u, meeting the following conditions:

a) $\frac{\partial x^k}{\partial q_i}$ and $\frac{\partial u}{\partial q_i}$ continuous functions of its arguments;

b) Rank of matrix $\left\|\frac{\partial x^k}{\partial q_i}\right\|$ equals to $n-1$.

The Cauchi problem for differential equation (2.1.3.1) is put in the following way: find such solution of differential equation (2.1.3.1) $u = u(x^1, x^2, ..., x^n)$, which at $x^k = x^k(q_1, q_2, ..., q_{n-1})$ coincides with $u(q_1, q_2, ..., q_{n-1})$, or in other words, it is necessary to find such solution to equation (2.1.3.1) which runs through variety of (2.1.3.5).

For solving the Cauchi problem, as earlier, we'll apply the characteristic system of (2.1.3.2) which along with variables x^k, u also contains variables p_k. Initial data for x^k and u are determined in the form of $n-1$ dimensional variety (2.1.3.5), and as far as initial conditions for p_k parameters are concerned, they are determined from the following system:

$$\frac{\partial u}{\partial q_i}=\sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial q_i} \quad i=1,2,...,n-1,$$
$$F\left[x^k(q_1,q_2,...,q_{n-1}),u(q_1,q_2,...,q_{n-1}),p_k(q_1,q_2,...,q_{n-1})\right]=0 \qquad (2.1.3.6)$$

The first $n-1$ equity of this system is called a strip condition, and parameters $x^k(q_1,q_2,...,q_{n-1})$, $u(q_1,q_2,...,q_{n-1})$, $p_k(q_1,q_2,...,q_{n-1})$ $(k=1,2,...,n)$ – are strip coordinates.

Let's assume that this is a system from n equations relative to n independent variables $p_k(q_1,q_2,...,q_{n-1})$ $k=1,2,...,n$ (function $x^k(q_1,q_2,...,q_{n-1})$, and $u(q_1,q_2,...,q_{n-1})$) are determined per (2.1.3.5), and has a single solution. Namely these functions determine the initial conditions for p_k variables.

Let

$$x^k=\psi^k(s,c_1,c_2,...,c_{2n+1}),\ u=\psi^0(s,c_1,c_2,...,c_{2n+1}),$$
$$p_k=\eta_k(s,c_1,c_2,...,c_{2n+1}) \quad k=1,2,...,n - \qquad (2.1.3.7)$$

is a common solution of system (2.1.3.2), depending on $c_1,c_2,...,c_{2n+1}$ arbitrary constants of integration. We'll select these constants as function of parameters $q_1,q_2,...,q_{n-1})$, thus, that at $s=0$ the following conditions could have taken place:

$$\psi^k(0,c_1,c_2,...,c_{2n+1})=x^k(q_1,q_2,...,q_{n-1}),$$
$$\psi^0(0,c_1,c_2,...,c_{2n+1})=u(q_1,q_2,...,q_{n-1}), \qquad (2.1.3.8)$$
$$\eta_k(0,c_1,c_2,...,c_{2n+1})=p_k(q_1,q_2,...,q_{n-1}), \quad k=1,2,...,n$$

System (2.1.3.7) is differentiated relative to $s,c_1,c_2,...,c_{2n+1}$ and is uniquely solved relative to constants $c_1,c_2,...,c_{2n+1}$ [5], consequently from (2.1.3.7) and (2.1.3.8) we'll get

$$x^k=\varphi^k(s,q_1,q_2,...,q_{n-1}),$$
$$u=\varphi^0(s,q_1,q_2,...,q_{n-1}), \quad k=1,2,...,n \qquad (2.1.3.9)$$

If variety (2.1.3.5) is such that condition of

$$\begin{vmatrix} \frac{\partial\varphi^1}{\partial s} & \frac{\partial\varphi^2}{\partial s} & .. & \frac{\partial\varphi^n}{\partial s} \\ \frac{\partial\varphi^1}{\partial q_1} & \frac{\partial\varphi^2}{\partial q_1} & .. & \frac{\partial\varphi^n}{\partial q_1} \\ ... & ... & .. & ... \\ \frac{\partial\varphi^1}{\partial q_{n-1}} & \frac{\partial\varphi^2}{\partial q_{n-1}} & .. & \frac{\partial\varphi^n}{\partial q_{n-1}} \end{vmatrix} = \begin{vmatrix} \frac{\partial F}{\partial P_1} & \frac{\partial F}{\partial P_2} & .. & \frac{\partial F}{\partial P_n} \\ \frac{\partial\varphi^1}{\partial q_1} & \frac{\partial\varphi^2}{\partial q_1} & .. & \frac{\partial\varphi^n}{\partial q_1} \\ ... & ... & .. & ... \\ \frac{\partial\varphi^1}{\partial q_{n-1}} & \frac{\partial\varphi^2}{\partial q_{n-1}} & .. & \frac{\partial\varphi^n}{\partial q_{n-1}} \end{vmatrix} \neq 0 \qquad (2.1.3.10)$$

is met in all of its points (it also is satisfied in some of its vicinity in force of continuity of determinant), than (2.1.3.9) determines solution of the Cauchi problem in parametric form. Values of $\frac{\partial F}{\partial p_k}$ in the left side of this inequality contain p_k $k=1,2,...,n$, which represent solution of system (2.1.3.6).

When $s=0$, as per (2.1.3.7) and (2.1.3.8):

$$\varphi^k(0,q_1,q_2,...,q_{n-1})=x^k(q_1,q_2,...,q_{n-1}),$$
$$\varphi^0(0,q_1,q_2,...,q_{n-1})=u(q_1,q_2,...,q_{n-1}), \quad k=1,2,...,n, \qquad (2.1.3.11)$$

i.e., functions (2.1.3.9) satisfy the initial conditions of the Cauchi problem.

Equations (2.1.3.7) and (2.1.3.8) determine also the functions:

$$p_k=\omega_k(s,q_1,q_2,...,q_{n-1}) \quad k=1,2,...,n, \qquad (2.1.3.12)$$

which at $s=0$, in accord to (2.1.3.7) and (2.1.3.8) satisfy the following initial conditions:

$$\omega_k(0,q_1,q_2,...,q_{n-1})=p_k(q_1,q_2,...,q_{n-1}) \quad k=1,2,...,n \qquad (2.1.3.13)$$

Let's show that (2.1.3.9) and (2.1.3.12) satisfy condition:

$$F\left(\varphi^1, \varphi^2, ..., \varphi^n, \varphi^0, \omega_1, \omega_2, ..., \omega_n\right) = 0 \qquad \textit{(2.1.3.14)}$$

Actually, in accordance with that $\varphi^k, \varphi^0, \omega_k \;\; k = 1, 2, ..., n$ - is solution of characteristic system (2.1.3.2), than equation (2.1.3.3) is valid for them, i.e.:

$$F\left[\varphi^k(s, q_1, q_2, ..., q_{n-1}),\; \varphi^0(s, q_1, q_2, ..., q_{n-1}),\right.$$
$$\left.\omega_k(s, q_1, q_2, ..., q_{n-1})\right] = const$$

Hence, from (2.1.3.11) and (2.1.3.13), at $s = 0$, we'll get:

$$F\left[x^k(q_1, q_2, ..., q_{n-1}),\; u(q_1, q_2, ..., q_{n-1}),\right.$$
$$\left.p_k(q_1, q_2, ..., q_{n-1})\right] = const$$

And in accord to (2.1.3.6), the left hand side of this equity is equal to zero, i.e. $const = 0$. Thus, functions x^k, u, p_k, determined in accord to equities (2.1.3.9) and (2.1.3.12) satisfy (2.1.3.14) for any value of arguments $s, q_1, q_2, ..., q_{n-1}$. If, by that, it can be proved that

$$\frac{\partial u}{\partial x^k} = p_k = \omega_k(s, q_1, q_2, ..., q_{n-1}) \;\; k = 1, 2, ..., n \qquad \textit{(2.1.3.15)}$$

for any value of arguments $s, q_1, q_2, ..., q_{n-1}$, than we'll get finally that x^k and u, determined by (2.1.3.9) represent a solution of the Cauchi problem. For this, let's consider the following expressions:

$$w_0 = \frac{\partial u}{\partial s} - \sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial s}, \quad w_i = \frac{\partial u}{\partial q_i} - \sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial q_i}, \qquad \textit{(2.1.3.16)}$$
$$i = 1, 2, ..., n-1$$

where, x^k, u and p_k are determined in accord to equities of (2.1.3.9) and (2.1.3.12).

In conformity to characteristic system (2.1.3.2) $w_0 = 0$. From (2.1.3.16) we'll have:

$$\frac{\partial w_i}{\partial s} - \frac{\partial w_0}{\partial q_i} = \sum_{j=1}^{n}\left(\frac{\partial p_j}{\partial q_i}\frac{\partial x^j}{\partial s} - \frac{\partial p_j}{\partial s}\frac{\partial x^j}{\partial q_i}\right).$$

If we substitute here the values of $\frac{\partial x_j}{\partial s}$ and $\frac{\partial p_j}{\partial s}$ from characteristic system of (2.1.3.2), than we'll get:

$$\frac{\partial w_i}{\partial s} = \sum_{j=1}^{n}\left(\frac{\partial p_j}{\partial q_i}\frac{\partial F}{\partial p_j} + \frac{\partial x^j}{\partial q_i}\frac{\partial F}{\partial q_i} + p_j\frac{\partial F}{\partial u}\frac{\partial x^j}{\partial q_i}\right), \; i = 1, 2, ..., n-1$$

From (2.1.3.14), on the other hand, we'll have:

$$\sum_{j=1}^{n}\left(\frac{\partial F}{\partial x^j}\frac{\partial x^j}{\partial q_i} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial q_i} + \frac{\partial F}{\partial p_j}\frac{\partial p_i}{\partial q_i}\right) = 0, \quad i = 1, 2, ..., n-1$$

These last two equities give:

$$\frac{\partial w_i}{\partial s} = \frac{\partial F}{\partial u}\left(\sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial q_i} - \frac{\partial u}{\partial q_i}\right), \quad i = 1, 2, ..., n-1$$

Hence, with allowance of (2.1.3.16) we'll get:

$$\frac{\partial w_i}{\partial s} + \frac{\partial F}{\partial u} w_i = 0, \quad i = 1, 2, ..., n-1 \qquad \textit{(2.1.3.17)}$$

In accordance with (2.1.3.6), functions $w_i(s, q_1, q_2, ..., q_{n-1}) \;\; (i = 1, 2, ..., n-1)$, satisfy homogeneous initial conditions:

$$w_i = 0 \quad \text{at} \quad s = 0, \quad i = 1, 2, ..., n-1 \qquad \textit{(2.1.3.18)}$$

from (2.1.3.17) and (2.1.3.18) it is clear that

$$w_i(s, q_1, q_2, ..., q_n) \equiv 0, \quad i = 1, 2, ..., n-1$$

consequently (2.1.3.16) will have the form of:

$$\frac{\partial u}{\partial s}=\sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial s},\ \frac{\partial u}{\partial q_i}=\sum_{j=1}^{n} p_j \frac{\partial x^j}{\partial q_i},\ i=1,2,...,n-1$$

From parametric dependence between u and x^k, determined by (2.1.3.9) we'll have:

$$\frac{\partial u}{\partial s}=\sum_{j=1}^{n} \frac{\partial u}{\partial x^j}\frac{\partial x^j}{\partial s},\ \frac{\partial u}{\partial q_i}=\sum_{j=1}^{n} \frac{\partial u}{\partial x^j}\frac{\partial x^j}{\partial q_i},\ i=1,2,...,n-1$$

Comparing the last two systems and with allowance of condition of (2.1.3.10) is valid, we'll get:

$$p_k(s,q_1,q_2,...,q_{n-1})=\frac{\partial u}{\partial x^k},\ k=1,2,...,n$$

Than from (2.1.3.14) we'll have:

$$F\left(x^1,x^2,...,x^n,u,\frac{\partial u}{\partial x^1},\frac{\partial u}{\partial x^2},...,\frac{\partial u}{\partial x^n}\right)=0$$

i.e., (2.1.3.9) satisfies the equation (2.1.3.1)

In accordance with (2.1.3.10) system of (2.1.3.9) is reversible through unique way relative to variables $s,q_1,q_2,...,q_{n-1}$. It means that in vicinity of variety (2.1.3.5) in which (2.1.3.10) is valid, the solution of Cauchi problem is unique.

Let the variety (2.1.3.5) is such that condition

$$\begin{vmatrix} \frac{\partial F}{\partial p_1} & \frac{\partial F}{\partial p_2} & .. & \frac{\partial F}{\partial p_n} \\ \frac{\partial x^1}{\partial q_1} & \frac{\partial x^2}{\partial q_1} & .. & \frac{\partial x^n}{\partial q_1} \\ ... & ... & .. & ... \\ \frac{\partial x^1}{\partial q_{n-1}} & \frac{\partial x^2}{\partial q_{n-1}} & .. & \frac{\partial x^n}{\partial q_{n-1}} \end{vmatrix}=0 \qquad (2.1.3.19)$$

is valid in its points, where $p_k(q_1,q_2,...,q_{n-1})$ $(k=1,2,...,n)$ represent solution of system (2.1.3.6). Owing to continuity of determinant, this condition is valid also for some vicinity of variety (2.1.3.5). It is clear from this equity that:

$$\frac{\partial F}{\partial p_k}=\sum_{i=1}^{n-1}\lambda_i(q_1,q_2,...,q_{n-1})\frac{\partial x^k}{\partial q_i},\ k=1,2,...,n \qquad (2.1.3.20)$$

where, $\lambda_i(q_1,q_2,...,q_{n-1})$ $(i=1,2,...,n-1)$ are some continuous functions. If, along with that, is also valid the condition of

$$\begin{aligned} &\sum_{k=1}^{n} p_k \frac{\partial F}{\partial p_k}=\sum_{i=1}^{n}\lambda_i \frac{\partial u}{\partial q_i} \\ &-\frac{\partial F}{\partial x^k}-\frac{\partial F}{\partial p_k}p_k=\sum_{i=1}^{n-1}\lambda_i\frac{\partial p_k}{\partial q_i},\ k=1,2,...,n \end{aligned} \qquad (2.1.3.21)$$

where p_k $(k=1,2,...,n)$ is solution of system (2.1.3.6), than the variety (2.1.3.5) is called the variety of characteristic strips.

It is proved [4] that if condition (2.1.3.19) is correct for variety (2.1.3.5), than for availability of Cauchi problem it is necessary and sufficient that (2.1.3.5) were variety of characteristic strips. In this case, the Cauchi problem has infinitely many solutions.

Thus:

a) If variety (2.1.3.5) is such that condition (2.1.3.10) is valid, than Cauchi problem for equation (2.1.3.1) has the only solution;
b) If variety (2.1.3.5) meets condition of (2.1.3.19), than in order that Cauchi problem has solution, it is necessary and sufficient that variety (2.1.3.5) be variety of characteristic

strips. In this case, the Cauchi problem has infinitely multiple solutions, and (2.1.3.5) is variety of ramification of solution.

2.2 KINEMATICS OF NONHOLONOMIC TRANSFORMATIONS

2.2.1 DIVISION OF METRICS INTO TIME AND SPACE PARTS

It was mentioned above that four-dimensional space-time variety this four-dimensional space signifying $+---$ may be applied to describe such a variety distinguish any reference system $SR(x^k, \vec{e}^k)$. From four-dimensionless numbers x^k, it is not possible to coordinates, and consequently, they can't explain time and distance in three-dimensional space. Realization of orientation in three-dimensional space in terms of time distance and direction, in these four numbers is an independent and not trivial problem.

For instance, to determine the flow of time in some point of three-dimensional space, it is necessary to have opportunity of fixing such point, i.e., in four-dimensional space-time variety such world line has to be distinguished that corresponds to a given point of three-dimensional space. Or perhaps, if it is required to determine distance between two points of three-dimensional space, than we'll have opportunity of fixing their coordinates simultaneously, having understood this word in definite meaning of this word. These requirements implementing correspondence between diverted meaning and usual (rational) notions of three-dimensional space and time, are realizable through application of intermediate, special reference system in which the space and time parts of metrics are separated. Such separation is realizable through various methods including the method of orthogonalization of time and space coordinates.

Thus, the real four-dimensional space-time variety, which shall be further dealt with, has signature of $+---$; it means that

$$g = \det(g_{ik}) < 0 \qquad (2.2.1.1)$$

Let's consider a fixed point of four-dimensional space-time variety with coordinates $x^k = const$; in this point g_{ik} are fixed numbers. Let's draw a quadratic form in this point

$$I = g_{ik}\zeta^i\zeta^k, \qquad (2.2.1.2)$$

where ζ^i is some contra-variant vector. As it is known, it is always possible to select [4] such orthogonal transformation of vector

$$\zeta^i = \alpha^i_p \zeta'^p \qquad (2.2.1.3)$$

where $\|\alpha^i_p\|$ is a transformation matrix, and ζ'^p - are new components of vector, that reduce quadratic form (2.2.1.2) of vector to a canonical form.

In order to determine such transformation, i.e. to define a matrix $\|\alpha^i_p\|$, let's put the following extreme problem: to find such ζ^i parameters of value, which satisfy additional conditions of

$$\delta_{ik}\zeta^i\zeta^k = 1 \qquad (2.2.1.4)$$

and minimize quantity I. Here

$$\delta_{ik} = \begin{cases} 1 & at \ i = k, \\ 0 & at \ i \neq 0 \end{cases} \qquad (2.2.1.5)$$

When applying the Lagrangian method of indefinite multitudes [7], it is easy to show that these extreme values of variables ζ^i satisfy the following system of linear algebraic equations of fourth order:

$$g_{ik}\zeta^k - \lambda\delta_{ik}\zeta^k = 0 \qquad (2.2.1.6)$$

which in totality to (2.2.1.4) represent a complete system of equations in relation to ζ^i and λ.

The (2.2.1.6) is a homogeneous system, that's why in order that it has a nontrivial solution, λ should satisfy the following algebraic equation:

$$\left|g_{ik} - \lambda\delta_{ik}\right| = 0 \qquad (2.2.1.7)$$

This equation has four roots $\lambda_{k'}$ $(k = 0,1,2,3)$ which are called the proper value of matrix $\left\|g_{ik}\right\|$. One of these roots is positive (let's note it as λ_0), whereas the rest roots are negative - $\lambda_{\alpha'} < 0$.

With allowance that $g_{ik} = g_{ki}$, it is easy to show [10] that all roots $\lambda_{k'}$ are valid, and consequently valid are all solutions of system (2.2.1.6) relative to $\zeta^i_{k'}$, corresponding to separate roots $\lambda_{k'}$ of equation (2.2.1.7). To four different solutions of $\lambda_{r'}$ of equation (2.2.1.7) correspond to four different solutions of system (2.2.1.6) $\zeta^i_{r'}$, provided the values $\zeta^i_{r'}$ for each fixed value r are determined with accuracy of arbitrary constant multiplier the value of which can be determined through application of rating condition of (2.2.1.4). In accord to symmetry of metric tensor $(g_{ik} = g_{ki})$, these solutions $\zeta^i_{r'}$ constitute the orthogonal system of four vectors $\zeta^i_{0'}, \zeta^i_{1'}, \zeta^i_{2'}$ and $\zeta^i_{3'}$ [10]:

$$g_{ik}\zeta^i_{r'}\zeta^k_{l'} = \delta_{ik}\zeta^i_{r'}\zeta^k_{l'} = 0 \quad \text{at} \quad r \neq l \qquad (2.2.1.8)$$

Conditions (2.2.1.4) and (2.2.1.8) can be unified in the form of a condition

$$\delta_{ik}\zeta^i_{r'}\zeta^k_{l'} = \delta_{rl} \qquad (2.2.1.9)$$

In case of multiple roots, i.e. when $\lambda_{r'} = \lambda_{l'}$ $r \neq l$ $(r \neq 0,\ l \neq 0)$, the system of equations (2.2.1.4) and (2.2.1.6) ambiguously determine the corresponding ζ^k values, and at the expense of remained degree of freedom it is always possible to achieve fulfillment of condition of (2.2.1.8).

All solutions of systems (2.2.1.4) and (2.2.1.6) determine the matrix $\left\|\zeta^i_{r'}\right\|$, based on which in infinitesimals vicinity of selected point $x^k = const$ of four-dimensional space-time variety, the linear transformation can be determined as per following equities:

$$d\underset{e}{x}^k = \zeta^k_{r'}d\underset{e}{x}'^r \qquad (2.2.1.10)$$

In accord to it, we'll have:

$$g_{ik}d\underset{e}{x}^i\, d\underset{e}{x}^k = g_{ik}\zeta^i_{r'}\zeta^k_{l'}dx'^r dx'^l \qquad (2.2.1.11)$$

With allowance of (2.2.1.6), we'll get:

$$g_{ik}d\underset{e}{x}^i\, d\underset{e}{x}^k = \sum_{l=0}^{3}\lambda_{l'}\delta_{ik}\zeta^i_{r'}\zeta^k_{l'}d\underset{e}{x}'^r\, d\underset{e}{x}'^l$$

This equity, in accordance with (2.2.1.8) and (2.2.1.4), is simplified and will have the following form:

$$g_{ik}d\underset{e}{x}^i\, d\underset{e}{x}^k = \lambda_{0'}\left(dx'^0\right)^2 + \lambda_{1'}\left(dx'^1\right)^2 + \lambda_{2'}\left(dx'^2\right)^2 + \lambda_{3'}\left(dx'^3\right)^2 \qquad (2.2.1.12)$$

Transformation of (2.2.1.10), which reduces the quadratic form of $g_{ik}d\underset{e}{x}^i\, d\underset{e}{x}^k$ to a canonical form of (2.2.1.12), is orthogonal, so as the matrix $\left\|\zeta^i_{r'}\right\|$ satisfies condition (2.2.1.9).

Along with transformation (2.2.1.10), the following transformation may be considered as well

$$\underset{e}{d x}^{k} = \zeta^{k}_{0'} \underset{e}{d x}'^{0} + a^{k}_{\mu} \underset{e}{d x}''^{\mu}, \qquad (2.2.1.13)$$

where,

$$\underset{e}{d x}'^{\mu} = b^{\mu}_{\nu} \underset{e}{d x}''^{\nu}, \quad a^{k}_{\mu} = \zeta^{k}_{\eta'} b^{\eta}_{\mu},$$

so that

$$\delta_{ik} \zeta^{i}_{0'} a^{k}_{\mu} = 0 \qquad \text{when} \qquad \mu = 1, 2, 3 \qquad (2.2.1.15)$$

In this case

$$g_{ik} \underset{e}{d x}^{i} \underset{e}{d x}^{k} = \lambda_{0'} \left(\underset{e}{d x}'^{0} \right)^{2} + \gamma_{\alpha\beta} \underset{e}{d x}''^{\alpha} \underset{e}{d x}''^{\beta}, \qquad (2.2.1.16)$$

where,

$$\gamma_{\alpha\beta} = g_{ik} a^{i}_{\alpha} a^{k}_{\beta}$$

With allowance that $\lambda_{\alpha'} < 0$, from (2.2.1.12) and (2.2.1.16) it is clear that $\|\gamma_{\alpha\beta}\|$ – is negatively defined matrix, that is why any vector, for which $\underset{e}{d x}'^{0} = 0$ and dx''^{μ} are arbitrary infinitesimals – is space-like.

So as the matrix elements $\|\zeta^{i}_{r'}\|$ generally don't satisfy conditions of

$$\frac{\partial \zeta^{k}_{r'}}{\partial x'^{l}} = \frac{\partial \zeta^{k}_{l'}}{\partial x'^{r}}$$

for any values of indices k, r and l, than locally linear transformation (2.2.1.10), in all four-dimensional space-time variety, represents nonholonomic transformation.

Through variation of infinitesimals of $dx^{k}_{l'}$, we'll describe some infinitively small interval of four-dimensional space-time variety near fixed point $x^{k} = const$. With allowance that in all points of this small interval of the value of components of metric tensor g_{ik} slightly differs from these very components in point $x^{k} = const$, than this interval of four-dimensional space-time variety, with high degree of accuracy may be considered as plane having constant metric $g_{ik} / _{x^{k}} = const$, , and tangential of real four-dimensional space-time variety.

2.2.2 DETERMINATION OF INFINITELY SMALL INTERVAL OF TIME AND INFINITELY SMALL DISTANCE

Dividing of metrics (interval) into time and space parts, defined through equity (2.2.1.16) (or equation (2.2.1.12)), is realized in some small vicinity of fixed point $x^{k} = const$, of four-dimensional space-time variety, $g_{ik}, \lambda_{0'}$ and $\gamma_{\alpha\beta}$ are the constant numbers in this vicinity, whereas $\underset{e}{d x}'^{0}$ and $\underset{e}{d x}''^{\alpha}$ are arbitrary infinitesimals which don't represent differentials of some variables x'^{0} and x''^{α}.

Let's assume that

$$\underset{e}{d x}''^{\alpha} = 0 \qquad (\text{or } \underset{e}{d x}'^{\alpha} = 0)$$

than in accordance with (2.2.1.16) (or (2.2.1.12) interval coincides with $\lambda_{0'}\left(d\underset{e}{x}'^{0}\right)^{2}$, and equalizing it to c^2dt^2, we'll determine the infinitesimal interval of time

$$dt = \frac{1}{c}\sqrt{\lambda_{0'}}\,d\underset{e}{x}'^{0}, \qquad (2.2.2.1)$$

referenced from some moment of time per hour placed in fixed point of three-dimensional space. Moment of reference of interval of time dt and location of hour in three-dimensional space correspond to selected fixed point $x^k = const$ of four-dimensional space-time variety.

In this connection, it should be noted that by means of nonholonomic transformations, it is possible to achieve fulfillment of definite requirements relative to vectors, such as, for instance $d\underset{e}{x}^{k}$, and not relative to coordinates. That is why, when applying the nonholonomic transformations it is not allowed to fix the location (coordinates) of bodies (clock) in standard three-dimensional space as well as it is not possible to fix the moment of time in some point of three-dimensional space. In considered case, only intervals of time and space between two points in infinitesimals interval of point $x^k = const$ can be determined. Determination of location (coordinates) of clock in three-dimensional space, as well fixing of the moment of time in selected point of three-dimensional space, corresponding to fixed point $x^k = const$ of four-dimensional space-time variety, can be realized by method of coordinate transformation, which is considered in next paragraph.

Summing up the infinitesimals dt corresponding to different points x^k of four-dimensional space-time variety does not determine the time interval readable per hour, so as it will be shown below, to various values of coordinates x^k; to say in general, correspond different clock located at different points of three-dimensional space. Such summation, of course, can determine some interval of time, that may have some sense in connection to same physical requirements.

Quite similarly, assuming that $d\underset{e}{x}'^{0} = 0$ from (2.2.1.12) and (2.2.1.16) we shall have

$$\begin{aligned} -d\ell^2 &= \lambda_{1'}\left(d\underset{e}{x}'^{1}\right)^2 + \lambda_{2'}\left(d\underset{e}{x}'^{2}\right)^2 + \lambda_{3'}\left(d\underset{e}{x}'^{3}\right)^2 = \\ &= \gamma_{\alpha\beta}\,d\underset{e}{x}''^{\alpha}\,d\underset{e}{x}''^{\beta} \end{aligned} \qquad (2.2.2.2)$$

These equities determine distance $d\ell$ between two points of infinitesimals of three-dimensional space corresponding to mentioned infinitesimals, plane space-time variety.

2.2.3 VELOCITY AND ACCELERATION

Assuming, that

$$x^k = x^k(p) \qquad (2.2.3.1)$$

where p - is some invariant parameter, world line of four-dimensional space-time variety, and $d\underset{e}{x}^{k}$ is infinitesimals tangent vector of world line in some of its point, corresponding to increment of dp to parameter p. In equities of (2.2.1.10), we'll substitute the values $d\underset{e}{x}^{k}$ correspondent by values

$$d\underset{e}{x}^k = \frac{d\underset{e}{x}^k}{dp}dp \tag{2.2.3.2}$$

and we'll get:

$$dx'^k = \eta^k dp \tag{2.2.3.3}$$

With allowance of this increment dp can be tied with interval of time dt:

$$dt = \frac{1}{c}\sqrt{\lambda_{0'}}\eta^0 dp \tag{2.2.3.4}$$

According to (2.2.3.3) and (2.2.3.4) to infinitesimals changes of time dt corresponds to infinitesimals changes of coordinates $d\underset{e}{x}^k$ in $SR(\vec{e}^k, x^k)$, $d\underset{e}{x}'^k$ in $SR(\vec{e}'^k, x^k)$ and $d\underset{e}{x}''^k$ in $SR(\vec{e}''^k, x^k)$. In connection with it, the notion of speed and acceleration of coordinate changes can be introduced:

$$\frac{d\underset{e}{x}'^k}{dt} \quad \text{or} \quad \frac{d\underset{e}{x}'^0}{dt}, \ \frac{d\underset{e}{x}''^\alpha}{dt} \tag{2.2.3.5}$$

It is clear that

$$\underset{e}{v}'^\alpha = \frac{d\underset{e}{x}'^\alpha}{dt} \quad \text{and} \quad \underset{e}{v}''^\alpha = \frac{d\underset{e}{x}''^\alpha}{dt} \tag{2.2.3.6}$$

Constitute the space-like vectors which, in accord to (2.2.1.14), are interconnected by transformation law

$$\underset{e}{v}'^\alpha = b^\alpha_\mu \underset{e}{v}''^\mu \tag{2.2.3.7}$$

Motion speed of the point in three-dimensional space is determined as absolute value of these vectors:

$$\begin{gathered} v'_e = \sqrt{-\lambda_{1'}\left(\underset{e}{v}'^1\right)^2 - \lambda_{2'}\left(\underset{e}{v}'^2\right)^2 - \lambda_{3'}\left(\underset{e}{v}'^3\right)^2} \\ \underset{e}{v}'' = \sqrt{-\gamma_{\alpha\beta}\underset{e}{v}''^\alpha \underset{e}{v}''^\beta}, \quad \underset{e}{v}' = \underset{e}{v}'' \end{gathered} \tag{2.2.3.8}$$

Assuming $\underset{e}{\tau}''^\alpha$ as space-like vector of unit vector

$$-\gamma_{\alpha\beta}\underset{e}{\tau}''^\alpha \underset{e}{\tau}''^\beta = 1,$$

then the speed projection $\underset{e}{v}''^\alpha$ on $\underset{e}{v}''^\alpha$ is equal to

$$\underset{e}{v}''_v = -\gamma_{\alpha\beta}\underset{e}{\tau}''^\alpha = \sqrt{-\gamma_{\alpha\beta}\underset{e}{v}''^\alpha \underset{e}{v}''^\beta}\cos\varphi, \tag{2.2.3.9}$$

where, φ is angle between vectors of τ''^α_e and v''^α_e. Hence

$$\cos\varphi = \frac{-\gamma_{\alpha\beta}\underset{e}{\tau}''^\alpha \underset{e}{v}''^\beta}{\sqrt{-\gamma_{\alpha\beta}\underset{e}{v}''^\alpha \underset{e}{v}''^\beta}} \tag{2.2.3.10}$$

So as $\left\|-\gamma_{\varepsilon\beta}\right\|$ is not negatively determined matrix, it is easy to show that $|\cos\varphi| \le 1$. Actually, from inequity

$$-\gamma_{\alpha\beta}\left(\eta\underset{e}{\tau}''^\alpha + \underset{e}{v}''^\alpha\right)\left(\eta\underset{e}{\tau}''^\beta + v''^\beta\right) \ge 0,$$

where η is an arbitrary parameter, obviously:

$$\eta^2 + 2\left(-\gamma_{\alpha\beta} \underset{e}{\tau}^{''\alpha} \underset{e}{v}^{''\beta}\right)\eta + \left(-\gamma_{\alpha\beta} \underset{e}{v}^{''\alpha} \underset{e}{v}^{''\beta}\right) \geq 0$$

This inequity means that discriminant of square trinomial (relative to η) is nonpositive value, i.e.

$$\left(-\gamma_{\alpha\beta} \underset{e}{\tau}^{''\alpha} \underset{e}{v}^{''\beta}\right)^2 + \gamma_{\alpha\beta} \underset{e}{v}^{''\alpha} \underset{e}{v}^{''\beta} \leq 0\,.$$

Hence, according to (2.2.3.10) we'll have $|\cos\alpha| \leq 1$.

(1,0,0), (0,1,0) and (0,0,1) three vectors correspondingly along reference points are $\vec{e}''^1$, $\vec{e}''^2$ and $\vec{e}''^3$. Lengths of these vectors correspondingly equal to $\sqrt{-\gamma_{11}}$, $\sqrt{-\gamma_{22}}$ and $\sqrt{-\gamma_{33}}$, that's why $\left(1/\sqrt{-\gamma_{11}}, 0, 0\right)$, $\left(0, 1/\sqrt{-\gamma_{22}}, 0\right)$ and $\left(0, 0, 1/\sqrt{-\gamma_{33}}\right)$ are unit vectors and projections of vector $\underset{e}{v}^{''\alpha}$ on reference points $\vec{e}''^1$, $\vec{e}''^2$ and $\vec{e}''^3$ correspondingly equal to:

$$\left(\underset{e}{v}''\right)_{1'} = -\frac{\gamma_{1\beta} \underset{e}{v}^{''\beta}}{\sqrt{-\gamma_{11}}}, \quad \left(\underset{e}{v}''\right)_{2'} = -\frac{\gamma_{2\beta} \underset{e}{v}^{''\beta}}{\sqrt{-\gamma_{22}}}, \quad \left(\underset{e}{v}''\right)_{3'} = -\frac{\gamma_{3\beta} \underset{e}{v}^{''\beta}}{\sqrt{-\gamma_{33}}}, \qquad (2.2.3.11)$$

and in the system of reference points $\vec{e}'^1$, $\vec{e}'^2$, $\vec{e}'^3$ these projection take the form of:

$$\left(\underset{e}{v}'\right)_{1'} = \sqrt{-\gamma_{11}}\, v'^1, \quad \left(\underset{e}{v}'\right)_{2'} = \sqrt{-\gamma_{22}}\, v'^2, \quad \left(\underset{e}{v}'\right)_{3'} = \sqrt{-\gamma_{33}}\, v'^3 \qquad (2.2.3.12)$$

Acceleration of change of coordinate is determined in accord to equities:

$$\underset{e}{a}^{'\alpha} = \frac{D\underset{e}{v}^{'\alpha}}{dt} = \underset{e}{v}^{'\alpha}_{,\mu} \underset{e}{v}^{'\mu} = \left(\frac{\partial \underset{e}{v}^{'\alpha}}{\partial \underset{e}{x}^{'\mu}} + H'^{\alpha}_{\mu\nu} \underset{e}{v}^{'\nu}\right) \underset{e}{v}^{'\mu},$$

$$\underset{e}{a}^{''\alpha} = \frac{D\underset{e}{v}^{''\alpha}}{dt} = \underset{e}{v}^{''\alpha}_{,\mu} \underset{e}{v}^{'\mu} = \left(\frac{\partial \underset{e}{v}^{''\alpha}}{\partial \underset{e}{x}^{''\mu}} + H''^{\alpha}_{\mu\nu} \underset{e}{v}^{''\nu}\right) \underset{e}{v}^{''\mu} \qquad (2.2.3.13)$$

$\underset{e}{a}^{'\alpha}$ and $\underset{e}{a}^{''\alpha}$ – are three-dimensional space-like vectors which are interconnected between them by transformation law ($\underset{e}{v}^{'\alpha}_{,\mu}$ and $\underset{e}{v}^{'\mu}$ is a tensor of second order and vector, and $\underset{e}{v}^{'\alpha}_{,\mu} \underset{e}{v}^{'\mu}$ is vector) -

$$a'^{\alpha}_{e} = b^{\alpha}_{\mu} \underset{e}{a}^{''\mu} \qquad (2.2.3.14)$$

All major relations for speed component, obtained above, are valid for acceleration component.

2.3 KINEMATICS OF HOLONOMIC TRANSFORMATIONS

2.3.1 TIME AND DISTANCE IN THREE-DIMENSIONAL SPACE

In the considered case, by transforming of coordinates $x'^k = x'^k(x^0, x^1, x^2, x^3)$, the reference system $SR(x^k, \vec{E}^k)$ is transformed into system of $SR(x'^k, \vec{E}'^k)$, provided the metrics in $SR(x^k, \vec{E}'^k)$ is divided into time and space parts in all points of space. This procedure is implemented by two methods, the first of which is more elementary and represents itself direct generalization of local method described in §2.2. Initial point of this case is equation (2.2.1.7).

Assuming $\underset{0}{\lambda}$ is positive root of this equation. After substitution from (2.2.1.4) and (2.2.1.6) we'll determine the corresponding values of $\underset{0}{\zeta}^k(x^0, x^1, x^2, x^3)$. In considered case we'll assume that these four functions satisfy the following equations:

$$\underset{0}{\zeta}_i = g_{ip}\,\underset{0}{\zeta}^0 = \mu \frac{\partial \varphi^0}{\partial x^i}, \qquad (2.3.1.1)$$

where, $\mu(x^0, x^1, x^2, x^3)$ and $\varphi^0(x^0, x^1, x^2, x^3)$ are some functions. These conditions can be re-written in other way if equation (2.2.1.6) is used, in particular:

$$\underset{0}{\lambda}\,\delta_{ip}\,\underset{0}{\zeta}^p = \mu \frac{\partial \varphi_0}{\partial x^i}. \qquad (2.3.1.2)$$

As it is known, the (2.3.1.1) and (2.3.1.2) are not always valid. In order to fulfill these conditions, it is necessary and sufficient that four functions $\underset{0}{\zeta}^k(x^0, x^1, x^2, x^3)$ satisfy the following equities:

$$\frac{\partial}{\partial x^k}\left(\frac{1}{\mu} g_{ip}\,\underset{0}{\zeta}^p\right) = \frac{\partial}{\partial x^i}\left(\frac{1}{\mu} g_{kp}\,\underset{0}{\zeta}^p\right), \qquad (2.3.1.3)$$

or

$$\frac{\partial}{\partial x^k}\left(\frac{\underset{0}{\lambda}}{\mu}\delta_{ip}\,\underset{0}{\zeta}^p\right) = \frac{\partial}{\partial x^i}\left(\frac{\underset{0}{\lambda}}{\mu}\delta_{kp}\,\underset{0}{\zeta}^p\right) \qquad (2.3.1.4)$$

for a certain function $\underset{0}{\mu}^k(x^0, x^1, x^2, x^3)$. When considering the matrix structure $\|\delta_{ik}\|$, it is clear from the last equity that $\underset{0}{\lambda}/\mu$ plays the role of integrating multiplier.

Consequently, in the first case we mean that conditions of (2.3.1.3) or (2.3.1.4) are fulfilled and functions $\zeta^k_{0'}$ will be represented in the form of (2.3.1.1).

Let's compose following differential equation in partial derivatives of first order:

$$g^{ik}\,\underset{0}{\zeta}_i \frac{\partial \varphi}{\partial x^k} = 0 \qquad (2.3.1.5)$$

As it was mentioned above, it has three independent solutions:

$$\varphi^\alpha(x^0, x^1, x^2, x^3) \qquad \alpha = 1, 2, 3,$$

which, in accordance to (2.3.1.1) and (2.3.1.5) are orthogonal to φ^0, in particular:

$$\frac{1}{\mu} g^{ik} \frac{\partial \varphi^0}{\partial x^i}\frac{\partial \varphi^\alpha}{\partial x^k} = 0 \qquad (2.3.1.6)$$

Let's introduce new coordinates:

$$\begin{aligned} x'^0 &= \varphi^0(x^0, x^1, x^2, x^3), \\ x'^\alpha &= \varphi^\alpha(x^0, x^1, x^2, x^3) \end{aligned} \qquad (2.3.1.7)$$

By force of independence of functions φ^α, the rank of matrix $\left\|\frac{\partial\varphi^\alpha}{\partial x^k}\right\|$ is equal to three. Than, in accord to (2.3.1.6) it is easy to show that four functions of $\varphi^k(x^0,x^1,x^2,x^3)-$ are also independent and consequently the condition of:

$$\left|\frac{\partial\varphi^k}{\partial x^i}\right|\neq 0 \qquad (2.3.1.8)$$

is valid.

Assuming that φ^k are dependent functions and there takes place the equity:

$$F(\varphi^0,\varphi^1,\varphi^2,\varphi^3)=0 \qquad (2.3.1.9)$$

Hence,

$$\frac{\partial F}{\partial\varphi^0}\frac{\partial\varphi^0}{\partial x^k}+\frac{\partial F}{\partial\varphi^\alpha}\frac{\partial\varphi^\alpha}{\partial x^k}=0 \qquad (2.3.1.10)$$

Let's multiply these equities by $g^{ik}\frac{\partial\varphi^0}{\partial x^i}$ and summarize in accordance to index k: with allowance (2.3.1.6) we'll get:

$$\frac{\partial F}{\partial\varphi^0}g^{ik}\frac{\partial\varphi^0}{\partial x^i}\frac{\partial\varphi^0}{\partial x^k}=0,$$

i.e.

$$\frac{\partial F}{\partial\varphi^0}=0$$

It means that the function F should not depend on φ^0. Than (2.3.1.10) will have such a form

$$\frac{\partial F}{\partial\varphi^\alpha}\frac{\partial\varphi^\alpha}{\partial x^k}=0.$$

On the other hand, the rank of matrix $\left\|\frac{\partial\varphi^\alpha}{\partial x^k}\right\|$ equals to three, than from here will yield that $\frac{\partial F}{\partial\varphi^\alpha}=0$ i.e. F doesn't depend on neither functions φ^k.

Thus, there is no such function F for which (2.3.1.9) occurs, and consequently $\varphi^0,\varphi^1,\varphi^2,\varphi^3$ are functionally independent, i.e. condition of (2.3.1.8) is valid.

In new coordinates x'^k we have:

$$g'^{ik}=\frac{\partial\varphi_i}{\partial x^p}\frac{\partial\varphi^k}{\partial x^q}g^{pq} \qquad (2.3.1.11)$$

From it,

$$g'^{00}=\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}g^{pq}. \qquad (2.3.1.12)$$

This equity in accord to (2.3.1.1) can be re-written thus:

$$g'^{00}=\frac{1}{\mu^2}q^{pq}g_{ps}g_{qt}\zeta^s_{0'}\zeta^t_{0'}=\frac{1}{\mu^2}g_{st}\zeta^s_{0'}\zeta^t_{0'}$$

Considering (2.2.1.4) and (2.2.1.6), we'll get:

$$g'^{00}=\frac{1}{\mu^2}\lambda_{0'},$$

but as $\lambda_{0'}>0$, than it is obvious that $g'^{00}>0$.

From (2.3.1.11) for $g'^{0\alpha}$ will have:

$$g'^{0\alpha} = \frac{\partial \varphi^0}{\partial x^p} \frac{\partial \varphi^\alpha}{\partial x^q} g^{pq}$$

Hence, as per (2.3.1.1) we'll get:

$$g'^{0\alpha} = \frac{1}{\mu} g^{pq} \underset{0'}{\zeta}{}_p \frac{\partial \varphi^\alpha}{\partial x^q},$$

i.e. $g'^{0\alpha} = 0$, as φ^α represents itself a solution to equation (2.3.1.5).

Thus,

$$\left\| g'^{ik} \right\| = \begin{Vmatrix} g'^{00} & 0 & 0 & 0 \\ 0 & g'^{11} & g'^{12} & g'^{13} \\ 0 & g'^{12} & g'^{22} & g'^{23} \\ 0 & g'^{13} & g'^{23} & g'^{33} \end{Vmatrix}, \qquad (2.3.1.13)$$

provided that $g'^{00} > 0$. Hence

$$\left\| g'_{ik} \right\| = \begin{Vmatrix} g'_{00} & 0 & 0 & 0 \\ 0 & g'_{11} & g'_{12} & g'_{13} \\ 0 & g'_{12} & g'_{22} & g'_{23} \\ 0 & g'_{13} & g'_{23} & g'_{33} \end{Vmatrix}, \qquad (2.3.1.14)$$

where $g'_{00} = \frac{1}{g'^{00}} > 0$, and $\left\| g'^{\alpha\beta} \right\|$ and $\left\| g'_{\alpha\beta} \right\|$ are non-positively defined matrices.

Thus:

a) x'^0 is time-like coordinate line;

b) x'^1, x'^2 and x'^3 are space-like coordinate lines;

c) x'^0 is a line perpendicular to all lines x'^α and interval

$$ds^2 = g'^{00} (dx'^0)^2 + g'_{\alpha\beta} dx'^\alpha dx'^\beta \qquad (2.3.1.15)$$

is divided into time and space parts, provided that x'^0 is time coordinate, and x'^α is a space coordinate.

Let's consider the second case, when conditions (2.3.1.3) or (2.3.1.4) do not take place. In this case there is no such function φ^0, which would satisfy the condition of (2.3.1.1). However, during this it is always possible to select such function φ^0, which satisfies condition of

$$g^{ik} \frac{\partial \varphi^0}{\partial x^i} \frac{\partial \varphi^0}{\partial x^k} > 0. \qquad (2.3.1.16)$$

Actually, assuming $\psi\left(x^0, x^1, x^2, x^3\right)$ is some function, than in accord to § 2.1, the differential equation

$$g^{ik} \frac{\partial \varphi^0}{\partial x^i} \frac{\partial \varphi^0}{\partial x^k} = \psi^2 \qquad (2.3.1.17)$$

always has solution φ^0 in rather general conditions in relation to function ψ, and so as $\psi^2 > 0$, than it is obvious that solution of equation (2.3.1.17) satisfies condition of (2.3.1.16).

If function $\varphi^0\left(x^0, x^1, x^2, x^3\right)$ is determined, then instead of (2.3.1.5), the following differential equation should be drawn up for partial derivatives of first order relative to $\varphi^\alpha\left(x^0, x^1, x^2, x^3\right)$:

$$g^{ik} \frac{\partial \varphi^0}{\partial x^i} \frac{\partial \varphi}{\partial x^k} = 0 \qquad (2.3.1.18)$$

which in accord to §2.1 has three independent solutions φ^α. The totality of functions φ^0 and φ^α determines transformation of coordinates (2.3.1.7) and for g'^{ik} will give expression (2.3.1.13).

The second case is more general in comparison to the first one, and may be applied as well to fulfill conditions of (2.3.1.3) or (2.3.1.4).

Indication of clock located in space point of $x'^\alpha = const$ is determined from (2.3.1.15) by equity:

$$t = t_0 \pm \frac{1}{c}\int_0^{x'^0} \sqrt{g'_{00}}\, d\xi, \quad x'^\alpha = const,$$

where, t_0 is initial indication of clock, i.e. reading of clock corresponding to value $x'^0 = 0$. Symbol before integral is selected in such a way, that to fulfill condition $t > t_0$. t is function of variables x'^α; integration is implemented along time-like line $x'^\alpha = const$, that's why the value t in different points of three-dimensional space will be various, i.e. clock located in different points show different times:

$$\begin{aligned} t\left(x'^0, x'^1, x'^2, x'^3\right) &= t_0\left(x'^1, x'^2, x'^3\right) \pm \\ &\pm \frac{1}{c}\int_0^{x'^0} \sqrt{g'_{00}\left(\xi, x'^1, x'^2, x'^3\right)}\, d\xi \end{aligned} \qquad (2.3.1.19)$$

In this equity the coordinates x'^α are the parameters.

Fixation of time coordinate $x'^0 = const$ means that from four-dimensional space-time variety, we've selected the standard three-dimensional space, provided the clock located at its different points, show different time depending on its coordinates x'^α in accord to the law of (2.3.1.19). In this connection the following issue is of interest: if clock with coordinates $x_1'^\alpha$, showing time t_1, than what would be indication of t_2 clock with coordinates $x_2'^\alpha$. From mathematical viewpoint, this problem is reduced to determination of functional dependence between t_1 and t_2. It is determined from (2.3.1.19) as a system of two equations:

$$\begin{aligned} t_1 &= t_0\left(x_1'^1, x_1'^2, x_1'^3\right) \pm \frac{1}{c}\int_0^{x'^0} \sqrt{g'_{00}\left(\xi, x_1'^1, x_1'^2, x_1'^3\right)}\, d\xi \\ t_2 &= t_0\left(x_2'^1, x_2'^2, x_{21}'^3\right) \pm \frac{1}{c}\int_0^{x'^0} \sqrt{g'_{00}\left(\xi, x_2'^1, x_2'^2, x_2'^3\right)}\, d\xi \end{aligned} \qquad (2.3.1.20)$$

These equities determine functional dependence between t_1 and t_2 in parametric form; x'^0 is a parameter.

Development of this or that dynamic physical phenomenon can be referred to reading of different clock located at various points of space. For instance, the investigated dynamic phenomenon can be referred to reading of certain clock in the fixed point of three-dimensional space. In connection with it, from practical viewpoint, in some cases it could seem convenient to introduce a standard time t_c; reading of any isolated clock, for instance, located at initial point or at infinity of reference system

$$t_c = t_{c0} \pm \frac{1}{c}\int_0^{x'^0} \sqrt{g'_{00}(\xi, 0, 0, 0)}\, d\xi \qquad (2.3.1.21)$$

This equity in totality with equation (2.3.1.19) determines the functional dependence between t_c and t.

The major equation (2.3.1.19) and all equations yielded from it, expressed through line parameters x'^k and g'_{00}, can be expressed as well through initial parameters x^k and g_{ik}. Actually, from (2.3.1.12) and (2.3.1.7) we have:

$$g'_{00} = \frac{1}{g'^{00}} = \frac{1}{g^{pq} \frac{\partial \varphi^0}{\partial x^p} \frac{\partial \varphi^0}{\partial x^q}}, \quad dx'^0 = \frac{\partial \varphi^0}{\partial x^k} dx^k \qquad (2.3.1.22)$$

The reverse transforming of transformation (2.3.1.7) (it is always available there) unambiguously determines the coordinates of initial and current points of integration $\left(x'^\alpha = const\right)$:

$$\begin{aligned} & x_0^k = x^k(0, const, const, const) - \text{initial point}, \\ & x^k = x^k(x'^0, const, const, const) - \text{current point} \end{aligned} \qquad (2.3.1.23)$$

These equities at every possible values of x'^α (i.e. *const)* determine the coordinates of x^k points of standard three-dimensional space of four-dimensional space-time variety at initial (the first four equations) and optional (the last four equations) moments of time. Taking it into account from (2.3.1.19) and (2.3.1.22) we shall have:

$$\begin{aligned} & t\left(x'^0, x_0^0, x_0^\alpha\right) = t_0\left(x_0^0, x_0^\alpha\right) \pm \frac{1}{c} \int_0^{x'^0} \frac{d\xi}{\sqrt{g^{pq} \frac{\partial \varphi^0}{\partial x^p} \frac{\partial \varphi^0}{\partial x^q}}}, \\ & x'^0(x_0^0, x_0^\alpha) = 0 \end{aligned} \qquad (2.3.1.24)$$

Here in sub-integral function x^0, x^α are determined by second line of equation system (2.3.1.23).

As well as earlier in (2.3.1.24) x'^0 plays a role of parameter, by means of which the synchronization of clock at various points of three-dimensional space can be realized.

If practically it is not feasible to transform a system (2.3.1.7), than the integration in (2.3.1.19) can be realized in a different way, in particular, assuming that $x'^\alpha = const$, after differentiation from (2.3.1.7) we'll have:

$$dx'^0 = \frac{\partial \varphi^0}{\partial x^k} dx^k, \quad 0 = \frac{\partial \varphi^\alpha}{\partial x^k} dx^k \qquad (2.3.1.25)$$

As was shown above, functions φ^α are independent, that’s why the rank of matrix $\left\| \frac{\partial \varphi^\alpha}{\partial x^k} \right\|$ equals to three, than from the last three equations of the system (2.3.1.25) three out of four differentials dx^0, dx^α can be represented through one differential, for example, it can be written

$$dx^0 = \bar{a}^0 dx^2, \quad dx^1 = \bar{a}^1 dx^2, \quad dx^3 = \bar{a}^3 dx^2$$

and realized the integrating along the parameter x^2. In this respect none of variables of x^k has any advantage over the other ones, each of them can be used as integration parameter. Further the integration will be accomplished along variable x^0. For this purpose, from the last three systems of equities (2.3.1.25), differentials dx^α are expressed through dx^0

$$dx^\alpha = a^\alpha dx^0, \qquad (2.3.1.26)$$

than from the first equity of the same system we'll have:

$$dx'^0 = \left(\frac{\partial \varphi^0}{\partial x^0} + a^\alpha \frac{\partial \varphi^0}{\partial x^\alpha} \right) dx^0. \qquad (2.3.1.27)$$

With allowance of this equity we'll get from (2.3.1.24):

$$t\left(x^0, x_0^k\right)=t_0\left(x_0^k\right)\pm\frac{1}{c}\int_{x_0^0}^{x^0}\frac{\left(\frac{\partial\varphi^0}{\partial x^0}+a^\alpha\frac{\partial\varphi^0}{\partial x^\alpha}\right)dx^0}{\sqrt{g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}}}, \qquad (2.3.1.28)$$

$$x'^0(x_0^0, x_0^\alpha)=0$$

This equity permits to realize the synchronization of clock by means of parameter x^0. It is the most common and as particular case, contains in it the known expression to calculate the time [2,3]:

$$cdt=\pm\frac{1}{\sqrt{g_{00}}}\left(g_{00}dx^0+g_{0\alpha}dx^\alpha\right). \qquad (2.3.1.29)$$

The (2.3.1.29) is obtained in definite restrictions, relative to system of coordinates and metric tensor, and if these restrictions are followed, than differential of equity (2.3.1.28) coincides with (2.3.1.29).

Suppose that system of coordinates and metric tensor of four-dimensional space-time variety are such that

a) x^0 is time-like line, i.e.

$$g_{00}>0, \qquad (2.3.1.30)$$

b) for g_{0k} components there exists an integrating multiplier μ i.e. they permit presentation of

$$g_{0k}=\mu\frac{\partial\varphi^0}{\partial x^k}, \qquad (2.3.1.31)$$

than the (2.3.1.29) is valid.

Let's introduce new coordinates x'^k by following equities:

$$\begin{aligned} x'^0 &= x'^0(x^0, x^\alpha), \\ x'^\alpha &= x'^\alpha(x^\alpha) \end{aligned} \qquad (2.3.1.32)$$

and select corresponding functions in such a way that (2.3.1.32) would accomplish division of metric into time and space parts. When x'^0 and x'^α are such functions, than $g'_{0\alpha}=0$ and from obvious equity:

$$g_{ik}=g'_{pq}\frac{\partial x'^p}{\partial x^i}\frac{\partial x'^q}{\partial x^k}$$

we'll have:

$$g_{0\alpha}=g'_{00}\frac{\partial x'^0}{\partial x^0}\frac{\partial x'^0}{\partial x^\alpha}, \qquad g_{00}=g'_{00}\left(\frac{\partial x'^0}{\partial x^0}\right)^2.$$

Hence, in accord to (2.3.1.30) we have:

$$g'_{00}>0,\ \frac{\partial x'^0}{\partial x^0}=\sqrt{\frac{g_{00}}{g'_{00}}}, \qquad g_{0\alpha}=\sqrt{g_{00}g'_{00}}\frac{\partial x'^0}{\partial x^0} \qquad (2.3.1.33)$$

Thus, when x^0 is time-like coordinate line and g_{ik} through transformation (2.3.1.32) is divisible, than g_{0k} should satisfy conditions of (2.3.1.31), i.e. the necessary conditions.

These conditions are sufficient as well, i.e. when g_{0k} satisfy these conditions, than there's transformation of form (2.3.1.32) which realizes the division of metric g_{ik}. Actually, if (2.3.1.31) has occurred, then μ and φ^0 are fully determined functions. Let's determine x'^0 and g'_{00} as per following equities:

$$x'^0=\varphi^0(x^0, x^\alpha), \quad g'_{00}=\mu^2/g_{00} \qquad (2.3.1.34)$$

By force of (2.3.1.30) $g'_{00} > 0$. Let's select arbitrary functions $\varphi^{\alpha}(x^1, x^2, x^3)$, for which $\left|\frac{\partial \varphi^{\alpha}}{\alpha x^{\beta}}\right| \neq 0$ and let's draw up transformation

$$x'^0 = \varphi^0(x^0, x^{\alpha}),\ \ x'^{\alpha} = \varphi^{\alpha}(x^1, x^2, x^3) \qquad \textit{(2.3.1.35)}$$

Having calculated $g'_{0\alpha}$ and g'_{00} we'll have:

$$g'_{0\alpha} = \mu \frac{\partial x^0}{\partial x'^0}\frac{\partial x'^0}{\partial x'^{\alpha}} = 0, \quad g'^0_{00} = \mu^2 / g_{00} \ ^{1} \qquad \textit{(2.3.1.35')}$$

Thus, in case when x^0 is time-like coordinate line, the necessary and sufficient conditions to divide metric into space and time parts, will be (2.3.1.31).

In system x'^0, x'^{α} the interval ds will have such a form:

$$ds^2 = g'_{00}\left(dx'^0\right)^2 + g'_{\mu\nu}dx'^{\mu}dx'^{\nu} \qquad \textit{(2.3.1.36)}$$

In this case for time we'll have:

$$cdt = \sqrt{g'_{00}}\,dx'^0 = \sqrt{g'_{00}}\left(\frac{\partial x'^0}{\partial x^0}dx^0 + \frac{\partial x'^0}{\partial x^{\alpha}}dx^{\alpha}\right)$$

which, in accord to (2.3.1.33) coincides with (2.3.1.29).

The distance between two points in three-dimensional space can be calculated as well, by meeting the conditions of (2.3.1.30) and (2.3.1.31). The corresponding expression will be required below.

For $g'_{\alpha\beta}$ we have:

$$g'_{\alpha\beta} = g'_{pq}\frac{\partial x'^p}{\partial x^{\alpha}}\frac{\partial x'^q}{\partial x^{\beta}} = g'_{\mu\nu}\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\frac{\partial x'^{\nu}}{\partial x^{\beta}} + g'_{00}\frac{\partial x'^0}{\partial x^{\alpha}}\frac{\partial x'^0}{\partial x^{\beta}}$$

With allowance of equation (2.3.1.33), we'll have:

$$g'_{\mu\nu}\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\frac{\partial x'^{\nu}}{\partial x^{\beta}} = g'_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \qquad \textit{(2.3.1.37)}$$

On the other hand for distance from (2.3.1.36) we'll have:

$$-d\ell^2 = g'_{\mu\nu}dx'^{\mu}dx'^{\nu} = g'_{\mu\nu}\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\frac{\partial x'^{\nu}}{\partial x^{\beta}}dx^{\alpha}dx^{\beta}$$

With allowance of (2.3.1.37) we'll have:

$$-d\ell^2 = \left(g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right)dx^{\alpha}dx^{\beta} \qquad \textit{(2.3.1.38)}$$

Thus, in conditions of (2.3.1.30) and (2.3.1.31) the known formula (2.3.1.29) and (2.3.1.38) are obtained for time and space. It is easy to show that in the same conditions, the general expression (2.3.1.28) after differentiation coincides with (2.3.1.29). Differential of equity (2.3.1.28) yields:

$$cdt = \pm\frac{\dfrac{\partial \varphi^0}{\partial x^0}dx^0 + \dfrac{\partial \varphi^0}{\partial x^{\alpha}}dx^{\alpha}}{\sqrt{g^{pq}\dfrac{\partial \varphi^0}{\partial x^p}\dfrac{\partial \varphi^0}{\partial x^q}}}, \qquad \textit{(2.3.1.39)}$$

but, so as

---

[1] here was applied the equity $\frac{\partial x^0}{\partial x'^0} = 1 \Big/ \frac{\partial x'^0}{\partial x^0}$, validity of which was proved in [10].

$$\frac{\partial\varphi^0}{\partial x^0}dx^0+\frac{\partial\varphi^0}{\partial x^\alpha}dx^\alpha=\sqrt{\frac{g_{00}}{g'_{00}}}dx^0+\frac{g_{0\alpha}}{\sqrt{g_{00}g'_{00}}}dx^\alpha,$$
$$g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}=g'^{00}=\frac{1}{g'_{00}},$$
than the last equity coincides indeed with (2.3.1.29).

From (2.3.1.15) for distance between two points with coordinates x'^α and $x'^\alpha+dx'^\alpha$ in three-dimensional space $x'^0=const$, we'll have:

$$-d\ell^2=g'_{\mu\nu}dx'^\mu dx'^\nu,\quad x'^0=const \qquad (2.3.1.39')$$

Assuming that $x_1'^\alpha$ and $x_2'^\alpha$ are two fixed points of three-dimensional space $x'^0=const$ and

$$x'^\alpha=x'^\alpha(p)- \qquad (2.3.1.40)$$

is some line running through these points $x_1'^\alpha=x'^\alpha(p_1)$ and $x_2'^\alpha=x'^\alpha(p_2)$, where p – is some invariant parameter. The length of line arc ℓ between these points is determined by application of (2.3.1.39') and is equal to

$$\ell=\int_{p_1}^{p_2}\sqrt{-g'_{\mu0}\frac{dx'^\mu}{dp}\frac{dx'^\nu}{dp}}dp \qquad (2.3.1.41)$$

So as the three-dimensional space is inserted into four-dimensional space-time variety, then its metric properties are determined in accord to (1.2.2.2) from [10], if assuming that

$$x'^0=const,\qquad x'^\alpha=\bar{x}^\alpha$$

These equations determine metric tensor of three-dimensional space, it is equal to $g'_{\alpha\beta}$, such result is natural.

Distance between two points $x_1'^\alpha$ and $x_2'^\alpha$ of three-dimensional space – is a length of arc of geodesic line of three-dimensional space between these points.

Thus, in order to determine distance between two points $x_1'^\alpha$ and $x_2'^\alpha$ of three-dimensional space, we have to:

a) construct a system of differential equations of geodesic line

$$\frac{d^2x'^\mu}{d\ell^2}+\Gamma'^\mu_{\alpha\beta}\frac{dx'^\alpha}{d\ell}\frac{dx'^\beta}{d\ell}=0,\quad x'^0=const, \qquad (2.3.1.42)$$

where $\Gamma'^\mu_{\alpha\beta}$ - are Christoffel symbols corresponding to components $g'_{\alpha\beta}$;

b) determine solution of this system, that satisfies the following boundary conditions:

$$\begin{aligned} x'^\alpha&=x_1'^\alpha \quad at \quad \ell=\ell_1,\\ x'&=x_2'^\alpha \quad at \quad \ell=\ell_2; \end{aligned} \qquad (2.3.1.43)$$

c) Calculate the integral (2.3.1.41) substituting here the solution of problem (2.3.1.42) (2.3.1.43).

Distance between two points of three-dimensional space can be expressed as well in initial parameters x^k and g_{ik}. With this purpose, the considered three-dimensional space will be represented in the following form:

$$\varphi^0(x^0,x^1,x^2,x^3)=const \qquad (2.3.1.44)$$

As was mentioned above, the x^0,x^1,x^2,x^3 are uniform dimensionless numbers, from which no space and time coordinates can be separately distinguished, that's why the investigated three-dimensional space can be described with the help of three variables, selected in any way from these four. For instance, when the x^0,x^2 and x^3, are taken as major parameters, than three-dimensional space (2.3.1.44) is represented in following way:

$$x^0 = \overline{x}^0, \qquad x^1 = \overline{\psi}(\overline{x}^0, \overline{x}^2, \overline{x}^3),$$
$$x^2 = \overline{x}^2, \qquad x^3 = \overline{x}^3,$$

where $x^1 = \overline{\psi}(x^0, x^2, x^3)$ is a solution of equation (2.3.1.44) relative to x^1. By that, the metric property of considered three-dimensional space can be expressed in variables x^0, x^2, x^3. However, with the aim to keep the homogeneity, this procedure will be implemented in variables x^1, x^2 and x^3 in particular, the considered three-dimensional space let's represent in following form:

$$x^0 = \overline{\psi}(\overline{x}^1, \overline{x}^2, \overline{x}^3), \qquad x^\alpha = \overline{x}^\alpha, \qquad \textit{(2.3.1.45)}$$

where, $x^0 = \overline{\psi}(x^1, x^2, x^3)$ is solution of equation (2.3.1.44) relative to x^0. By application of equity (1.2.2.2) from [10], we'll receive:

$$\gamma_{\alpha\beta} = \frac{\partial x^p}{\partial \overline{x}^\alpha}\frac{\partial x^q}{\partial \overline{x}^\beta} g_{pq} = \frac{\partial x^\mu}{\partial \overline{x}^\alpha}\frac{\partial x^\nu}{\partial \overline{x}^\beta} g_{\mu\nu} + \left(\frac{\partial x^0}{\partial \overline{x}^\alpha}\frac{\partial x^\mu}{\partial \overline{x}^\beta} + \right.$$
$$\left. + \frac{\partial x^0}{\partial \overline{x}^\beta}\frac{\partial x^\mu}{\partial \overline{x}^\alpha}\right) g_{0\mu} + \frac{\partial x^0}{\partial \overline{x}^\alpha}\frac{\partial x^\mu}{\partial \overline{x}^\beta} g_{00}$$

With allowance that $x^\alpha = \overline{x}^\alpha$

$$\gamma_{\alpha\beta} = g_{\alpha\beta} + \left(\frac{\partial x^0}{\partial x^\alpha} g_{0\beta} + \frac{\partial x^0}{\partial x^\beta} g_{0\alpha}\right) + \frac{\partial x^0}{\partial x^\alpha}\frac{\partial x^0}{\partial x^\beta} g_{00}$$

If we substitute here

$$\frac{\partial x^0}{\partial x^\alpha} = -\frac{\partial \varphi^0}{\partial x^\alpha} \Big/ \frac{\partial \varphi^0}{\partial x^0},$$

which represents consequence of equity (2.3.1.44), than we'll get:

$$\gamma_{\alpha\beta} = g_{\alpha\beta} - \frac{\dfrac{\partial \varphi^0}{\partial x^\alpha} g_{0\beta} + \dfrac{\partial \varphi^0}{\partial x^\beta} g_{0\alpha}}{\dfrac{\partial \varphi^0}{\partial x^0}} + \frac{\dfrac{\partial \varphi^0}{\partial x^\alpha}\dfrac{\partial \varphi^0}{\partial x^\beta}}{\left(\dfrac{\partial \varphi^0}{\partial x^0}\right)^2} g_{00}, \qquad \textit{(2.3.1.46)}$$

$$\varphi^0(x^0, x^1, x^2, x^3) = const$$

This equity determines metric of three-dimensional space inserted into four-dimensional space-time variety in the most general form. If the system of coordinates x^k and metric tensor g_{ik} meet conditions of (2.3.1.30) and (2.3.1.31), than in accord to (2.3.1.33)

$$\frac{\partial \varphi^0}{\partial x^k} = \frac{1}{\sqrt{g'_{00} g_{00}}} g_{0k}$$

Let's substitute this expression in (2.3.1.46) and we'll get:

$$\gamma_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{0\alpha} g_{0\beta}}{g_{00}}$$

which coincides with (2.3.1.38).

Having the metric tensor $\gamma_{\alpha\beta}$ of three-dimensional space, the infinitesimals length of $d\ell$ can be determined

$$-d\ell^2 = \gamma_{\alpha\beta} d\overline{x}^\alpha d\overline{x}^\beta$$

But so as $\overline{x}^\alpha = x^\alpha$, that is why

$$-d\ell^2 = \gamma_{\alpha\beta} dx^\alpha dx^\beta \qquad \textit{(2.3.1.47)}$$

Values of $\gamma_{\alpha\beta}$ determined in accord to (2.3.1.46), depend in principle on four variables x^0, x^α, which are interconnected between each other by condition (2.3.1.44), where *const* is a fixed value of parameter x'^0, according to which the synchronization of clock in three-dimensional space is realized. It should be noted that the condition

$$\varphi^0(x^0, x^1, x^2, x^3) = x'^0 = f(t_c) \qquad (2.3.1.48)$$

(see (2.3.1.44)) in each moment of standard time t_c from four-dimensional space-time variety distinguishes the usual three-dimensional space.

Assuming $t_c = const$ (i.e. at $x'^0 = const$) is some line,

$$x^k = x^k(p)$$

belonging to three-dimensional space

$$\varphi^0\left[x^0(p), x^\alpha(p)\right] = f(t_c),$$

is running through points x_1^k and x_2^k of four-dimensional space-time variety, provided

$$x_1^k = x^k(p_1), \qquad x_2^k = x^k(p_2),$$
$$\varphi^0(x_1^0, x_1^\alpha) = f(t_c), \quad \varphi^0(x_2^0, x_2^\alpha) = f(t_c),$$

Than the arc length of this line between these points, in accord to (2.3.1.47) is determined by equity:

$$\ell = \int_{p_1}^{p_2} \sqrt{-\gamma_{\mu\nu}\frac{dx^\mu}{dp}\frac{dx^\nu}{dp}}\,dp \qquad (2.3.1.49)$$

In sub-integral expression the arguments x^k of components $\lambda_{\mu\nu}$ are changed through $x^k(p)$.

Let's determine distance between two points x_1^k and x_2^k in accord to length of geodesic line belonging to considered three-dimensional space, which is the smallest in comparison to lengths of arcs of other lines, that run through these points. System of differential equations this geodesic line is determined from conditions of minimal value *l,* determined by (2.3.1.49), during fulfillment of additional restricting condition (2.3.1.48).

As it is known [4] the corresponding system of differential equations, has a form of:

$$\frac{d}{dp}\left(\frac{\partial L}{\partial \dot{x}^k}\right) - \frac{\partial L}{\partial x^k} = 0,$$
$$L = \sqrt{-\gamma_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta} + \lambda\varphi^0(x^0, x^1, x^2, x^3) \qquad (2.3.1.50)$$

$\dot{x}^\alpha = \frac{dx^\alpha}{dp}$, λ is a Lagrangian multiplier.

Quite similarly to clause 1.1.3, from here we'll sought system of differential equations, in particular for variables of x^α these equations are as follows:

$$\frac{d^2x^\alpha}{dp^2} + \tilde{\Gamma}^\alpha_{\mu\nu}\frac{dx^\mu}{dp}\frac{dx^\nu}{dp} - \frac{dx^\alpha}{dp}\frac{\frac{d^2\ell}{dp^2}}{\frac{d\ell}{dp}} + \lambda\gamma^{\alpha\mu}\frac{\partial\varphi^0}{\partial x^\mu}\frac{d\ell}{dp} = 0 \qquad (2.3.1.51)$$

where $\tilde{\Gamma}^\alpha_{\mu\nu}$ are Christoffel symbols, comprised of $\gamma_{\alpha\beta}$ and x^α.

From (2.3.1.50) it is obvious that L doesn't depend on $\frac{dx^0}{dp}$, that's why the corresponding x^0 Eulerian-Lagrangian equation is leading to equation of $\frac{\partial L}{\partial x^0} = 0$, i.e.

$$\frac{-\partial\gamma_{\mu\nu}}{\partial x^0}\frac{dx^\mu}{dp}\frac{dx^\nu}{dp}\frac{dp}{d\ell}+2\lambda\frac{\partial\varphi^0}{\partial x^0}=0 \qquad (2.3.1.52)$$

The equations (2.3.1.48), (2.3.1.51) and (2.3.1.52) at $t_c = const$ $(x'^0 = const)$ constitute a full system of differential equations relative to the sought parameters $x^k(p)$ and λ determining the geodesic line of three-dimensional space for a given moment of standard time t_c. In case when $p = \ell$, we'll get:

$$\frac{d^2x^\alpha}{d\ell^2}+\tilde{\Gamma}^\alpha_{\mu\nu}\frac{dx^\mu}{d\ell}\frac{dx^\nu}{d\ell}+\lambda\gamma^{\alpha\mu}\frac{\partial\varphi^0}{\partial x^\mu}=0,$$
$$-\frac{\partial\gamma_{\mu\nu}}{\partial x^0}\frac{dx^\mu}{d\ell}\frac{dx^\nu}{d\ell}+2\lambda\frac{\partial\varphi^0}{\partial x^0}=0 \qquad (2.3.1.53)$$
$$\varphi^0(x^0,x^1,x^2,x^3)=f(t_c)$$

This system in totality with boundary conditions:

$$x^k = x_1^\alpha \quad at \quad \ell=\ell_1, \quad x^k = x_2^k \quad at \quad \ell=\ell_2, \qquad (2.3.1.54)$$

(x_1^0 and x_2^0 are determined correspondingly with equities $\varphi^0(x_1^0, x_1^\alpha) = f(t_c)$ and $\varphi^0(x_2^0, x_2^\alpha) = f(t_c)$) determines the sought geodesic line $x^k = x^k(\ell, t_c)$, passing through points x_1^k and x_2^k for each moment of standard time t_c. By substituting these value $x^k(\ell, t_c)$ in (2.3.1.49) will determine distance between points x_1^k and x_2^k in three-dimensional space at the moment of time t_c.

2.3.2 SPEED AND ACCELERATION OF MOVING POINT

Under the motion of some point in three-dimensional space we mean such a state at which its coordinates are changed in time. Changing of coordinates point can be related to different time, for instance, coordinates can be presented as functions of standard time t_c or time t, readable at certain fixed point of three-dimensional space, or at the point in which it is found during process of motion. All these times are in unambiguous functional dependence on time coordinate x'^0, consequently we'll think that coordinates of moving point are the function of parameter x'^0

$$x^k = \omega^k(x'^0) \qquad (2.3.2.1)$$

Moreover, these functions should satisfy the following conditions:

$$\varphi^0\left[\omega^0(x'^0), \omega^1(x'^0), \omega^2(x'^0), \omega^3(x'^0)\right] = x'^0 \qquad (2.3.2.2)$$

Assuming that to two values x'^0 and $x'^0 + dx'^0$ of parameter x'^0 correspond values of coordinates x^k and $x^k + dx^k$ of moving point, than infinitively small distance passed by point, in accord to (2.3.1.47) is equal to

$$dl = \sqrt{-\gamma_{\mu\nu}d\omega^\mu\omega^\nu}\Big/_{x^k=\omega^k(x'_0)} \qquad (2.3.2.3)$$

Interval of standard time dt_c can be determined as well, which corresponds to changing of dx'^0. With this purpose, the equation (2.3.1.24) should be differentiated along x'^0 and determine from (2.3.1.23) the functional dependence of $x^k = x^k(x'^0,0,0,0) = \; = \psi_c^k(x'^0)$, corresponding to standard time

$$dt_c = \frac{1}{c}\frac{dx'^0}{\sqrt{g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}}\Big/_{x^k = \psi_c^k(x'^0)}} \tag{2.3.2.4}$$

By that, the speed v_i of moving point by standard time is determined through equation:

$$\begin{aligned} v_c = \frac{d\ell}{dt_c} = \sqrt{-\gamma_{\mu\nu}\frac{d\omega^\mu}{dt_c}\frac{d\omega^\nu}{dt_c}}\Big/_{x^k = \omega^k(x'^0)} = \\ = c\sqrt{-\gamma_{\mu\nu}\frac{d\omega^\mu}{dx'^0}\frac{d\omega^\nu}{dx'^0}}\Big/_{x^k = \omega_k(x'^0)} \times \\ \times\sqrt{g^{pq}\frac{\partial\varphi^0}{dx^p}\frac{d\varphi^0}{dx^q}}\Big/_{x^k = \psi_c^k(x'^0)} \end{aligned} \tag{2.3.2.5}$$

In parameters x'^k and g'_{ik} the (2.3.2.5) will have such a form:

$$v_c = \frac{\sqrt{-g'_{\mu\nu}\frac{dx'^\mu}{dx'^0}\frac{dx'^\nu}{dx'^0}}\Big/_{x'^\alpha = \omega'^\alpha(x'^0)}}{\sqrt{g'_{00}}\Big/_{x'^\alpha = 0}}, \tag{2.3.2.6}$$

where

$$\begin{aligned} \omega'^\alpha(x'^0) = \varphi^\alpha\left[\omega^0(x'^0), \omega^1(x'^0), \omega^2(x'^0), \omega^3(x'^0)\right], \\ 0 = \varphi^\alpha\left[\psi_c^0(x'^0), \psi_c^1(x'^0), \psi_c^2(x'^0), \psi_c^3(x'^0)\right] \end{aligned} \tag{2.3.2.7}$$

In case of light beam

$$g_{pq}dx^p dx^q = g'_{00}(dx'^0)^2 + g'_{\mu\nu}dx'^\mu dx'^\nu = 0$$

Hence and from (2.3.2.6) it is obvious that $v_c = c$.

Thus, the light beam in the space with arbitrary admitted metric (when metric of four-dimensional space-time variety has signature $+---$) is moving at the speed of c.

Similarly the value of speed can be calculated related to different times using the fact that these times are in unambiguous functional dependence on parameter x'^0.

Coordinates x^α mentioned above have been used to describe the three-dimensional space $\varphi^0(x^0, x^1, x^2, x^3) = const$ and accordingly with it was determined metric tensor of three-dimensional space $\gamma_{\alpha\beta}$. In the next paragraph will be shown that by transformation of only coordinates x^α, the values of $\gamma_{\alpha\beta}$ constitute a three-dimensional covariant tensor of second order, whereas the dt_c is invariant. Consequently, the totality of values

$$\frac{dx^\alpha}{dt_c} = \frac{d\omega^\alpha}{dt_c} \; -$$

is a three-dimensional vector, the length of the latter, according to (2.3.2.5) is equal to velocity v_c. These values determine the coordinates changing speed x^1, x^2, x^3 of moving point and so as x^α are dimensionless (non-metric) numbers, then dx^α / dt_c have no other physical sense.

Assuming that τ^α is an unit vector in three-dimensional space, determined along trajectory $x^k = \omega^k(x'^0)$ of moving point:

$$-\gamma_{\alpha\beta}\tau^\alpha\tau^\beta = 1 \quad at \quad x^k = \omega^k(x'^0) \tag{2.3.2.8}$$

Projection of motion speed on v^α is determined according to formulae:

$$v_{cv} = -\gamma_{\alpha\beta}\tau^\alpha\frac{d\omega^\beta}{dt_c}\Big/_{x^k = \omega^k(x'^0)} \tag{2.3.2.9}$$

By application of those formulae let's determine the speed projection on coordinate lines. For this purpose we'll introduce three vectors (1,0,0), (0,1.0) and (0,0,1), which are tangent coordinate lines. The lengths of these vectors are correspondingly equal to $\sqrt{-\gamma_{11}}, \sqrt{-\gamma_{22}}, \sqrt{-\gamma_{33}}$, that is why after normalizing we'll get the following unit vectors:

$$\tau_{1'}^{\alpha} = \left(\frac{1}{\sqrt{-\gamma_{11}}}, 0, 0 \right), \; v_{2'}^{\alpha} = \left(0, \frac{1}{\sqrt{-\gamma_{22}}}, 0 \right), \quad \tau_{3'}^{\alpha} = \left(0, 0, \frac{1}{\sqrt{-\gamma_{33}}} \right) \qquad (2.3.2.10)$$

Then in accord to (2.3.2.9), we have:

$$v_{c1} = -\frac{\gamma_{1\beta} \frac{d\omega^{\beta}}{dt_c}}{\sqrt{-\gamma_{11}}}, \quad v_{c2} = -\frac{\gamma_{2\beta} \frac{d\omega^{\beta}}{dt_c}}{\sqrt{-\gamma_{22}}}, \quad v_{c3} = -\frac{\gamma_{3\beta} \frac{d\omega^{\beta}}{dt_c}}{\sqrt{-\gamma_{33}}} \qquad (2.3.2.11)$$

In case when x^{α} are orthogonal coordinates, these equities are simplified:

$$v_{c1} = \sqrt{-\gamma_{11}} \frac{d\omega^{\beta}}{dt_c}, \quad v_{c2} = \sqrt{-\gamma_{22}} \frac{d\omega^{2}}{dt_c}, \quad v_{c3} = \sqrt{-\gamma_{33}} \frac{d\omega^{3}}{dt_c} \qquad (2.3.2.12)$$

In plane three-dimensional space, by application of Cartesian system of coordinates

$$\left\| \gamma_{\alpha\beta} \right\| = \begin{Vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

and

$$v_{c1} = \frac{dx}{dt}, \quad v_{c2} = \frac{dy}{dt}, \quad v_{c3} = \frac{dz}{dt},$$

dx^{α}/dt_c acquire definite physical sense, they coincide with projections of speed along coordinate lines. By application of spherical system of coordinates

$$\left\| \gamma_{\alpha\beta} \right\| = \begin{Vmatrix} -1 & 0 & 0 \\ 0 & -r^2 & 0 \\ 0 & 0 & -r^2 \sin^2 \vartheta \end{Vmatrix}$$

that’s why

$$v_{c1} = \frac{dr}{dt_c}, \; v_{c2} = r\frac{d\vartheta}{dt_c}, \; v_{c3} = r \sin \vartheta \frac{d\varphi}{dt_c},$$

Angle can also be determined, this angle comprises direction of motion with some unit vector τ^{α}, in particular:

$$\cos \varphi_v = -\frac{\gamma_{\alpha\beta} \tau^{\alpha} \frac{dx^{\beta}}{dt_c}}{\sqrt{-\gamma_{\alpha\beta} \frac{dx^{\alpha}}{dt_c} \frac{dx^{\beta}}{dt_c}}} \qquad (2.3.2.13)$$

$\left\| -\gamma_{\alpha\beta} \right\|$ is nonnegatively determined matrix, consequently $\left| \cos \varphi_0 \right| \leq 1$.

In case when v^{α} coincides with vectors (2.3.2.10) –

$$\cos\varphi_1 \cdot v_c = -\frac{\gamma_{1\beta}}{\sqrt{-\gamma_{11}}}\frac{dx^{\beta}}{dt_c},$$
$$\cos\varphi_2 \cdot v_c = -\frac{\gamma_{2\beta}}{\sqrt{-\gamma_{22}}}\frac{dx^{\beta}}{dt_c}, \qquad (2.3.2.14)$$
$$\cos\varphi_3 \cdot v_c = -\frac{\gamma_{3\beta}}{\sqrt{-\gamma_{33}}}\frac{dx^{\beta}}{dt_c}$$

where φ_1, φ_2 and φ_3 are angles comprised by direction of motion with coordinate lines.

From (2.3.2.11) and (2.3.2.14) it is obvious that:

$$v_{c\alpha} = v_c \cos\varphi_\alpha \qquad (2.3.2.15)$$

These equities establish contact between absolute value of speed v_c, its projections $v_{c\alpha}$ on coordinate lines and angles.

With allowance, that

$$\frac{dx^{\alpha}}{dt_c} = \frac{dx^{\alpha}}{d\ell}\frac{d\ell}{dt_c} = \frac{dx^{\alpha}}{d\ell}v_c$$

from (2.3.2.14) we'll determine angles φ_α through line elements

$$\cos\varphi_\alpha = -\frac{\gamma_{\alpha\beta}\dfrac{dx^{\alpha}}{d\ell}}{\sqrt{-\gamma_{\alpha\alpha}}} \qquad (2.3.2.16)$$

These equities are valid for any line, independently from the fact, whether some point is moving along it or not. $\dfrac{dx^{\alpha}}{dt_c}$ represents tangent vector of line motion (trajectory) of a point [10].

By application of three-dimensional vector $\dfrac{dx^{\alpha}}{dt_c}$, other three-dimensional vector can be formed as well:

$$\frac{D}{dt_c}\left(\frac{dx^{\alpha}}{dt_c}\right) = \frac{d}{dt_c}\left(\frac{dx^{\alpha}}{dt_c}\right) + \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{dt_c}\frac{dx^{\nu}}{dt_c} =$$
$$= \left\{\frac{d}{d\ell}\left(\frac{dx^{\alpha}}{dt_c}\right) + \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{dt_c}\frac{dx^{\nu}}{d\ell}\right\}\frac{d\ell}{dt_c} - \qquad (2.3.2.17)$$

the components of which specify acceleration of changing of coordinates of moving point. The absolute value of this vector

$$a_c = \sqrt{-\gamma_{\mu\nu}\frac{D}{dt_c}\left(\frac{dx^{\mu}}{dt_c}\right)\frac{D}{dt_c}\left(\frac{dx^{\nu}}{dt_c}\right)} \qquad (2.3.2.18)$$

we'll call acceleration of moving point according to standard time t_c.

Let's admit that τ^{α} is three-dimensional vector determined along trajectory, then projection of acceleration on τ^{α} will be determined from equity:

$$a_{cv} = -\gamma_{\alpha\beta}v^{\alpha}\frac{D}{dt_c}\left(\frac{dx^{\beta}}{dt_c}\right) = -\gamma_{\alpha\beta}v^{\alpha}\left(\frac{d^2x^{\beta}}{dt_c} + \Gamma^{\beta}_{\mu\nu}\frac{dx^{\mu}}{dt_c}\frac{dx^{\nu}}{dt_c}\right) \qquad (2.3.2.19)$$

Acceleration projection on coordinate lines, in accord to (2.3.2.10) are relevantly equal to:

$$a_{cv} = -\frac{\gamma_{\alpha\beta}}{\sqrt{-\gamma_{\alpha\alpha}}}\left(\frac{d^2x^{\beta}}{dt_c{}^2} + \Gamma^{\beta}_{\mu\nu}\frac{dx^{\mu}}{dt_c}\frac{dx^{\nu}}{dt_c}\right) \qquad (2.3.2.20)$$

Angle that constitutes acceleration with direction v^{α}, is equal to

$$\cos\varphi_v = -\frac{\gamma_{\alpha\beta}\tau^{\alpha}\frac{D}{dt_c}\left(\frac{dx^{\beta}}{dt_c}\right)}{\sqrt{-\gamma_{\alpha\beta}\frac{D}{dt_c}\left(\frac{dx^{\alpha}}{dt_c}\right)\frac{D}{dt_c}\left(\frac{dx^{\beta}}{dt_c}\right)}} \qquad (2.3.2.21)$$

As $\left\|-\gamma_{\alpha\beta}\right\|$ is nonnegatively determined matrix, then $\left|\cos\varphi_0\right| \le 1$.

In conformity with (2.3.2.18) and (2.3.2.21) we'll have:

$$a_{c\alpha} = a_c \cos\varphi_v \qquad (2.3.2.22)$$

These equities establish relation between absolute value of acceleration, its projections and angles, comprised of acceleration with coordinate lines.

In above obtained equities relative to acceleration, vector $\frac{dx^{\alpha}}{dt_c}$ is equal to

$$c\frac{d\omega^{\alpha}}{dx'^0}\sqrt{g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}}\Bigg/ x^k = \psi^k(x'^0) \qquad (2.3.2.23)$$

and $\frac{d^2x^{\alpha}}{dt_c{}^2}$ -

$$c^2\frac{d}{dx'^0}\left\{\frac{d\omega^{\alpha}}{dx'^0}\sqrt{g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}}\Bigg/ x^k = \psi^k(x'^0)\right\}\times$$
$$\times\sqrt{g^{pq}\frac{\partial\varphi^0}{\partial x^p}\frac{\partial\varphi^0}{\partial x^q}}\Bigg/ x^k = \psi^k(x'^0) \qquad (2.3.2.24)$$

Let's rewrite the (2.3.2.17) as follows

$$\frac{D}{dt_c}\left(\frac{dx^{\alpha}}{dt_c}\right) = \frac{D}{d\ell}\left(\frac{dx^{\alpha}}{d\ell}v_c\right)v_c = \frac{D}{dl}\left(\frac{dx^{\alpha}}{d\ell}\right)v_c{}^2 +$$
$$+\frac{dx^{\alpha}}{d\ell}\frac{d^2\ell}{dt_c{}^2} \qquad (2.3.2.25)$$

Here $\frac{dx^{\alpha}}{d\ell}$ is unit tangent, and $\frac{D}{d\ell}\left(\frac{dx^{\alpha}}{d\ell}\right)$ is normal vector to trajectory of motion point. Taking into account that these vectors are orthogonal

$$-\gamma_{\mu\nu}\frac{dx^{\mu}}{d\ell}\frac{D}{d\ell}\left(\frac{dx^{\nu}}{d\ell}\right) = 0,$$

from (2.3.2.18) we'll have:

$$a_c^2 = \left[-\gamma_{\mu\nu}\frac{D}{d\ell}\left(\frac{dx^{\mu}}{d\ell}\right)\frac{D}{d\ell}\left(\frac{dx^{\nu}}{d\ell}\right)\right]v_c^4 +$$
$$+\left[-\gamma_{\mu\nu}\frac{dx^{\mu}}{d\ell}\frac{dx^{\nu}}{d\ell}\right]\left(\frac{d^2\ell}{dt_c{}^2}\right)^2$$

Also with allowance that [10]

$$-\gamma_{\mu\nu}\left(\frac{dx^{\mu}}{d\ell}\right)\frac{dx^{\nu}}{d\ell} = 1,$$

$$-\gamma_{\mu\nu}\frac{D}{d\ell}\left(\frac{dx^{\mu}}{d\ell}\right)\frac{D}{d\ell}\left(\frac{dx^{\nu}}{d\ell}\right)=\frac{1}{\rho^2},$$

where, ρ - is first curvature of trajectory, we'll obtain:

$$a_c{}^2=\left(\frac{v_c^2}{\rho}\right)^2+\left(\frac{d^2\ell}{dt_c}\right)^2 \qquad (2.3.2.26)$$

This formula coincides (by form) with a formula known in kinematics and is its generalization. It is valid in any three-dimensional space as well as for any system of coordinates. If the point is moving along geodesic line, then

$$\frac{D}{d\ell}\left(\frac{dx^{\mu}}{d\ell}\right)=0, \qquad \frac{1}{\rho}=0$$

and from (2.3.2.26) we'll have

$$a_c=\frac{d^2\ell}{dt_c{}^2}, \qquad (2.3.2.27)$$

but if the point moves at zero linear acceleration $\frac{d^2\ell}{dt_c{}^2}=0$, than

$$a_c=\frac{v^2}{\rho} \qquad (2.3.2.28)$$

In the first case a_c is absolute value of vector $\frac{dx^{\alpha}}{d\ell}\frac{d^2\ell}{dt_c{}^2}$, representing a tangent trajectory, and consequently is called a tangent acceleration, whereas in the second case – a_c is absolute value of vector $\frac{D}{d\ell}\left(\frac{dx^{\alpha}}{d\ell}\right)v_c$, which is perpendicular to vector $\frac{dx^{\alpha}}{d\ell}$ [10], that’s why this acceleration is directed along the normal of trajectory.

2.4. TRANSFORMATION LAWS

2.4.1 TRANSFORMATION OF COORDINATES x^{α}

The kinematics values $dt_c, d\ell, v_c, v_a$, etc. have been determined above, which during transformation of coordinate system are transformed in certain way. Their transformation laws depend not only upon structure of kinematics values themselves, but depend as well on law of coordinates transformation. In this connection we'll consider two types of system coordinates:

$$\tilde{x}^0=x^0, \quad \tilde{x}^{\alpha}=\tilde{x}^{\alpha}(x^1,x^2,x^3) \qquad (2.4.1.1)$$

and

$$x''^k=x''^k(x'^0,x'^1,x'^2,x'^3) \qquad (2.4.1.2)$$

when x'^0 and x''^0 are time, and x'^{α} and x''^{α} are space coordinates. In the first case only coordinates x^{α} (x^0 remains unchanged) are transformed, but in the second case – all four are transformed. It should be remembered that transformation (2.4.1.1) can’t be considered only as transformation of space coordinates, as in general case x^0 and x^{α} are neither time nor space coordinates accordingly.

First let’s investigate transformation laws of kinematic values related to transformation of system coordinates (2.4.1.1).

In accord to (2.3.1.39), the differential of standard time is determined in accord to following equity:

$$dt_c = \frac{1}{c} \frac{\dfrac{\partial \varphi^0}{\partial x^0} + a^\mu \dfrac{\partial \varphi^0}{\partial x^\mu}}{\sqrt{g^{pq} \dfrac{\partial \varphi^0}{\partial x^p} \dfrac{\partial \varphi^0}{\partial x^q}}} \Bigg/ _{x'^\alpha = 0}, \qquad (2.4.1.3)$$

where

$$a^\mu = a^\mu_\nu \frac{\partial \varphi^\nu}{\partial x^0}, \qquad \left\| a^\mu_\nu \right\| = \left\| \frac{\partial \varphi^\mu}{\partial x^\nu} \right\|^{-1} \qquad (2.4.1.4)$$

It is obvious that denominator on the right hand side of equity (2.4.1.3) is invariant value relation to transformation (2.4.1.1). Let's determine transformation laws for value a^μ.

In conformity with (2.4.1.4)

$$\frac{\partial \varphi^\mu}{\partial x^0} = a^\nu \frac{\partial \varphi^\mu}{\partial x^\nu} \qquad (2.4.1.5)$$

Besides,

$$\frac{\partial \varphi^\mu}{\partial x^0} = \frac{\partial \varphi^\mu}{\partial \tilde{x}^0}, \qquad \frac{\partial \varphi^\mu}{\partial x^\nu} = \frac{\partial \varphi^\mu}{\partial \tilde{x}^\alpha} \frac{\partial \tilde{x}^\alpha}{\partial x^\nu}$$

i.e., by transformation of coordinates (2.4.1.1) $\frac{\partial \varphi^\mu}{\partial x^0}$ and $\frac{\partial \varphi^\mu}{\partial x^\nu}$ are transformed both scalar and covariant vector accordingly, that is why from (2.4.1.5) it is obvious that a^μ is a three-dimensional vector relative to transformation (2.4.1.1). With allowance by it that $\frac{\partial \varphi^0}{\partial x^0}$ is scalar, and $\frac{\partial \varphi^0}{\partial x^\mu}$ is three-dimensional covariant vector relative to transformation (2.4.1.1), then we'll obtain that numerator on the right side of equation (2.4.1.3) is an invariant value as well.

Thus, relative to transformation of system of coordinate (2.4.1.1), the value dt_c determined by (2.4.1.3), is a scalar value.

The following expression has been yielded above for $d\ell^2$:

$$d\ell^2 = -\gamma_{\mu\nu} dx^\mu dx^\nu, \qquad (2.4.1.6)$$

where, $\gamma_{\mu\nu}$ is determined in accord to equity (2.3.1.46).

From equations:

$$\tilde{g}_{\mu\nu} = \frac{\partial x^p}{\partial \tilde{x}^\mu} \frac{\partial x^q}{\partial \tilde{x}^\nu} g_{pq} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta},$$

$$\tilde{g}_{0\mu} = \frac{\partial x^p}{\partial \tilde{x}^0} \frac{\partial x^q}{\partial \tilde{x}^\mu} g_{pq} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} g_{0\alpha}, \qquad \tilde{g}_{00} = \frac{\partial x^p}{\partial \tilde{x}^0} \frac{\partial x^q}{\partial \tilde{x}^0} g_{pq} = g_{00}$$

which are obtained with allowance of (2.4.1.1), it is evident that with respect to the transformation system of coordinates under consideration $g_{\mu\nu}$ is the three-dimensional covariant tensor, $g_{0\mu}$ is the three-dimensional covariant vector and g_{00} - the scalar. Than, since $\frac{\partial \varphi^0}{\partial x^\mu}$ is the three-dimensional covariant vector, it follows from (2.3.1.46) that $\gamma_{\mu\nu}$ is the three-dimensional covariant tensor of second order, and $d\ell$ is the scalar with respect to transformation (2.4.1.1). $\frac{dx^\alpha}{dt_c}$, as well as $\frac{D}{dt_c}\left(\frac{dx^\alpha}{dt_c}\right)$ are the three-dimensional contravariant vectors with respect to these very transformations, since dt_c is the invariant. In this connection the velocity v_c, acceleration a_c and their projections v_{cv} and a_{cv} on some direction, defined by the vector v^{α} are also the scalar values. The validity of this suggestion follows from the structure of these values.

2.4.2.TRANSFORMATION OF x'^k COORDINATES DIVIDED INTO TIME AND SPACE PARTS

As it has been shown above, there is an infinite number of functions $\varphi^0(x^0,x^1,x^2,x^3)$ and thus, $\varphi^{\alpha}(x^0,x^1,x^2,x^3)$ functions which determine a new system of coordinates x'^k with time x'^0 and space x'^{α} coordinates. These functions depend on one arbitrary function $\psi(x^0,x^1,x^2,x^3)$, to different forms of which correspond the different coordinate systems x'^k. If we restrict ourselves to continuous ψ -functions, then, as is known [9], a great number of such functions have a continuum power, that is why a great number of corresponding functions $\varphi^k(x^0,x^1,x^2,x^3)$ and thus a great number of various coordinate systems under consideration also have a continuum power.

All similar coordinate systems are in unambiguous accordance with each other, in fact, if x'^k and x''^k are two coordinate systems which correspond to two functions ψ' and ψ'', we can write

$$x'^k = \varphi'^k(x^0,x^1,x^2,x^3), \quad x''^k = \varphi''^k(x^0,x^1,x^2,x^3) \qquad (2.4.2.1)$$

Hence, in accord to unambiguity of functions φ'^k and φ''^k it follows that

$$x''^k = x''^k(x'^0,x'^1,x'^2,x'^3) \qquad (2.4.2.2.)$$

x'^k and x''^k being in unambiguous functional relation.

From the kinematic viewpoint each such a coordinate systems differs from each other by definite properties according to which it is possible to single out one isolated coordinate system characterized by concepts reasonable for us. This system should be recognized as rational and all other systems are considered with respect to it. A comparative analysis of the coordinate systems determined by equities (2.4.2.1) will allow one to elaborate a clear physical essence of these systems.

Since condition (2.3.1.8) holds for the functions φ'^k and φ''^k, not only (2.4.2.2) but also the inverse functional relation is valid

$$x'^k = x'^k(x''^0,x''^1,x''^2,x''^3) \qquad (2.4.2.3)$$

(2.4.2.1) can be regarded as a parametric representation of transformations (2.4.2.2) and (2.4.2.3); it is determined by the conditions:

$$g^{ik}\frac{\partial\varphi'^0}{\partial x^i}\frac{\partial\varphi'^0}{\partial x^k} = (\psi')^2, \quad g^{ik}\frac{\partial\varphi'^0}{\partial x^i}\frac{\partial\varphi'^{\alpha}}{\partial x^k} = 0,$$
$$g^{ik}\frac{\partial\varphi''^0}{\partial x^i}\frac{\partial\varphi''^0}{\partial x^k} = (\psi'')^2, \quad g^{ik}\frac{\partial\varphi''^0}{\partial x^i}\frac{\partial\varphi''^{\alpha}}{\partial x^k} = 0 \qquad (2.4.2.4)$$

Thus it is evident that any transformation of the (2.4.2.2) or (2.4.2.3) type is determined by two functions $\psi'(x^0,x^1,x^2,x^3)$ and $\psi''(x^0,x^1,x^2,x^3)$. In the multitude of these transformations there is also an identity transformation relevant to identical values of the functions ψ' and ψ''.

Combination of two transformations (2.4.2.2.)

$$
\begin{aligned}
x''^{k} &= x''^{k}(x'^{0}, x'^{1}, x'^{2}, x'^{3}) \\
x'''^{k} &= \varphi'''^{k}(x''^{0}, x''^{1}, x''^{2}, x''^{3})
\end{aligned}
\qquad (2.4.2.5)
$$

can be taken as one transformation

$$
x'''^{k} = \bar{x}^{k}(x'^{0}, x'^{1}, x'^{2}, x'^{3}) \qquad (2.4.2.6)
$$

when the first transformation of the system (2.4.2.5) is determined by equations (2.4.2.4) and the second one by the equations:

$$
\begin{aligned}
g^{ik}\frac{\partial\varphi''^{0}}{\partial x^{i}}\frac{\partial\varphi''^{0}}{\partial x^{k}} &= (\psi'')^{2}, \quad g^{ik}\frac{\partial\varphi''^{0}}{\partial x^{i}}\frac{\partial\varphi''^{\alpha}}{\partial x^{k}} = 0, \\
g^{ik}\frac{\partial\varphi'''^{0}}{\partial x^{i}}\frac{\partial\varphi'''^{0}}{\partial x^{k}} &= (\psi''')^{2}, \quad g^{ik}\frac{\partial\varphi'''^{0}}{\partial x^{i}}\frac{\partial\varphi'''^{\alpha}}{\partial x^{k}} = 0
\end{aligned}
\qquad (2.4.2.7)
$$

As for the transformation (2.4.2.6) obtained from (2.4.2.5) by exclusion of the variables x''^{k}, it is determined from (2.4.2.4) and (2.4.2.7) also by exclusion of parameters with two primes, i.e. it is determined by the conditions:

$$
\begin{aligned}
g^{ik}\frac{\partial\varphi'^{0}}{\partial x^{i}}\frac{\partial\varphi'^{0}}{\partial x^{k}} &= (\psi')^{2}, \quad g^{ik}\frac{\partial\varphi'^{0}}{\partial x^{i}}\frac{\partial\varphi'^{\alpha}}{\partial x^{k}} = 0, \\
g^{ik}\frac{\partial\varphi'''^{0}}{\partial x^{i}}\frac{\partial\varphi'''^{0}}{\partial x^{k}} &= (\psi''')^{2}, \quad g^{ik}\frac{\partial\varphi'''^{0}}{\partial x^{i}}\frac{\partial\varphi'''^{0}}{\partial x^{k}} = 0
\end{aligned}
\qquad (2.4.2.8)
$$

and therefore belongs to the class of transformations of (2.4.2.2).

Summarizing these results one can conclude that the transformations of (2.4.2.2) type satisfy the following requirements:

a) There exists an inverse transformation of transformation (2.4.2.2);

b) There exists an identity transformation;

c) Combination of two of transformations of (2.4.2.2) class also belongs to the same class. Therefore, the multitude of (2.4.2.2) transformations constitutes a group of transformations depending on two arbitrary functions $\psi'(x^{0}, x^{1}, x^{2}, x^{3})$ and $\psi''(x^{0}, x^{1}, x^{2}, x^{3})$.

The transformation laws of time and distance of during the coordinate transformation (2.4.2.2) can be established using (2.3.1.39) and (2.3.1.47).

Let $\varphi'^{k}(x^{0}, x^{1}, x^{2}, x^{3})$ and $\varphi''^{k}(x^{0}, x^{1}, x^{2}, x^{3})$ are two function systems corresponding to the $\psi'(x^{0}, x^{1}, x^{2}, x^{3})$ functions and $\psi''(x^{0}, x^{1}, x^{2}, x^{3})$, then, according to (2.3.1.39) we have:

$$
\begin{aligned}
cdt' &= \frac{\dfrac{\partial\varphi'^{0}}{\partial x^{0}} + a'^{\mu}\dfrac{\partial\varphi'^{0}}{\partial x^{\mu}}}{\sqrt{g^{pq}\dfrac{\partial\varphi'^{0}}{\partial x^{p}}\dfrac{\partial\varphi'^{0}}{\partial x^{q}}}}dx^{0}, \\
\varphi'^{\alpha}(x^{0}, x^{1}, x^{2}, .x^{3}) &= c'^{\alpha} = const, \\
cdt'' &= \frac{\dfrac{\partial\varphi''^{0}}{\partial x^{0}} + a''^{\mu}\dfrac{\partial\varphi''^{0}}{\partial x^{\mu}}}{\sqrt{g^{pq}\dfrac{\partial\varphi''^{0}}{\partial x^{p}}\dfrac{\partial\varphi''^{0}}{\partial x^{q}}}}dx^{0}, \\
\varphi''^{\alpha}(x^{0}, x^{1}, x^{2}, .x^{3}) &= c''^{\alpha} = const
\end{aligned}
\qquad (2.4.2.9)
$$

The constants c'^{α} and c''^{α} in the right-hand side of these equities are dependent quantities; of them only c'^{α} (or c''^{α}) can be selected arbitrarily, while the second three constants should be determined according to these values. To be specific, let us assume that c'^{α} are the fixed known values, whereas the conditions

$$\varphi'^{\alpha}(x^0, x^1, x^2, x^3) = c'^{\alpha} \qquad (2.4.2.10)$$

determine the world line of some point of three-dimensional space in the four-dimensional space-time variety. The three-dimensional space itself is determined from the following equity

$$\varphi'^{0}(x^0, x^1, x^2, x^3) = c'^{0} = const \qquad (2.4.2.11)$$

The arbitrary constant c'^0 can take a zero value, which corresponds to fixation of three-dimensional space at the initial instant of time. Simultaneous fulfillment of conditions (2.4.2.10) and (2.4.2.11) fixes a point in the four-dimensional space-time variety x_0^k which corresponds to fixation of a point in three-dimensional space at the fixed instant of time. If we substitute these x_0^k coordinate values into the function $\varphi''^{\alpha}(x^0, x^1, x^2, x^3)$, we can define the fixed values of these functions which determine the values of the sought c''^{α} , i.e.

$$c''^{\alpha} = \varphi''^{\alpha}(x_0^0, x_0^1, x_0^2, x_0^3)$$

Thus the c'^{α} and c''^{α} constants are related to each other by the following conditions:

$$\begin{aligned} &\varphi'^{k}(x_0^0, x_0^1, x_0^2, x_0^3) = c'^{k}, \\ &\varphi''^{\gamma}(x_0^0, x_0^1, x_0^2, x_0^3) = c''^{\gamma} \end{aligned} \qquad (2.4.2.12)$$

while the constant c'^0 determines the instant of time at which the interval dt' is calculated.

Conditions (2.4.2.9) determine the relation between dt' and dt'' in the parametric form; dx^0 is the parameter and excluding it the sought time interval transformation law for the given point of three-dimensional space at a fixed instant of time can be established during transformation of coordinates system (2.4.2.2).

According to (2.3.1.47)

$$\begin{aligned} -d\ell'^2 = \gamma'_{\mu\nu} dx^{\mu} dx^{\nu},\ \varphi'^{0}(x^0, x^1, x^2, x^3) = c'^{0} = const, \\ -d\ell''^2 = \gamma''_{\mu\nu} dx^{\mu} dx^{\nu},\ \varphi''^{0}(x^0, x^1, x^2, x^3) = c''^{0} = const, \end{aligned} \qquad (2.4.2.13)$$

where, according to (2.3.1.46):

$$\begin{aligned} \gamma'_{\mu\nu} &= g_{\mu\nu} - \frac{g_{0\mu}\dfrac{\partial\varphi'^0}{\partial x^{\nu}} + g_{0\nu}\dfrac{\partial\varphi'^0}{\partial x^{\mu}}}{\dfrac{\partial\varphi'^0}{\partial x^0}} + g_{00}\frac{\dfrac{\partial\varphi'^0}{\partial x^{\mu}}\dfrac{\partial\varphi'^0}{\partial x^{\nu}}}{\left(\dfrac{\partial\varphi'^0}{\partial x^0}\right)^2}, \\ \gamma''_{\mu\nu} &= g_{\mu\nu} - \frac{g_{0\mu}\dfrac{\partial\varphi''^0}{\partial x^{\nu}} + g_{0\nu}\dfrac{\partial\varphi''^0}{\partial x^{\mu}}}{\dfrac{\partial\varphi'^0}{\partial x^0}} + g_{00}\frac{\dfrac{\partial\varphi''^0}{\partial x^{\mu}}\dfrac{\partial\varphi''^0}{\partial x^{\nu}}}{\left(\dfrac{\partial\varphi''^0}{\partial x^0}\right)^2} \end{aligned} \qquad (2.4.2.14)$$

The c'^0 and c'^0 constants that fix three-dimensional space in the four-dimensional space-time variety at a certain instant of time are the dependent values. To fix the point in three-dimensional space where the length of the infinitely small element $d\ell'$ is established, the condition $\varphi'^0(x^0, x^1, x^2, x^3) = c'^0$ should also be added to the condition $\varphi'^{\alpha}(x^0, x^1, x^2, x^3) = c'^{\alpha}$. All these conditions determine the point in three-dimensional space at a fixed instant of time where the length $d\ell'$ is established. It is evident that the same conditions determine the value x_0^k. Substituting them into $\varphi''^0(x^0, x^1, x^2, x^3)$, can be defined c''^0, i.e.

$$c''^{0} = \varphi''^{0}(x_0^0, x_0^1, x_0^2, x_0^3)$$

Thus, the functional relation between c'^0 and c''^0 is determined from the conditions:

$$\varphi'^k(x_0^0, x_0^1, x_0^2, x_0^3) = c'^k$$
$$\varphi''^0(x_0^0, x_0^1, x_0^2, x_0^3) = c''^0 \qquad (2.4.2.15)$$

Knowing c'^k and c''^0, let us define $d\ell'$ and $d\ell''$ in the selected point of three-dimensional space for a fixed instant of time. The dx^k values as characteristic elements of the one-dimensional variety can be represented by one parameter

$$dx^k = b^k(x_0^0, x_0^1, x_0^2, x_0^3)dp \qquad (2.4.2.16)$$

Then, according to (2.4.2.13), we have:

$$-d\ell'^2 = \gamma'_{\mu\nu}(x_0^0, x_0^1, x_0^2, x_0^3)b^\mu(x_0^0, x_0^1, x_0^2, x_0^3)\times$$
$$\times b^\nu(x_0^0, x_0^1, x_0^2, x_0^3)dp^2,$$
$$-d\ell''^2 = \gamma''_{\mu\nu}(x_0^0, x_0^1, x_0^2, x_0^3)b^\mu(x_0^0, x_0^1, x_0^2, x_0^3)\times \qquad (2.4.2.17)$$
$$\times b^\nu(x_0^0, x_0^1, x_0^2, x_0^3)dp^2$$

Hence, excluding the parameter dp, let's determine the transformation law for an infinitely small length at the given point of three-dimensional space at a fixed instant during realization of transformation of coordinate system (2.4.2.2).

The velocity transformation law can be established on the basis of use of (2.3.2.5), in particular, its parametric expression has the form:

$$v'_c = \sqrt{-\gamma'_{\mu\nu}\frac{dx^\mu}{dt'_c}\frac{dx^\nu}{dt'_c}}\Bigg/ x^k = \omega'^k(x'^0)\, ,$$
$$v''_c = \sqrt{-\gamma''_{\mu\nu}\frac{dx^\mu}{dt''_c}\frac{dx^\nu}{dt''_c}}\Bigg/ x^k = \omega''^k(x''^0) \qquad (2.4.2.18)$$

where the point motion laws

$$x^k = \omega'^k(x'^0) \qquad \text{and} \qquad x^k = \omega''^k(x''^0)$$

are established from the following equities:

$$x'^0 = \varphi'^0(x^0, x^1, x^2, x^3),\ c'^\alpha(x'^0) = \varphi'^\alpha(x^0, x^1, x^2, x^3)\,, \qquad (2.4.2.19)$$
$$x''^0 = \varphi''^0(x^0, x^1, x^2, x^3),\ c''^\alpha(x'^0) = \varphi''^\alpha(x^0, x^1, x^2, x^3)\,, \qquad (2.4.2.20)$$

where $c'^\alpha(x'^0)$ and $c''^\alpha(x''^0)$ are the given functions of x'^0 and x''^0 respectively. They characterize the point motion law in the corresponding three-dimensional spaces. These functions are dependent: indeed, if one excludes the variables x^k from (2.4.2.19) and (2.4.2.10), the following equations can be obtained

$$x''^0 = \varphi''^0\left\{\psi'^0\left[x'^0, c'^\alpha(x'^0)\right], \psi'^\alpha\left[x'^0, c'^\alpha(x'^0)\right]\right\},$$
$$c''^\alpha(x''^0) = \varphi''^\alpha\left\{\psi'^0\left[x'^0, c'^\alpha(x'^0)\right], \psi'^\alpha\left[x'^0, c'^\alpha(x'^0)\right]\right\}, \qquad (2.4.2.21)$$

where $x^0 = \psi'^0\left[x'^0, c'^\alpha(x'^0)\right]$, $x^\alpha = \psi'^\alpha\left[x'^0, c'^\alpha(x'^0)\right]$ - the solutions of system (2.4.2.19) with respect to x^0 and x^α - must be identical. In other words, if the functions $c'(x'^0)$ characterizing the point motion along the coordinates x'^α are properly selected, the functions $c''^\alpha(x''^0)$ must be determined from (2.4.2.21).

According to (2.4.2.21) the values v'_c and v''_c determined from (2.4.2.18) depend on one variable, either on x'^0, or on x''^0. Excluding this variable, let's determine the velocity transformation law during the coordinate system transformation (2.4.2.2).

Quite similarly, the transformation law for the velocity projection on some direction v^α is quite similarly determined (in the parametric form) from the following relations:

$$v'_{cv} = -\gamma'_{\alpha\beta} v^{\alpha} \left. \frac{dx^{\beta}}{dt'_c} \right/ x^k = \omega'^k (x'^0)\text{'}$$
$$v''_{cv} = -\gamma''_{\alpha\beta} v^{\alpha} \left. \frac{dx^{\beta}}{dt''_c} \right/ x^k = \omega''^k (x''^0)\text{'} \qquad (2.4.2.22)$$

Similarly for the acceleration transformation law we have:

$$a'_c = \sqrt{-\gamma'_{\alpha\beta} \left(\frac{d^2 x^{\alpha}}{dt'^2_c} + \Gamma'^{\alpha}_{\mu\nu} \frac{dx^{\mu}}{dt'_c} \frac{dx^{\nu}}{dt'_c} \right)} \Bigg/ x^k = \omega'^k (x'^0) \times$$
$$\times \sqrt{\left(\frac{d^2 x^{\beta}}{dt'^2_c} + \Gamma'^{\beta}_{\xi\eta} \frac{dx^{\xi}}{dt'_c} \frac{dx^{\eta}}{dt'_c} \right)} \Bigg/ x^k = \omega'^k (x'^0)\text{'}$$
$$a''_c = \sqrt{-\gamma''_{\alpha\beta} \left(\frac{d^2 x^{\alpha}}{dt''^2_c} + \Gamma''^{\alpha}_{\mu\nu} \frac{dx^{\mu}}{dt''_c} \frac{dx^{\nu}}{dt''_c} \right)} \Bigg/ x^k = \omega''^k (x''^0) \times \qquad (2.4.2.23)$$
$$\times \sqrt{\left(\frac{d^2 x^{\beta}}{dt''^2_c} + \Gamma''^{\beta}_{\xi\eta} \frac{dx^{\xi}}{dt''_c} \frac{dx^{\eta}}{dt''_c} \right)} \Bigg/ x^k = \omega''^k (x''^0)\text{'}$$

and for the transformation law for the acceleration projection on some direction v^{α} we have:

$$a'_{cv} = -\gamma'_{\alpha\beta} v^{\alpha} \left(\frac{d^2 x^{\beta}}{dt'^2_c} + \Gamma'^{\beta}_{\mu\nu} \frac{dx^{\mu}}{dt'_c} \frac{dx^{\nu}}{dt'_c} \right) \Bigg/ x^k = \omega'^k (x'^0)\text{'}$$
$$a''_{cv} = -\gamma''_{\alpha\beta} v^{\alpha} \left(\frac{d^2 x^{\beta}}{dt''^2_c} + \Gamma''^{\beta}_{\mu\nu} \frac{dx^{\mu}}{dt''_c} \frac{dx^{\nu}}{dt''_c} \right) \Bigg/ x^k = \omega''^k (x''^0) \qquad (2.4.2.24)$$

In [10] the problems of kinematics of a moving point in uniformly accelerated and uniformly rotated coordinate system are discussed. To study these problems general results obtained in the present paper were used. Below, in studies of the central symmetry GEH field the relevant problems of kinematics will be considered.

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CHAPTER III

ELEMENTS OF RELATIVISTIC DYNAMICS OF MATERIAL BODY

The concept "material body dynamics" makes sense if one assumes the existence of a concept “material body” independent of the concept "field". In many cases this classical concept is associated with substantial inconveniences resulting in a crisis situation in physics. Nevertheless, it is rather attractive because it enables one to determine (in many cases with very high accuracy) approximate finite solutions of complex problems allowing one to create definite physical models of the physical phenomenon under consideration.

In chapter IV the basic elements of the unified GEH field theory will be given which, when using the Einstein’s ideas, must define the laws of material body motion, determining a material body by using the concept of a unified GEH field.

The considered problems of dynamic based on the classical notion of the material body are one of the stages of the inductive judgment used in the present work to explain a physical essence of the parameters used to explain the unified GEH field and allow one to establish the relationship between these parameters and the usual classical parameters used in mechanics and electricity.

3.1. THE LAW OF MATERIAL BODY MOTION

3.1.1.THE LAW OF UNCHARGED MATERIAL BODY MOTION

In the next chapter of the present work, using the GEH field concept, it will be shown that any material body is accompanied by electric phenomena. In some cases the total electric charge of a material body may appear to be equal to zero, but its separate parts have a particular electric charge. The concept of an uncharged material body used here is classical and means the absence of the total electric charge in the material body representing the point objective reality.

Let us formulate the law of uncharged material body motion as the following principle: An uncharged material body moves along the nonisotropic geodesic line of the four-dimensional space-time variety.

According to the results of chapter I, the equation for uncharged material body motion in $SR(x^k, \underset{k}{\vec{E}})$ has the form:

$$\frac{d^2x^k}{ds^2} + \Gamma^k_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds} = 0\,, \qquad (3.1.1.1)$$

where s is the length of geodesic line. It is covariant in all reference systems of $SR(x^k, \underset{k}{\vec{E}})$ - type. Taking into account that ds is the invariant, the equations for uncharged material body motion in $SR(x^k, \underset{k}{\vec{e}})$ are defined from (3.1.1.1) using the relationship between the relevant parameters in $SR(x^k, \underset{k}{\vec{E}})$ and $SR(x^k, \underset{k}{\vec{e}})$ (here, according to the results of chapter I, transformation coefficients a^k_i are determined from the equities $a^k_i = \alpha^k_i$) we obtain, in particular,

$$\frac{d_e}{ds}\left(\frac{d_e x^k}{ds}\right)+\alpha_i^k\alpha_r^i\left(\frac{\partial \underset{p}{e}^r}{\partial \underset{e}{x}^q}+\alpha_p^j\alpha_q^\ell\Gamma_{j\ell}^r\right)\frac{d_e x^p}{ds}\frac{d_e x^q}{ds}=0 \qquad (3.1.1.2)$$

where

$$\Gamma_{j\ell}^k=\frac{1}{2}\underset{p}{e}^k\,\underset{q}{e}^r\,\underset{e}{g}^{pq}\left\{\alpha_\ell^m\frac{\partial\left(\alpha_j^s\alpha_r^t\,\underset{e}{g}_{st}\right)}{\partial \underset{e}{x}^m}+\alpha_j^m\frac{\partial\left(\alpha_\ell^s\alpha_r^t\,\underset{e}{g}_{st}\right)}{\partial \underset{e}{x}^m}-\right.$$

$$\left.-\alpha_r^m\frac{\partial\left(\alpha_j^s\alpha_\ell^t\,\underset{e}{g}_{st}\right)}{\partial \underset{e}{x}^m}\right\} \qquad (3.1.1.3)$$

Quite similarly, the equation of geodesic line in $SR(x^k,\underset{k}{\vec{e}}')$ has the following form

$$\frac{d_e}{ds}\left(\frac{d_e x'^k}{ds}\right)+\alpha_i'^k\alpha_r'^i\left(\frac{\partial \underset{p}{e}'^r}{\partial \underset{e}{x}'^q}+\alpha_p'^j\alpha_q'^\ell\Gamma_{j\ell}'^r\right)\frac{d_e x'^p}{ds}\frac{d_e x'^q}{ds}=0 \qquad (3.1.1.4)$$

And here the transformation coefficients responsible for transition from $SR(x^k,\underset{k}{\vec{E}})$ to $SR(x^k,\underset{k}{\vec{e}}')$ are equal to $a_i^k=\alpha_i'^k$. It should be mentioned, that (3.1.1.2) holds not only during using of transformations belonging to the EH group, but also for any nonholonomic transformations.

It follows from (3.1.1.1), (3.1.1.2) and (3.1.1.4) that geodesic line equations in the form of (3.1.1.2) are invariant with respect to arbitrary nonholonomic transformations, including those which comprise the EH group of nonholonomic transformations.

Indeed, using the nonholonomic transformation with the coefficients $a_i^k=\alpha_i^k$, let us go back from (3.1.1.2) to (3.1.1.1) from which, using the nonholonomic transformation with the coefficients $a_i^k=\alpha_i'^k$, we arrive at (3.1.1.4). The combination of these two nonholonomic transformations is also a nonholonomic transformation which transforms (3.1.1.2) into (3.1.1.4). i.e. (3.1.1.2) is an invariant form of the geodesic line with respect to nonholonomic transformations.

If the two transformations mentioned here belong to the EH group, the resulting transformation also belongs to the EH group and therefore (3.1.1.2) is invariant with respect to the EH group of nonholonomic transformations.

(3.1.1.2) is a general form of geodesic line equations in any $SR(x^k,\underset{k}{\vec{e}})$ and is reduced to (3.1.1.1) in $SR(x^k,\underset{k}{\vec{E}})$. Here $\underset{i}{e}^k=\delta_i^k$ and $\alpha_i^k=\delta_i^k$, $\frac{\partial \underset{p}{e}^r}{\partial \underset{e}{x}^q}=0$, $\alpha_i^k\alpha_r^i=\delta_r^k$, $\alpha_p^i\alpha_q^\ell\Gamma_{i\ell}^r=\Gamma_{pq}^r$ i.e. *(3.1.1.2)* coincides with (3.1.1.1).

To study the practical problems, it is more convenient to use equations (3.1.1.1). The solution of this system must satisfy the initial conditions:

$$x^k=x_0^k \quad \text{and} \quad \frac{dx^k}{ds}=\dot{x}_0^k \quad \text{at} \quad s=0 \qquad (3.1.1.5)$$

where x_0^k and $\dot{x}_0^k$ are the given numbers satisfying the condition

$$g_{ik}(x_0^0,x_0^1,x_0^2,x_0^3)\dot{x}_0^i\dot{x}_0^k=1 \qquad (3.1.1.6)$$

In the rational coordinate system x'^k with the definite $\psi^2(x^0,x^1,x^2,x^3)$ the conditions $g'_{0\alpha}=0$ and $g'_{00}=\psi^2>0$ are valid. The x^k and x'^k coordinate systems are related to the transformations

$$x'^k = x'^k(x^0,x^1,x^2,x^3) \qquad (3.1.1.7)$$

Functions $x'^k(x^0,x^1,x^2,x^3)$ are defined by solving the above-mentioned differential equations:

$$g^{pq}\frac{\partial x'^0}{\partial x^p}\frac{\partial x'^0}{\partial x^q}=\psi^2(x^0,x^1,x^2,x^3),$$
$$g^{pq}\frac{\partial x'^0}{\partial x^p}\frac{\partial x'^\alpha}{\partial x^q}=0, \qquad (3.1.1.8)$$

if particular initial conditions are fulfilled. x'^0 is the time x'^α and is the space coordinates. In the rational coordinate system a free material body moves along the geodesic line; the motion laws are described by solving the following Cauchi problem:

$$\frac{d^2x'^k}{ds^2}+\Gamma'^k_{pq}\frac{dx'^p}{ds}\frac{dx'^q}{ds}=0$$
$$x'^k=x'^k_0 \quad \text{and} \quad \frac{d\dot{x}'^k}{ds}=\dot{x}'^k_0 \quad \text{at} \quad s=0, \qquad (3.1.1.9)$$

with

$$g_{ik}(x'^0_0,x'^1_0,x'^2_0,x'^3_0)\dot{x}'^i_0\dot{x}'^k_0=1 \qquad (3.1.1.10)$$

Let x''^k be another, also divided coordinate system corresponding to the function $\overline{\psi}^2$. The relationship between x^k and x''^k is determined by solving the system of first-order differential equations

$$g^{pq}\frac{\partial x''^0}{\partial x^p}\frac{\partial x''^0}{\partial x^q}=\overline{\psi}^2(x''^0),$$
$$g^{pq}\frac{\partial x''^0}{\partial x^p}\frac{\partial x''^\alpha}{\partial x^q}=0, \qquad (3.1.1.10')$$

if particular initial conditions are fulfilled. The right-hand side of this system depends on the required function x''^0. Therefore system (3.1.1.10) differs from that considered above in section §2.1, however, as is known, the existence and uniqueness theorem of solution also is valid for system (3.1.1.10). And in the case under consideration the conditions $g''_{00}(x''^0)=\overline{\psi}^2\left(x''^0\right)$ and $g''_{0\alpha}=0$ are also valid.

By solving the system (3.1.1.10), the relevant initial conditions being fulfilled, four functions are determined:

$$x''^k=x''^k(x^0,x^1,x^2,x^3), \qquad (3.1.1.11)$$

which in totality with (3.1.1.7) determine the relationship between the coordinates x'^k and x''^k

$$x'^k=\tilde{x}'^k(x''^0,x''^1,x''^2,x''^3) \qquad (3.1.1.12)$$

Here $\frac{dx'^k}{ds}$ is transformed as a vector

$$\frac{\partial x''^k}{\partial s}=\frac{\partial x''^k}{\partial x'^p}\frac{dx'^p}{ds} \qquad (3.1.1.13)$$

In particular, if $s=0$, we have:

$$\dot{x}_0''^k = \left(\frac{\partial x''^k}{\partial x'^p}\right)_{x_0'^p} \dot{x}_0'^p, \qquad (3.1.1.14)$$

where $\left(\frac{\partial x''^k}{\partial x'^p}\right)_{x_0'^p}$ is the value of $\frac{\partial x''^k}{\partial x'^p}$ at $x'^p = x_0'^p$.

One can always select initial conditions and the right-hand side of system (3.1.1.10) so as to meet the conditions

$$\dot{x}_0''^k = \left(\frac{\partial x''^k}{\partial x'^p}\right)_{x_0'^p} \dot{x}_0'^p = 0, \qquad (3.1.1.14')$$

then the law of a free neutral body in the system x''^k is described by the following Cauchi problem:

$$\frac{d^2 x''^k}{ds^2} + \Gamma''^k_{pq}\frac{dx''^p}{ds}\frac{dx''^q}{ds} = 0,$$

$$x''^k = x_0''^k \text{ and } \frac{dx''^k}{ds} = 0,\ \frac{dx''^k}{ds} = \dot{x}_0''^k \text{ at } s = 0, \qquad (3.1.1.15)$$

$$\left[g''_{00}(x_0''^0)(\dot{x}_0''^0)^2 = 1\right]$$

Taking into account that $g''_{0\alpha} = 0$, the validity of the following equities can easily be shown:

$$\Gamma''^\alpha_{00} = 0,\ \Gamma''^\alpha_{00} = \frac{d}{dx''^0}\left(\ln\sqrt{g''_{00}}\right)$$

Therefore (3.1.1.15) can be rewritten as follows:

$$\frac{d^2 x''^\gamma}{ds^2} + \Gamma''^\gamma_{pq}\frac{dx''^\alpha}{ds}\frac{dx''^\beta}{ds} + 2\Gamma''^\gamma_{0\alpha}\frac{dx''^0}{ds}\frac{dx''^\alpha}{ds} = 0, \qquad (3.1.1.16)$$

$$x''^\gamma = x_0''^\gamma,\ \dot{x}''^\gamma = 0 \text{ at } s = 0,$$

and

$$\frac{d^2 x''^0}{ds^2} + \Gamma''^0_{\alpha\beta}\frac{dx''^\alpha}{ds}\frac{dx''^\beta}{ds} + 2\Gamma''^0_{0\alpha}\frac{dx''^0}{ds}\frac{dx''^\alpha}{ds} +$$

$$+\Gamma''^0_{00}\left(\frac{dx''^0}{ds}\right)^2 = 0, \qquad (3.1.1.17)$$

$$x''^0 = x_0''^0,\ \left(\frac{dx''^0}{ds}\right)^2 = \frac{1}{g_{00}(x_0''^0)} \text{ at } s = 0,$$

i.e. it is split up into two Cauchi problems. In the Cauchi problem (3.1.1.16) $\frac{dx''^0}{ds}$ should be replaced by its value determined from the condition $g''_{\alpha\beta}\dot{x}''^\alpha\dot{x}''^\beta + g''_{00}(\dot{x}''^0)^2 = 1$. It is evident that if $x''^\alpha = x_0''^\alpha$, (3.1.1.16) is satisfied and since this Cauchi problem has the unique solution, $x''^\alpha = x_0''^\alpha = const$ is its solution. If these values x''^α are substituted into (3.1.1.13) we obtain

$$\frac{d^2 x''^0}{ds^2} + \frac{d}{dx''^0}\left(\ln\sqrt{g''_{00}}\right)\left(\frac{dx''^0}{ds}\right)^2 = 0,$$

$$x''^0 = x_0''^0 \text{ and } \frac{dx''^0}{ds} = \pm\frac{1}{\sqrt{g''_{00}(x_0''^0)}} \text{ with } s = 0 \qquad (3.1.1.18)$$

The first integral of this problem has the following form:

$$\sqrt{g''_{00}(x''^0)}dx''^0 = ds, \qquad (3.1.1.19)$$

i.e. $s = ct$ (s is the time in the $x_0''^{\gamma}$ point with the accuracy of factor c).

Thus, it is always possible to select such x''^k system of coordinates, where a neutral material body is at rest. If we substitute $x''^{\alpha} = x_0''^{\alpha}$ and $x''^0 = \zeta(s)$ from (3.1.1.19) into (3.1.1.12), then (3.1.1.12) will determine the law of body motion along x'^k coordinates, in particular, $x'^k = \tilde{x}'^k\left(\zeta(s), x_0''^1, x_0''^2, x_0''^3\right)$. The nature of the relationship between the x'^k and x''^k systems in all points will be difficult to explain, and in the vicinity of the body itself the system x''^{γ} moves relative to x'^{γ} according to the above-mentioned law. When a satellite is moving in the near space x'^{γ} is a system connected with the earth, and x''^{α} - a system, connected with the satellite.

3.1.2. THE LAW OF CHARGED BODY MOTION [1]

When considering the EH group of transformations the functions φ_i and ψ_i were introduced. The physical meaning (in the notions usual for us) of these parameters can be explained by using the equations for charged material body motion. This method is completely equivalent to that used by Einstein, which he used to explain the physical essence of the elements of the metric tensor g_{ik}. Hereinafter similar to the relativistic theory of gravitational field we assume that the tensor g_{ik} characterizes the gravitational field.

Let us formulate the law of charged material body motion as the following principle: a charged material body moves along the non-isotropic line of the four-dimensional space-time variety described by the equations:

$$\underset{e}{u}^i{}_{,p}\,\underset{e}{u}^p = 0\,, \qquad (3.1.2.1)$$

where $\underset{e}{u}^i = \dfrac{d_e x^i}{ds}$, and ds is the elementary line arc length. Using the notation

$$\frac{d_e{}^2 x^i}{ds^2} = \underset{e}{u}^p \frac{\partial \underset{e}{u}^i}{\partial \underset{e}{x}^p} = \frac{d_e x^p}{ds}\frac{\partial}{\partial \underset{e}{x}^p}\left(\frac{d_e x^i}{ds}\right), \qquad (3.1.2.2)$$

(3.1.2.1) can be rewritten in the following form:

$$\frac{d_e{}^2 x^i}{ds^2} + H^i_{pq}\frac{d\,\underset{e}{x}^p}{ds}\frac{d\,\underset{e}{x}^q}{ds} = 0\,, \qquad (3.1.2.3)$$

or

$$\frac{d_e{}^2 x^i}{ds^2} + \Gamma^i_{pq}\frac{d_e x^p}{ds}\frac{d_e x^q}{ds} + \frac{1}{2\eta}\left(\underset{q}{\psi}\frac{d_e x^p}{ds}\right)F^i_p\frac{d_e x^p}{ds} = 0 \qquad (3.1.2.4)$$

This is precisely the system of equations for the charged continuous body motion (pseudogeodesic system of equations) with the electric and gravitational charge density ratio equal to:

$$\frac{\rho_e}{\rho_g} = -\frac{c}{2\eta}\psi_p\frac{d_e x^p}{ds}.$$

In that part of the space where the condition

$$\frac{1}{\eta}\psi_p\frac{d_e x^p}{ds} = 2a = const \qquad (3.1.2.5)$$

is valid ($a = -Q/Mc^2$, where Q and M are the charge and the mass of a moving particle) system (3.1.2.4) will take the form:

$$\frac{d_e{}^2 x^i}{ds^2} + \Gamma^i_{pq} \frac{d_e x^p}{ds} \frac{d_e x^q}{ds} + aF^i_p \frac{d_e x^p}{ds} = 0 \qquad (3.1.2.6)$$

This system coincides with the motion equation system (in nonholonomic form) for a charged material body in the curved space in the presence of the electromagnetic field. To be sure therein it is sufficient to rewrite it in $SR(x^k, \underset{k}{\vec{E}})$. Due to the covariance of system (3.1.2.6) with respect to nonholonomic transformations belonging to the EH group, it also retains its form in $SR(x^k, \underset{k}{\vec{E}})$, only the infinitesimals $d \underset{e}{x^i}$ and $d^2 \underset{e}{x^i}$ should be replaced by usual differentials dx^i and $d^2 x^i$. Then the system (3.1.2.6) in $SR(x^k, \underset{k}{\vec{E}})$ will take the following form:

$$\frac{d_e{}^2 x^i}{ds^2} + \Gamma^i_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds} + aF^i_p \frac{dx^p}{ds} = 0 \qquad (3.1.2.7)$$

This is the system of the charged material body motion equations in the external gravitational and electromagnetic field with the potential φ_i and the covariant strength tensor

$$F_{ij} = \frac{\partial \varphi_i}{\partial x^j} - \frac{\partial \varphi_j}{\partial x^i}.$$

Thus the equivalence requirement for the reference system $SR(x^k, \underset{k}{\vec{e}})$ or the covariance requirement for physical laws with respect to nonholonomic transformations has resulted to introduction of new functions φ_i which can be taken as electromagnetic field potentials. On the other hand, in chapter I it was shown that during transition from $SR(x^k, \underset{k}{\vec{e}})$ to $SR(x^k, \underset{k}{\vec{e}'})$ g_{ik} and φ_i are transformed not as independent parameters, but as components of a unified value. In this connection g_{ik} and φ_i can be considered as gravitational and electromagnetic potential components (gravitational potential, electromagnetic potential) of the unified gravitational-electromagnetic (GEH) field.

Let the reference system $SR(x^k, \underset{k}{\vec{E}})$ be divided, then charged body motion equations (3.1.2.3) can be rewritten as:

$$\begin{aligned} &\frac{d^2 x^\gamma}{ds^2} + H^\gamma_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} + H^\gamma_{0\alpha} \frac{dx^0}{ds} \frac{dx^\alpha}{ds} + \\ &+ H^\gamma_{\alpha 0} \frac{dx^0}{ds} \frac{dx^\alpha}{ds} + H^\gamma_{00} \left(\frac{dx^0}{ds} \right)^2 = 0, \qquad (3.1.2.8) \\ &\frac{d^2 x^0}{ds^2} + H^0_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds} = 0 \end{aligned}$$

This system, similar to the case of the neutral body, has the solution $x^\alpha = x^\alpha_0$, $x^0 = x^0(s)$ satisfying the initial conditions:

$$x^i = x^i_0, \quad \dot{x}^\alpha = \dot{x}^\alpha_0 = 0, \quad \dot{x}^0 = \dot{x}^0_0 = \pm \frac{1}{\sqrt{g_{00}}} \quad at \quad s = 0, \qquad (3.1.2.9)$$

only if the conditions

$$H^\alpha_{00} = 0 \qquad (3.1.2.10)$$

are valid.

Taking into account that $g_{0\alpha} = 0, \; g_{00} > 0$ and

$$H_{00}^{\alpha} = \frac{1}{2} g^{\alpha p} \left(2 \frac{\partial g_{0p}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^p} \right) + \frac{1}{2\eta} g^{\alpha p} g_{0q} \psi^q F_{0p} \,,$$

condition (3.1.2.10) reduces to the equation:

$$\frac{\partial}{\partial x^{\alpha}} (\ln g_{00}) - \frac{\psi^0}{2\eta} F_{0\alpha} = 0 \qquad \textit{(3.1.2.11)}$$

These conditions are necessary and sufficient for the Cauchi problem (3.1.2.8), (3.1.2.9) to have the solution $x^{\alpha} = x_0^{\alpha} = \ = const, \ \ x^0 = x^0(s)$, which corresponds to the motionless state of a charged body in three-dimensional space.

3.2. CONSERVATION LAWS

3.2.1. CONSERVATION LAWS FOR UNCHARGED MATERIAL BODY

In accord to the previous section the uncharged material body moves along the geodesic line of four-dimensional space-time variety the equation of which in $SR(x^k, \underset{k}{\vec{E}})$ has the form of (3.1.1.1) . As it was mentioned in chapter I, one of the integrals of motion of this system is the following equation:

$$g_{ik} \frac{dx^i}{ds} \frac{dx^k}{ds} = 1 \qquad \textit{(3.2.1.1)}$$

Geometrically it expresses the fact that $\dfrac{dx^i}{ds}$ is a unit tangent vector of the geodesic line.

It can be also defined other integrals of uncharged material body motion. With this aim and to explain the physical essence of the motion integral (3.2.1.1) we introduce a new x'^k coordinate system where the metric tensor g'_{ik} is divided into time and space parts.. As was shown in chapter II, this procedure is always feasible when defining the solutions $\varphi^0(x^0, x^1, x^2, x^3)$ and $\varphi^{\alpha}(x^0, x^1, x^2, x^3)$ of the relevant differential equations in the first-order partial derivatives. New x'^k coordinates are determined from the equities:

$$x'^k = \varphi^k (x^0, x^1, x^2, x^3) \qquad \textit{(3.2.1.2)}$$

The metric tensor in the system of x'^k coordinate will have the following form:

$$(g'_{ik}) = \begin{pmatrix} g'_{00} & 0 & 0 & 0 \\ 0 & g'_{11} & g'_{12} & g'_{13} \\ 0 & g'_{12} & g'_{22} & g'_{23} \\ 0 & g'_{13} & g'_{23} & g'_{33} \end{pmatrix} \qquad \textit{(3.2.1.3)}$$

Besides, in x'^k instead of (3.2.1.1) the equity

$$g'_{00} \left(\frac{dx'^0}{ds} \right)^2 + g'_{\alpha\beta} \frac{dx'^{\alpha}}{ds} \frac{dx'^{\beta}}{ds} = 1 \qquad \textit{(3.2.1.4)}$$

is valid.

Hence, taking into account that

$$\begin{aligned} ds^2 &= g'_{00} \left(dx'^0 \right)^2 + g'_{\alpha\beta} dx'^{\alpha} dx'^{\beta} , \\ g'_{00} \left(dx'^0 \right)^2 &= c^2 \left(dt' \right)^2 , \end{aligned} \qquad \textit{(3.2.1.5)}$$

we get

$$\frac{1}{c^2}E_k'^2 - p'^2 = m^2c^2, \qquad (3.2.1.6)$$

where

$$E_k = mc^2\sqrt{g'_{00}}\frac{dx'^0}{ds} = \frac{mc^2}{\sqrt{1-v'^2/c^2}} \; - \qquad (3.2.1.7)$$

is the kinetic energy (at $v'/c << 1$ we have $E'^k \approx mc^2 + \frac{1}{2}mv'^2$),

$$v' = \sqrt{-g'_{\alpha\beta}\frac{dx'^\alpha}{ds}\frac{dx'^\beta}{ds}} \; - \qquad (3.2.1.8)$$

is the absolute value of the motion velocity,

$$p' = \sqrt{-g'_{\alpha\beta}p'^\alpha p'^\beta} \; - \qquad (3.2.1.9)$$

is the absolute value of the material body momentum

$$p'^\alpha = mc\frac{dx'^\alpha}{ds} = \frac{m\frac{dx'^\alpha}{dt}}{\sqrt{1-v'^2/c^2}} \qquad (3.2.1.10)$$

Thus, according to (3.2.1.6), the motion integral (3.2.1.1) expresses physically the relation between the kinetic energy and the absolute momentum value of the moving material body.

In the new x'^k coordinate system the geodesic line equations have the following form:

$$\frac{d^2x'^k}{ds^2} + \Gamma'^k_{pq}\frac{dx'^p}{ds}\frac{dx'^q}{ds} = 0 \qquad (3.2.1.11)$$

Let us consider the fourth equation of this system:

$$\frac{d^2x'^0}{ds^2} + \Gamma'^0_{pq}\frac{dx'^p}{ds}\frac{dx'^q}{ds} = 0, \qquad (3.2.1.12)$$

where

$$\left(\Gamma'^0_{ik}\right) = \begin{pmatrix} \frac{1}{2}g'^{00}\frac{\partial g'_{00}}{\partial x'^0} & \frac{1}{2}g'^{00}\frac{\partial g'_{00}}{\partial x'^1} & \frac{1}{2}g'^{00}\frac{\partial g'_{00}}{\partial x'^2} & \frac{1}{2}g'^{00}\frac{\partial g'_{00}}{\partial x'^3} \\ & -\frac{1}{2}g'^{00}\frac{\partial g'_{11}}{\partial x'^0} & -\frac{1}{2}g'^{00}\frac{\partial g'_{12}}{\partial x'^0} & -\frac{1}{2}g'^{00}\frac{\partial g'_{13}}{\partial x'^0} \\ & & -\frac{1}{2}g'^{00}\frac{\partial g'_{22}}{\partial x'^0} & -\frac{1}{2}g'^{00}\frac{\partial g'_{23}}{\partial x'^0} \\ & & & -\frac{1}{2}g'^{00}\frac{\partial g'_{33}}{\partial x'^0} \end{pmatrix} \qquad (3.2.1.13)$$

Taking this into account, (3.2.1.12) takes the form:

$$\frac{d^2x'^0}{ds^2} + g'^{00}\frac{dg'_{00}}{ds}\frac{dx'^0}{ds} - \frac{1}{2}g'^{00}\frac{dg'_{pq}}{dx'^0}\frac{dx'^p}{ds}\frac{dx'^q}{ds} = 0 \qquad (3.2.1.14)$$

It is clear from the structure of the metric tensor g'_{ik} that $g'^{00} = \frac{1}{g'^{00}}$, therefore

$$g'_{00}\frac{d^2x'^0}{ds^2} + \frac{dg'_{00}}{ds}\frac{dx'^0}{ds} - \frac{1}{2}\frac{dg'_{pq}}{dx'^0}\frac{dx'^p}{ds}\frac{dx'^q}{ds} = 0,$$

or

$$\frac{d}{ds}\left(mc^2 g'_{00}\frac{dx'^0}{ds}\right) = \frac{mc^2}{2}\frac{\partial g'_{pq}}{\partial x'^0}\frac{dx'^p}{ds}\frac{dx'^q}{ds} \qquad (3.2.1.15)$$

If the four-dimensional space-time variety metric does not change in time (static gravitational field), i.e. if g'_{pq} does not depend on x'^0, (3.2.1.15) reduces to

$$\frac{d}{ds}\left(mc^2 g'_{00}\frac{dx'^0}{ds}\right)=0 \tag{3.2.1.16}$$

Hence,

$$mc^2 g'_{00}\frac{dx'^0}{ds}=const \tag{3.2.1.17}$$

where m is the material body rest mass (classical notion).

Using equations (3.2.1.5) and (3.2.1.8), the latter equity takes the form:

$$\frac{mc^2\sqrt{g'_{00}}}{\sqrt{1-v'^2/c^2}}=const \tag{3.2.1.18}$$

In the areas of three-dimensional space with weak gravitational field, the assessment is valid [2]

$$g'_{00}\approx 1+\frac{u}{c^2} \tag{3.2.1.19}$$

where u is the gravitational field potential (classical notion). Then, from (3.2.1.18) we have:

$$mc^2+\frac{mv^2}{2}+mu=const \tag{3.2.1.20}$$

This is precisely the mechanical energy law in classical physics. Therefore, let us call the quantity

$$p'_0=mc^2 g'_{00}\frac{dx'^0}{ds}=\frac{mc^2\sqrt{g'_{00}}}{\sqrt{1-v'^2/c^2}} \tag{3.2.1.21}$$

the total mechanical energy of a moving material body in general case.

Thus, the equity (3.2.1.17) expresses the total mechanical energy law for a moving uncharged material body with allowance of relativistic effects.

(3.2.1.17) can be rewritten in the initial x^k coordinate system parameters:

$$mc^2 g_{pq}\frac{\partial x^p}{\partial x'^0}\frac{\partial x^q}{\partial x'^0}\frac{\partial x'^0}{\partial x^r}\frac{dx^r}{ds}=const \tag{3.2.1.22}$$

According to (3.2.1.2), the transformation coefficients $\frac{\partial x^k}{\partial x'^0}$ can be determined through $\varphi^k(x^0,x^1,x^2,x^3)$ from the following system:

$$\frac{\partial x'^k}{\partial x^r}\frac{\partial x^p}{\partial x'^k}=\delta_r^p \tag{3.2.1.23}$$

Hence,

$$\frac{\partial x^p}{\partial x'^k}=\frac{A\left(\frac{\partial x'^p}{\partial x^k}\right)}{\det\left(\frac{\partial x'^q}{\partial x^r}\right)} \tag{3.2.1.24}$$

where $A\left(\frac{\partial x'^k}{\partial x^r}\right)$ is the algebraic complement to the $\frac{\partial x'^k}{\partial x^i}$ element in the determinant $\det\left(\frac{\partial x'^k}{\partial x^i}\right)$. After substitution the (3.2.1.22) is reduced to the following equity:

$$p'_0 = mc^2 g_{pq} \frac{\partial x'^0}{\partial x^r} \frac{A\left(\frac{\partial x'^p}{\partial x^0}\right) A\left(\frac{\partial x'^q}{\partial x^0}\right)}{\left[\det\left(\frac{\partial x'^k}{\partial x^i}\right)\right]^2} \frac{dx^r}{ds} = const \qquad (3.2.1.25)$$

This is precisely the sought motion integral of system (3.1.1.1) in x^k variables, expressing the mechanical energy law of an uncharged material body moving in the external static gravitational field.

In the case when g'_{ik} depend on x'^0, from (3.2.1.15) we have:

$$\frac{dp'_0}{ds} = \frac{mc^2}{2} \frac{\partial g'_{pq}}{\partial x'^0} \frac{dx'^p}{ds} \frac{dx'^q}{ds}, \qquad (3.2.1.26)$$

or

$$dp'_0 = da'_0 , \qquad (3.2.1.27)$$

where

$$da'_0 = \frac{mc^2}{2} \frac{\partial g'_{pq}}{\partial x'^0} \frac{dx'^p}{ds} \frac{dx'^q}{ds} ds \qquad (3.2.1.28)$$

Thus, the change in the total mechanical energy of a moving material body is equal to the work da'_0, fulfilled by the dynamic gravitational field. It is connected with the variation of the gravitational field in time $\left(\frac{\partial g'_{pq}}{\partial x'^0}\right)$; the gravitational field either gives the energy to the gravitational body (at $da'_0 > 0$), or does not take from it part of the mechanical energy (at $da'_0 < 0$). From the viewpoint of classical physics the electromagnetic field must not participate in this exchange since it has no direct effect on an uncharged material body. and besides, in classical physics gravitational and electromagnetic fields are independent objective realities. From the viewpoint of a unified field, in conditions of dynamics, the electromagnetic field exerts an indirect effect on an uncharged material body; this is evident from the fact that the time change in parameters φ_i causes the temporal change in parameters g_{ik}, this change, according to (3.2.1.28), is related to the gravitational field work.

With allowance of the structure of the g'_{ik} and Γ'_{ik} parameters the following expression for da'_0 can easily be obtained:

$$da'_0 = \frac{mc^2}{1 - v'^2/c^2} \left(\Gamma'^0_{00} - \frac{1}{c^2} g'_{00} \Gamma'^0_{\alpha\beta} v'^\alpha v'^\beta \right) ds \qquad (3.2.1.29)$$

From (3.2.11) for $k = 1,2,3$ we have:

$$\frac{d^2 x'^\alpha}{ds^2} + \Gamma'^\alpha_{pq} \frac{dx'^p}{ds} \frac{dx'^q}{ds} = 0, \qquad (3.2.1.30)$$

where, according to (3.2.1.3) -

$$\Gamma'^\alpha_{pq} = \frac{1}{2} g'^{\alpha\beta} \left(\frac{\partial g'_{p\beta}}{\partial x'^q} + \frac{\partial g'_{q\beta}}{\partial x'^p} - \frac{\partial g'_{pq}}{\partial x'^\beta} \right)$$

After substitution from (3.2.1.30) we have:

$$\frac{d^2 x'^\alpha}{ds^2} + g'^{\alpha\beta} \frac{dg'_{p\beta}}{ds} \frac{\partial x'^p}{\partial s} = \frac{1}{2} g'^{\alpha\beta} \frac{\partial g'_{pq}}{\partial x'^\beta} \frac{dx'^p}{ds} \frac{dx'^q}{ds}$$

Hence,

$$\frac{dp'_\alpha}{ds} = \frac{mc}{2} \frac{\partial g'_{pq}}{\partial x'^\alpha} \frac{dx'^p}{ds} \frac{dx'^q}{ds} , \qquad (3.2.1.31)$$

where

$$p'_\alpha = mcg'_{\alpha\beta}\frac{dx'^\beta}{ds} \qquad (3.2.1.32)$$

They are components of the moving material body momentum.

If we have such gravitational field where the condition

$$\frac{\partial g'_{pq}}{\partial x'^\alpha}\frac{dx'^p}{ds}\frac{dx'^q}{ds} = 0, \qquad (3.2.1.33)$$

is valid, then, according to (3.2.1.31)

$$p'_\alpha = const, \qquad (3.2.1.34)$$

i.e. the momentum conservation law is realized. Otherwise, the infinitely small change in the moving material body momentum is equal to:

$$dp'_\alpha = da'_\alpha, \qquad (3.2.1.35)$$

where

$$da'_\alpha = \frac{mc}{2}\frac{\partial g'_{pq}}{\partial x'^\alpha}\frac{dx'^p}{ds}\frac{dx'^q}{ds}ds \qquad (3.2.1.36)$$

If, instead of the parameter s, we use the time t', then from (3.2.1.32) for the moving material body momentum in $SR(x'^k, \underset{k}{\vec{E}})$ we will get:

$$p'_\alpha = \frac{mg'_{\alpha\beta}\dfrac{dx'^\beta}{dt'}}{\sqrt{1-v'^2/c^2}} \qquad (3.2.1.37)$$

Along with the parameter p'_0 determined according to (3.2.1.21), we can introduce the parameter

$$P'^0 = mc^2\frac{dx'}{ds} = \frac{mc^2}{\sqrt{g'_{00}}}\frac{1}{\sqrt{1-v'^2/c^2}} \qquad (3.2.1.37)$$

3.2.2. CONSERVATION LAWS FOR A CHARGED MATERIAL BODY

If in (3.1.2.7) a will be replaced by value

$$a = -\frac{1}{mc^2}q, \qquad (3.2.2.1)$$

then the system of equations for charged material body motion will take the following form:

$$\frac{d^2x^k}{ds^2} + \Gamma^k_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds} - \frac{q}{mc^2}F^k_p\frac{dx^p}{ds} = 0 \qquad (3.2.2.2)$$

Here q is the charge of a material body.

Let us multiply (3.2.2.2) by $g_{ik}\dfrac{dx^i}{ds}$ and summarize over the k index, we'll obtain:

$$\frac{d}{ds}\left(g_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds}\right) - \frac{q}{mc^2}F_{pi}\frac{dx^p}{ds}\frac{dx^i}{ds} = 0.$$

Since F_{ik} is the antisymmetric tensor (in $SR(x^k, \underset{k}{\vec{E}})$), then from this we have:

$$g_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds} = 1 \qquad (3.2.2.3)$$

Quite similar to the above, in the reference system with the metric tensor divided into time and space parts the equation of the (3.2.2.2) system at $k=0$ gives:

$$\frac{d}{ds}\left(mc^2 g'_{00}\frac{dx'^0}{ds}\right)=\frac{mc^2}{2}\frac{\partial g'_{pq}}{\partial x'^0}\frac{dx'^p}{ds}\frac{dx'^q}{ds}-qF'_{p0}\frac{dx'^p}{ds}=0 \quad (3.2.2.4)$$

If the unified GEH field is static -

$$\frac{\partial g'_{pq}}{\partial x'^0}=0,\ \frac{\partial \varphi'_i}{\partial x'^0}=0,\ F'_{\alpha 0}=-\frac{\partial \varphi'_0}{\partial x'^\alpha} \quad (3.2.2.5)$$

and (3.2.2.4) takes the following form:

$$\frac{d}{ds}\left(mc^2 g'_{00}\frac{dx'^0}{ds}+q\varphi'_0\right)=0, \quad (3.2.2.6)$$

or

$$mc^2 g'_{00}\frac{dx'^0}{ds}+q\varphi'_0=const \quad (3.2.2.7)$$

the $q\varphi'_0$ in classical physics is the charge energy q in the static potential field with the potential φ'_0. Then

$$P'_0=mc^2 g'_{00}\frac{dx'^0}{ds}+q\varphi'_0=\frac{mc^2\sqrt{g'_{00}}}{\sqrt{1-v'^2/c^2}}+q\varphi'_0 \quad (3.2.2.8)$$

is the total energy of a charged material body – mechanical energy + electric energy and (3.2.2.7) represents the energy conservation law.

In case of the dynamic GEH field from (3.2.2.4) we have:

$$dP'_0=dA'_0 \quad (3.2.2.9)$$

where

$$dA'_0=\left(\frac{mc^2}{2}\frac{\partial g'_{pq}}{\partial x'^0}\frac{dx'^p}{ds}\frac{dx'^q}{ds}+q\frac{\partial \varphi'_\alpha}{\partial x'^0}\frac{dx'^\alpha}{ds}\right)ds, \quad (3.2.2.10)$$

or

$$dA'_0=\frac{mc^2}{1-v'^2/c^2}\left[\Gamma'^0_{00}-\frac{1}{c^2}g'_{00}\Gamma'^0_{\alpha\beta}v'^\alpha v'^\beta\right]ds+ \\ +q\frac{\partial \varphi'_\alpha}{\partial x'^0}\frac{dx'^\alpha}{ds}ds, \quad (3.2.2.11)$$

i.e. the change in the total energy dP'_0 of a charged material body, moving in the dynamic GEH field is equal to the elementary work dA'_0 fulfilled by the GEH field.

Quite similarly to the previous one, from (3.2.2.2) in $SR(x'^k,\underset{k}{\vec{E}})$ with $k=1,2,3$ we obtain:

$$\frac{dP'_\alpha}{ds}=\frac{mc}{2}\frac{\partial g'_{pq}}{\partial x'^\alpha}\frac{dx'^p}{ds}\frac{dx'^q}{ds}+\frac{q}{c}\frac{\partial \varphi'_p}{\partial x'^\alpha}\frac{dx'^p}{ds}, \quad (3.2.2.12)$$

where

$$P'_\alpha=p'_\alpha+\frac{q}{c}\varphi'_\alpha \quad (3.2.2.13)$$

P'_α is the total momentum of a moving charged material body. Following (3.2.2.12) the infinitely small change in the total momentum equals to

$$\delta A'_\alpha=\left(\frac{mc}{2}\frac{\partial g'_{pq}}{\partial x'^\alpha}\frac{dx'^p}{ds}\frac{dx'^q}{ds}+\frac{q}{c}\frac{\partial \varphi'_p}{\partial x'^\alpha}\frac{dx'^p}{ds}\right)ds, \quad (3.2.2.14)$$

and if the GEH field is such that $\delta A'_\alpha=0$,

$$P'_\alpha = mcg'_{\alpha\beta}\frac{dx'^\beta}{ds} + \frac{q}{c}\varphi'_\alpha = \frac{m}{\sqrt{1-v'^2/c^2}}g'_{\alpha\beta}\frac{dx'^\beta}{dt'} + \frac{q}{c}\varphi'_\alpha = const, \qquad (3.2.2.15)$$

i.e. the law of conservation of the moving charged material body momentum is realized.

Along with P'_0 and P'_α let us introduce, the parameters P'^0 and P'^α determined by the following equities:

$$P'^0 = p'^0 + qg'^{00}\varphi'_0 \ , \quad P'^\alpha = p'^\alpha + \frac{q}{c}g^{\alpha\beta}\varphi'_\beta \qquad (3.2.2.16)$$

In $SR(x'^k, \underset{k}{\vec{E}})$ the (3.2.2.3) has a form:

$$g'_{00}\left(\frac{dx'^0}{ds}\right)^2 + g'_{\alpha\beta}\frac{dx'^\alpha}{ds}\frac{dx'^\beta}{ds} = 1 \ ,$$

i.e.

$$\frac{1}{c^2}g'_{00}\left(P'^0\right)^2 + g'_{\alpha\beta}P'^\alpha P'^\beta = m^2c^2 \ , \qquad (3.2.2.17)$$

or

$$\frac{1}{c^2}P'^0P'_0 + P'^\alpha P'_\alpha = m^2c^2$$

With allowance of (3.2.2.16) the condition (3.2.2.17) will take the form:

$$\frac{1}{c^2}g'_{00}\left(p'^0 - qg'^{00}\varphi'_0\right)^2 + g'_{\alpha\beta}\left(p'^\alpha - \frac{q}{c}g'^{\alpha\mu}\varphi'_\mu\right) \times \left(p'^\beta - \frac{q}{c}g'^{\beta\nu}\varphi'_\nu\right) = m^2c^2, \qquad (3.2.2.18)$$

or

$$\frac{1}{c^2}\left(P'^0 - qg'^{00}\varphi'_0\right)\left(P'_0 - q\varphi'_0\right) + \left(P'^\alpha - \frac{q}{c}g'^{\alpha\beta}\varphi'_\beta\right) \times \left(P'_\alpha - \frac{q}{c}\varphi'_\alpha\right) = m^2c^2$$

3.3. SOME COMMENTS

3.3.1. BEHAVIOUR OF DYNAMIC CHARACTERISTICS OF A MATERIAL BODY WITH RESPECT TO HOLONOMIC TRANSFORMATIONS

In the previous section the parameters p', p'^α and P'^0, P'^α, determined by the following equations were introduced:

$$p'^0 = mc^2\frac{dx'^0}{ds}, \quad p'^\alpha = mc^2\frac{dx'^\alpha}{ds}, \qquad (3.3.1.1)$$

$$P'^0 = p'^0 + qg'^{00}\varphi'_0 \ , \quad P'^\alpha = p'^\alpha + \frac{q}{c}g'^{\alpha\beta}\varphi'_\beta \qquad (3.3.1.2)$$

From equities (3.3.1.1) it is clear that the totality of parameters $\frac{1}{c}p'^0$, p'^α constitutes a four-dimensional contravariant vector which we call a four-dimensional contravariant momentum of a moving uncharged material body. Taking into account the structure of the metric tensor g'_{ik} the equations (3.3.1.2.) can be rewritten as:

$$P'^0 = p'^0 + qg'^{0p}\varphi'_p \ , \quad P'^\alpha = p'^\alpha + \frac{q}{c} g'^{\alpha p}\varphi'_p \qquad (3.3.1.3)$$

from which it is clear that the totality of the parameters $\frac{1}{c}P'^0, P'^\alpha$, is also a four-dimensional contravariant vector which we'll call a total four-dimensional contravariant momentum of a moving charged material body.

The parameters p', p'_α and P'_0, P'_α are related with the above-mentioned parameters by the following equities:

$$\begin{aligned} p'_0 = g'_{00} p'^0 = g'_{0p} p'^p \ , \quad p'_\alpha = g'_{\alpha\beta} p'^\beta = g'_{\alpha p} p'^p , \\ P'_0 = g'_{00} P'^0 = g'_{0p} P'^p \ , \quad P'_\alpha = g'_{\alpha\beta} P'^\beta = g'_{\alpha p} P'^p \end{aligned} \qquad (3.3.1.4)$$

Hence it is clear that $\frac{1}{c}p'_0, p'_\alpha$ and $\frac{1}{c}P'_0, P'_\alpha$ are four-dimensional covariant vectors; the first is the covariant four-dimensional momentum of an uncharged material body, and the second – the four-dimensional covariant momentum of a charged body.

Hence it is easy to establish the laws of energy and momentum transformation at general holonomic transformations. In particular, in $SR(x^k, \underset{k}{\vec{E}})$ we have:

$$\frac{\partial x^k}{\partial x'^0}\frac{1}{c}p'^0 + \frac{\partial x^k}{\partial x'^\alpha}p'^\alpha = mc\frac{dx^k}{ds} \qquad (3.3.1.5)$$

- for the contravariant momentum,

$$\frac{\partial x'^0}{\partial x^k}\frac{1}{c}p'_0 + \frac{\partial x'^\alpha}{\partial x^k}p'_\alpha = mcg_{kp}\frac{dx^p}{ds} \qquad (3.3.1.6)$$

- for the covariant momentum.

Since the metric in $SR(x^k, \underset{k}{\vec{E}})$ is not divided into time-space parts, here it is impossible to select a temporal component of the four-dimensional momentum which after being multiplied by c would give an energy of a moving uncharged material body. Similarly, it is impossible to determine momentum components. If $SR(x^k, \underset{k}{\vec{E}})$ is such that its metric is divided (here $g_{\alpha 0} = 0$), at $k = 0$ (3.3.1.6.) determines the energy p_0, and at $k = 1,2,3$ – the momentum components p_α of the material body in $SR(x^k, \underset{k}{\vec{E}})$:

$$\frac{1}{c}p_0 = mcg_{00}\frac{dx^0}{ds}, \ p_\alpha = mcg_{\alpha\beta}\frac{dx^\beta}{ds} \qquad (3.3.1.7)$$

This form of determination of the energy and momentum coincides with that determined by (3.2.1.21) and (3.2.1.32). This is quite natural, since in all the reference systems with the divided metrics the energy and momentum must be determined identically. Quite similar situation is observed for P^k and P_k vectors.

The equities

$$\begin{aligned} p'_0 = mc^2 g'_{00}\left[x'^0(s), x'^1(s), x'^2(s), x'^3(s)\right]\frac{dx'^0}{ds}, \\ p_0 = mc^2 g_{00}\left[x^0(s), x^1(s), x^2(s), x^3(s)\right]\frac{dx^0}{ds}, \end{aligned} \qquad (3.3.1.8)$$

where $x^k(s)$ and $x'^k(s)$ are the solutions of differential equations (3.1.1.1) and (3.2.1.1), respectively determine (in parametric form) a functional relation between p_0 and p'_0, i.e.

determine the law of material body energy transformation during the transition from $SR(x^k, \underset{k}{\vec{E}})$ to $SR(x'^k, \underset{k}{\vec{E}}')$ with the divided metrics. s is the parameter.

In the case under consideration the last three equations of system (3.3.1.7) are the consequence of system (3.3.1.6) at $k = 1, 2, 3$, therefore

$$p_\alpha = \frac{\partial x'^0}{\partial x^\alpha} \frac{1}{c} p'_0 + \frac{\partial x'^\beta}{\partial x^\alpha} p'_\beta \qquad (3.3.1.9)$$

From (3.2.2.18) we have

$$p'^0 = c\sqrt{g'^{00}\left(m^2c^2 - g'_{\alpha\beta}\, p'^\alpha p'^\beta\right)} \qquad (3.3.1.10)$$

Hence, following (3.3.1.4)

$$p'_0 = g'_{00}\, p'^0 = c\sqrt{g'_{00}\left(m^2c^2 - g'_{\alpha\beta}\, p'^\alpha p'^\beta\right)} \qquad (3.3.1.11)$$

Substituting this value p'_0 into (3.3.1.9), we get:

$$p_\alpha = \frac{\partial x'^0}{\partial x^\alpha} \sqrt{g'_{00}\left(m^2c^2 - g'_{\alpha\beta}\, p'^\alpha p'^\beta\right)} + \frac{\partial x'^\beta}{\partial x^\alpha} p'_\beta \qquad (3.3.1.12)$$

That last equities establish the functional relation between p_α and p'_α, i.e. the law of momentum component conversion during transition from $SR(x'^k, \underset{k}{\vec{E}}')$ to $SR(x^k, \underset{k}{\vec{E}})$ with divided metric tensors.

Thus, the independent of transformation energy and momentum is observed when holonomic transformation connects two reference systems with metric tensors divided into time and space parts.

Here the same situation is also observed for P_0 and P_α, in particular

$$\begin{aligned} P_0 &= mc^2 g_{00}\left[x^0(s), x^1(s), x^2(s), x^3(s)\right]\frac{dx^0}{ds} + \\ &+ q\varphi_0\left[x^0(s), x^1(s), x^2(s), x^3(s)\right], \\ P'_0 &= mc^2 g'_{00}\left[x'^0(s), x'^1(s), x'^2(s), x'^3(s)\right]\frac{dx'^0}{ds} + \\ &+ q\varphi'_0\left[x'^0(s), x'^1(s), x'^2(s), x'^3(s)\right] \end{aligned} \qquad (3.3.1.13)$$

determine the law of charged material body energy transformation, whereas the equity

$$P_\alpha = \frac{\partial x'^0}{\partial x^\alpha}\left\{\sqrt{g'_{00}\left(m^2c^2 - g'_{\alpha\beta}\left(P'^\alpha - \frac{q}{c} g'^{\alpha\mu}\varphi'_\mu\right)\left(P'^\beta - \frac{q}{c} g'^{\beta\nu}\varphi'_\nu\right)\right)} - \frac{q}{c}\varphi'_0\right\} + \frac{\partial x'^\beta}{\partial x^\alpha} P'_\beta \qquad (3.3.1.14)$$

is the law of charged material body momentum transformation.

Equations of uncharged material body motion (3.1.2.27) and (3.2.1.35) retain their form in all $SR(x^k, \underset{k}{\vec{E}})$ with divided metrics; besides, $\frac{1}{c} p'_0$ and p'_α constitute a four-dimensional vector, therefore it is clear that the totality of parameters $\frac{1}{c} da'_0$ and da'_α is also a four-dimensional vector with respect to transformations, connecting the reference system to the divided metric tensors. The same situation is also observed for the quantities $\frac{1}{c} dA'_0$ and dA'_α.

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CHAPTER IV

EQUATIONS OF GRAVITATIONAL-ELECTROMAGNETIC (GEH) FIELD

In chapter one we have considered the basic elements of nonholonomic geometry which is a generalization of Riemannian geometry in terms of equivalence of reference system $SR(x^k, \underset{k}{\vec{e}})$ interrelated by nonholonomic transformations belonging to the EH group. The results given in the chapter will further be used to build a GEH field system of equations. Here the well-known variation principle used in the Einstein's relativistic theory of gravitational field will be applied.

As is known, one of the main requirements of this theory consists of equivalence of all $SR(x^k, \underset{k}{\vec{E}})$. This means that the Einstein's gravitational field equations in all $SR(x^k, \underset{k}{\vec{E}})$ are equal. Here, instead of this requirement, we use a more general requirement consisting of equivalence of the reference system of $SR(x^k, \underset{k}{\vec{e}})$ selected by us. According to this requirement, GEH field equations in all reference systems $SR(x^k, \underset{k}{\vec{e}})$ interrelated by EH group nonholonomic transformations, are similar. The remaining requirements and methods used here completely coincide with the relevant requirements and methods used in the Einstein's relativistic theory of gravitational field.

4.1. THE ACTION FUNCTION AND THE SYSTEM OF EQUATIONS OF GEH FIELD

4.1.1. THE ACTION FUNCTION OF GEH FIELD

Similar to the Einstein's relativistic theory of gravitational field the GEH equation can be derived using the extremal principle with respect to the field action function

$$\underset{e}{S} = \int \sqrt{-\underset{e}{g}}\, \underset{e}{L}\, d^4 \underset{e}{x}\,, \qquad (4.1.1.1)$$

where $d^4 \underset{e}{x} = d\underset{e}{x}^0\, d\underset{e}{x}^1\, d\underset{e}{x}^2\, d\underset{e}{x}^3$, L is the Lagrangian density and integration[1] is performed in the whole space filled with the GEH field. Main requirements satisfied by the quantities L and S for the gravitational field consist of invariance of these quantities with regard to all the

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[1] $d\underset{e}{x}^i$ are not differentials of the quantities x^i, therefore the integral in the right-hand side of this equation is actually the Stiltess integral.

reference systems $SR(x^k, \underset{k}{\vec{E}})$. Let us generalize this condition for our case and demand that L be invariant with respect to all reference systems $SR(x^k, \underset{k}{\vec{e}})$ interrelated by EH group nonholonomic transformations. Since $\sqrt{-\underset{e}{g}}\, d^4 \underset{e}{x}$ is also an invariant value, it is clear that $\underset{e}{L}$ the invariance results in invariance of the action function $\underset{e}{S}$ as well, in covariance of the system of GEH field equations with regard to nonholonomic transformations belonging to the EH group.

The Lagrangian function must be composed of the parameters $\underset{e}{g}_{ij}$ and φ_i characteristic for the GEH field and of their first-order derivatives. The latter requirement provides the second order of the system of GEH field equations, which, in its turn, is the main requirement ensuring the correct asymptotic behavior of this system. Indeed, for a weak field the system of GEH field equations must coincide with classical systems of gravitational and electromagnetic field equations containing second-order equations with regard to potentials.

As has been repeatedly mentioned above, in all sections of the present chapter we use the methods of the general relativity theory and of the relativistic theory of gravitational field, generalizing them for nonholonomic transformations belonging to the EH group. In this connection, to determine the Lagrangian function for the GEH field, we shall try to use the Lagrangian function of gravitational field. It is known that [2,3,4] for the gravitational field

$$L = -\gamma R\,, \qquad (4.1.1.2)$$

where, γ is the dimensional coefficient, and k is the scalar curvature. Though R contains the second derivatives with respect to g_{ij}, this choice of the Lagrangian function does not violate the main requirement. The matter is that in the action function the second derivatives under the integral enter as an addend $div\vec{A}$, which is rejected when using the extremal principle with allowance of boundary conditions and thus does not change the character of the system of equations obtained in the rest part of the Lagrangian function containing only first derivatives. Generalizing (4.1.1.2), in the case under consideration, the Lagrangian function can be obtained with the nonholonomic scalar curvature. However, since the asymptotic value of the action function

$$S = -\gamma \int \sqrt{-\underset{e}{g}}\, \underset{e}{R}\, d^4 \underset{e}{x} \qquad (4.1.1.3)$$

does not coincide with the action function of the pure electromagnetic field in the flat space, an additional addend should be introduced.

By direct calculations can be easily shown that with a very weak GEH field gravitational component, i.e. at $g_{ij} \approx g^0_{ij}$ where[1] $g^0_{11} = g^0_{22} = g^0_{33} = -1,\ g^0_{00} = 1,\ g^0_{ij} = 0$ at $i \neq j$, the following equation is valid:

$$L = \gamma\left(-\frac{1}{16\eta^2}\psi^p\psi_p F^n_m F^m_n + \frac{1}{8\eta^2} g^{pq}\psi^m\psi^n F_{pm}F_{qn} \right) \qquad (4.1.1.4)$$

The first addend in the right-hand side with an accuracy of the multiplier $\frac{1}{\eta^2}\psi^p\psi_p$ coincides with the Lagrangian function of the pure electromagnetic field, i.e. electromagnetic field in the flat space (in the classical sense), whereas the second addend is not reduced to it. In

---

[1] This condition can hold in the special reference system $SR(x^k, \underset{k}{\vec{E}})$.

this connection we determine the Lagrangian function of the GEH field from the following equity:

$$L = -\gamma\left(\underset{e}{R} + \frac{1}{8\eta^2}\underset{e}{g}^{pq}\underset{e}{\psi}^m\underset{e}{\psi}^n\underset{e}{F}_{pm}\underset{e}{F}_{qn}\right), \qquad (4.1.1.5)$$

and demand that the function ψ^k satisfy the following condition:

$$\psi^p\psi_p \to const \quad \text{at} \quad g_{ij} \to g_{ij}^0 \qquad (4.1.16)$$

The invariance addend of

$$\frac{1}{8\eta^2}\underset{e}{g}^{pq}\underset{e}{\psi}^m\underset{e}{\psi}^n\underset{e}{F}_{pm}\underset{e}{F}_{qn}$$

follows from the law of transformation for the quantities F_{ij} (see (1.2.2.9)), in particular, the following equation takes place:

$$\frac{1}{8\eta^2}\underset{e}{g}^{pq}\underset{e}{\psi}^m\underset{e}{\psi}^n\underset{e}{F}_{pm}\underset{e}{F}_{qn} = \frac{1}{8(\eta-\beta)^2}\underset{e}{g}'^{pq}\underset{e}{\psi}'^m\underset{e}{\psi}'^n \times \times \underset{e}{F}'_{pm}\underset{e}{F}'_{qn} \qquad (4.1.1.7)$$

The Lagrangian function thus selected satisfies all requirements including the asymptotic requirement at $g_{ij} \to g_{ij}^0$. Using (4.1.1.5) the GEH field action function will take the form:

$$S = -\gamma\int\sqrt{-\underset{e}{g}}\left(\underset{e}{R} + \frac{1}{8\eta^2}\underset{e}{g}^{pq}\underset{e}{\psi}^m\underset{e}{\psi}^n\underset{e}{F}_{pm}\underset{e}{F}_{qn}\right)d^4\underset{e}{x} \qquad (4.1.1.8)$$

Using the equities

$$\underset{e}{g}^{ik}{}_{,j} = 0, \qquad \sqrt{-\underset{e}{g}}H^r_{r\,j} = \frac{\partial\sqrt{-\underset{e}{g}}}{\partial \underset{e}{x}^j}$$

for $-\sqrt{\underset{e}{g}}\,\underset{e}{R}$ we obtain:

$$-\sqrt{\underset{e}{g}}\,\underset{e}{R} = \frac{\partial}{\partial \underset{e}{x}^m}\left[\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\left(H^m_{pq} - \delta^m_q H^n_{pn}\right)\right] + \sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\times$$
$$\times\left(H^n_{pr}H^r_{qn} - H^r_{pq}H^n_{nr}\right) - \frac{1}{2\eta}\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\underset{e}{\psi}^r H^n_{pn}\underset{e}{F}_{qr} + \qquad (4.1.1.9)$$
$$+\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}H^r_{pn}H^n_{qr}$$

The first addend in the right-hand side of this equity has the form of $div\vec{A}$, however, since the derivatives in this term are generalized, the variation of the relevant integral is not zero. Let us transform this addend. With this aim, let us rewrite it in $SR(x^k, \underset{k}{\vec{E}})$ as follows (see (1.2.1.20)):

$$\frac{\partial}{\partial \underset{e}{x}^m}[...] = \underset{m}{e}^l\frac{\partial}{\partial x^l}[...]$$

In addition, according to (1.2.1.30) $d^4\underset{e}{x} = \alpha d^4x$, therefore

$$\frac{\partial}{\partial \underset{e}{x}^m}[...]d^4\underset{e}{x} = \frac{\partial}{\partial x^l}\left[\sqrt{-\underset{e}{g}}\,\alpha\,\underset{m}{e}^l\,\underset{e}{g}^{pq}\left(H^m_{pq} - \delta^m_q H^n_{pn}\right)\right]d^4x -$$
$$-\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\left(H^m_{pq} - \delta^m_q H^n_{pn}\right)\frac{\partial\left(\alpha\,\underset{m}{e}^l\right)}{\partial x^l}d^4x,$$

where $\alpha = \det\left(\alpha^i_k\right)$. Taking into account that (see (1.2.2.23))

$$\frac{\partial\left(\alpha \underset{m}{e}^{l}\right)}{\partial x^{l}}=\alpha E_{lm}^{l},$$

the last addend in the right-hand side of the last equity takes the following form:

$$-\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\left(H_{pq}^{m}-\delta_{q}^{m}H_{pn}^{n}\right)E_{lm}^{l}d^{4}x,$$

and the action function is defined from the equity:

$$S=\gamma\int\frac{\partial}{\partial x^{l}}\left[\sqrt{-\underset{e}{g}}\,\alpha\,\underset{m}{e}^{l}\,\underset{e}{g}^{pq}\left(H_{pq}^{m}-\delta_{q}^{m}H_{pn}^{n}\right)\right]d^{4}x+$$
$$+\gamma\int\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\left[H_{pr}^{n}H_{qn}^{r}-H_{pq}^{r}H_{nr}^{n}-\left(H_{pq}^{m}-\delta_{q}^{m}H_{pn}^{n}\right)E_{lm}^{l}-\right. \qquad (4.1.1.10)$$
$$\left.-\frac{1}{2\eta}\underset{e}{\psi}^{r}H_{pn}^{n}\underset{e}{F}_{qr}+H_{pn}^{r}E_{nr}^{n}-\frac{1}{8\eta^{2}}\underset{e}{\psi}^{r}\underset{e}{\psi}^{l}\underset{e}{F}_{pr}\underset{e}{F}_{ql}\right]d^{4}\underset{e}{x}$$

This equity determines the explicit form of the GEH field action function. It is very complex, and therefore, the system of differential equations with regard to $\underset{e}{g}_{ij}$ and $\underset{e}{\varphi}_{i}$ is also complex. In this connection, in most of cases to study the GEH field it is expedient to use directly the action function rather than the system of equations. Below the central symmetry GEH field will be investigated immediately when using the action function (2.1.1.10).

In $SR(x^{k},\underset{k}{\vec{E}})$ the GEH field action function is simplified and has the following form:

$$S=\gamma\int\frac{\partial}{\partial x^{r}}\left[\sqrt{-g}\,g^{pq}\left(H_{pq}^{r}-\delta_{q}^{r}H_{pn}^{n}\right)\right]d^{4}x+$$
$$+\gamma\int\sqrt{-g}\,g^{pq}\left(H_{pr}^{n}H_{qn}^{r}-H_{pq}^{r}H_{nr}^{n}-\frac{1}{2\eta}\psi^{r}H_{pn}^{n}F_{qr}-\right. \qquad (4.1.1.11)$$
$$\left.-\frac{1}{8\eta^{2}}\psi^{r}\psi^{l}F_{pr}F_{ql}\right)d^{4}x$$

Here integration is performed according to Riemann.

4.1.2. SYSTEM OF EQUATIONS RELETIVE TO $\underset{e}{g}_{ij}$ AND $\underset{e}{\varphi}_{i}$

Equations relative to $\underset{e}{g}_{ij}$ and $\underset{e}{\varphi}_{i}$ are obtained from the extremal principle

$$\delta S=0.$$

The first integral variation in the right-hand side of (4.1.1.11) is equal to zero and to define the variation of the subintegral expression of the second addend, let's calculate at first the following values

$$\delta\sqrt{-\underset{e}{g}}=\frac{1}{2\sqrt{-\underset{e}{g}}}\delta\left(-\underset{e}{g}\right)=\frac{1}{2}\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\,\delta\underset{e}{g}_{pq},$$
$$\underset{e}{g}^{kp}\,\delta\underset{e}{g}_{pi}+\underset{e}{g}_{pi}\,\delta\underset{e}{g}^{kp}=0 \qquad (4.1.2.1)$$

Hence

$$\delta\underset{e}{g}^{ik}=-\underset{e}{g}^{ip}\,\underset{e}{g}^{kq}\,\delta\underset{e}{g}_{pq} \qquad (4.1.2.2)$$

Besides,

$$\delta H^{k}_{ij} = -g^{ks} H^{t}_{ij}\delta g_{st} + N^{kstl}_{ij}\frac{\partial}{\partial \underset{e}{x}^{l}}(\delta g_{st}) + M^{kst}_{ij}\frac{\partial}{\partial \underset{e}{x}^{t}}(\delta\varphi_s), \quad (4.1.2.3)$$

where

$$N^{kstl}_{ij} = \frac{1}{2}\left(\underset{e}{g}^{ks}\delta^{t}_{i}\delta^{l}_{j} + \underset{e}{g}^{ks}\delta^{t}_{j}\delta^{l}_{i} - \underset{e}{g}^{kl}\delta^{s}_{i}\delta^{t}_{j}\right), \quad (4.1.2.4)$$

$$M^{kst}_{ij} = \frac{1}{4\eta}\left[\left(\delta^{s}_{i}\delta^{t}_{j} - \delta^{s}_{j}\delta^{t}_{i}\right)\underset{e}{\psi}^{k} - \underset{e}{g}^{ks}\left(\underset{e}{\psi}_{i}\delta^{t}_{j} + \underset{e}{\psi}_{j}\delta^{t}_{i}\right) + \right.$$
$$\left. + \underset{e}{g}^{kt}\left(\underset{e}{\psi}_{i}\delta^{s}_{j} + \underset{e}{\psi}_{j}\delta^{s}_{i}\right)\right]$$

Let's substitute these values into the right-hand side of the equity:

$$0 = \delta S = \gamma\delta\int\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{pq}\left[H^{n}_{pr}H^{r}_{qn} - H^{r}_{pq}H^{n}_{nr} - \left(H^{m}_{pq} - \right.\right.$$
$$\left. - \delta^{m}_{q}H^{n}_{pn}\right)E^{r}_{rm} - \frac{1}{2\eta}\underset{e}{\psi}^{r}H^{n}_{pn}\underset{e}{F}_{qr} + H^{r}_{pn}E^{n}_{qr} - \quad (4.1.2.5)$$
$$\left. - \frac{1}{8\eta^{2}}\underset{e}{\psi}^{r}\underset{e}{\psi}^{l}F_{pr}F_{ql}\right]d^{4}\underset{e}{x},$$

and allowing for the additional condition (1.2.2.5) and equating the coefficients before $\delta\underset{e}{g}_{ij}$ and $\delta\underset{e}{\varphi}_{i}$, we obtain:

$$\frac{\partial}{\partial \underset{e}{x}^{p}}\left(H^{p}_{ij} + H^{p}_{ji} - \delta^{p}_{j}H^{q}_{qi} - \delta^{p}_{i}H^{q}_{qj}\right) + H^{p}_{ij}H^{q}_{qp} + H^{p}_{ji}H^{q}_{qp} -$$
$$- H^{q}_{ip}H^{p}_{qj} - H^{q}_{jp}H^{p}_{qi} + H^{p}_{iq}E^{q}_{jp} + H^{p}_{jq}E^{q}_{ip} = h_{ijpq}\rho^{pq}, \quad (4.1.2.6)$$

$$\frac{\partial}{\partial x^{m}}\left[-\frac{1}{4\eta}\underset{e}{g}^{pi}\underset{e}{g}^{qm}\underset{e}{\psi}_{n}\underset{e}{\psi}^{n}\underset{e}{F}_{pq} + \left(\underset{e}{g}^{pi}\underset{e}{\psi}^{m} - \underset{e}{g}^{pm}\underset{e}{\psi}^{i}\right)E^{n}_{np} + \right.$$
$$\left. + \frac{1}{2}\underset{e}{g}^{pi}\underset{e}{g}^{qm}\underset{e}{\psi}_{n}E^{n}_{pq} + \frac{1}{2}\left(\underset{e}{g}^{pi}E^{m}_{pq} - \underset{e}{g}^{pm}E^{i}_{pq}\right)\underset{e}{\psi}^{q}\right] =$$
$$= \left(H^{l}_{lm} + E^{\ell}_{\ell m}\right)\left[\frac{1}{4\eta}\underset{e}{g}^{pi}\underset{e}{g}^{qm}\underset{e}{\psi}_{n}\underset{e}{\psi}^{n}\underset{e}{F}_{pq} + \left(\underset{e}{g}^{pi}\underset{e}{\psi}^{m} - \right.\right. \quad (4.1.2.7)$$
$$\left. - \underset{e}{g}^{pm}\underset{e}{\psi}^{i}\right)E^{n}_{np} + \frac{1}{2}\underset{e}{g}^{pi}\underset{e}{g}^{qm}\underset{e}{\psi}_{n}E^{n}_{pq} +$$
$$\left. + \frac{1}{2}\left(\underset{e}{g}^{pi}E^{m}_{pq} - \underset{e}{g}^{pm}E^{i}_{pq}\right)\underset{e}{\psi}^{q}\right] + \lambda\psi^{i},$$

$$\frac{\partial L^{*}}{\partial\psi^{k}} = \lambda\varphi_{k},$$

where λ is the Lagrangian multiplier,

$$L^{*} = \gamma\sqrt{-g}\,\underset{e}{g}^{pq}\left[H^{n}_{pr}H^{r}_{qn} - H^{r}_{pq}H^{n}_{nr} - \left(H^{m}_{pq} - \delta^{m}_{q}H^{n}_{pn}\right)E^{l}_{lm} - \right.$$
$$\left. - \frac{1}{2\eta}\underset{e}{\psi}^{r}H^{n}_{pn}F_{qr} + H^{r}_{pn}E^{n}_{nr} - \frac{1}{8\eta^{2}}\psi^{r}\psi^{l}\underset{e}{F}_{pr}\underset{e}{F}_{ql}\right],$$

$$h_{ijpq} = \frac{1}{4}\left(\underset{e}{g}_{ip}\underset{e}{g}_{jq} + \underset{e}{g}_{iq}\underset{e}{g}_{jp} - \underset{e}{g}_{ij}\underset{e}{g}_{pq}\right),$$

$$h^{ijpq} = \frac{1}{4}\left(\underset{e}{g}^{ip}\underset{e}{g}^{jq} + \underset{e}{g}^{iq}\underset{e}{g}^{jp} - \underset{e}{g}^{ij}\underset{e}{g}^{pq}\right),$$

$$
\begin{aligned}
\rho^{ij} = & \frac{\partial}{\partial \underset{e}{x}^{r}}\left[\frac{1}{2}E^{n}_{np}\left(h^{ijpr}-4\underset{e}{g}^{pr}\underset{e}{g}^{ij}\right)-4\underset{e}{g}^{pr}\underset{e}{g}^{qi}E^{j}_{pq}\right]+\\
&+\frac{1}{2}h^{ijpq}\left(E^{m}_{mq}H^{n}_{np}+E^{m}_{mq}H^{n}_{pn}-E^{n}_{qr}H^{r}_{pn}\right)+\\
&+2\underset{e}{g}^{pq}\underset{e}{g}^{ri}\left(E^{n}_{nr}H^{j}_{pq}+E^{n}_{nq}H^{j}_{pr}-E^{n}_{qr}H^{j}_{pn}\right)+\\
&+\frac{1}{2}\left(H^{m}_{mr}+E^{m}_{mr}\right)\left[\left(h^{ijpr}-4\underset{e}{g}^{pr}\underset{e}{g}^{ij}\right)E^{n}_{np}-8g^{pr}g^{qi}E^{j}_{pq}\right]- \qquad (4.1.2.8)\\
&-\frac{\partial}{\partial \underset{e}{x}^{r}}\left[\frac{1}{\eta}\left(\underset{e}{g}^{pr}\underset{e}{\psi}^{i}\underset{e}{F}{}^{j}_{p}-\underset{e}{g}^{pi}\underset{e}{\psi}^{j}\underset{e}{F}{}^{r}_{p}+\underset{e}{g}^{ij}\underset{e}{\psi}^{p}\underset{e}{F}{}^{r}_{p}\right)\right]-\\
&-\frac{1}{16\eta^{2}}h^{ijpq}\underset{e}{\psi}^{r}\underset{e}{\psi}^{l}\underset{e}{F}{}_{pr}\underset{e}{F}{}_{q\ell}+\frac{2}{\eta}g^{pq}g^{ri}\left(H^{j}_{pr}\underset{e}{\psi}^{n}\underset{e}{F}{}_{qn}-\right.\\
&\left.-H^{n}_{nq}\underset{e}{\psi}^{j}\underset{e}{F}{}_{pr}-H^{j}_{pn}\underset{e}{\psi}^{n}\underset{e}{F}{}_{qr}\right)-\frac{1}{\eta}\left(H^{m}_{mr}+E^{m}_{mr}\right)\times\\
&\times\left(\underset{e}{g}^{pr}\underset{e}{\psi}^{i}\underset{e}{F}{}^{j}_{p}-\underset{e}{g}^{ij}\underset{e}{\psi}^{p}\underset{e}{F}{}^{r}_{p}+\underset{e}{g}^{pi}\underset{e}{\psi}^{j}\underset{e}{F}{}^{r}_{p}\right)
\end{aligned}
$$

This system of equations for $\underset{e}{g}_{ij}$ and $\underset{e}{\varphi}_{i}$ is covariant relative to nonholonomic transformations belonging to the EH group, i.e. it retains its form in all $SR(x^{k},\underset{k}{\vec{e}})$, which are interrelated by nonholonomic transformations of the given type.

With allowance that the systems (4.1.2.6) and (4.1.2.7) contain the generalized derivatives and today the theory of differential equations with generalized derivatives does not exist, it is more practical to write these systems in $SR(x^{k},\underset{k}{\vec{E}})$. In this case $\underset{i}{e}^{k}=\alpha^{k}_{i}=\delta^{k}_{i}$, $E^{k}_{ij}=0$ and from (4.1.2.6) – (4.1.2.8) we obtain:

$$
\begin{aligned}
&\frac{\partial}{\partial x^{p}}\left(H^{p}_{ij}+H^{p}_{ji}-\delta^{p}_{i}H^{q}_{qj}-\delta^{p}_{j}H^{q}_{qi}\right)+H^{p}_{ij}H^{q}_{qp}+H^{p}_{ji}H^{q}_{qp}-\\
&-H^{q}_{ip}H^{p}_{qj}-H^{q}_{jp}H^{p}_{qi}=-\frac{1}{\eta}h_{ijst}\left[\frac{\partial}{\partial x^{r}}\left(g^{pr}\psi^{s}F^{t}_{p}-g^{ps}\psi^{t}F^{r}_{p}+\right.\right.\\
&\left.+g^{st}\psi^{p}F^{r}_{p}\right)+H^{m}_{mr}\left(g^{pr}\psi^{s}F^{t}_{p}-g^{ps}\psi^{t}F^{r}_{p}+g^{st}\psi^{p}F^{r}_{p}\right)- \qquad (4.1.2.9)\\
&-2g^{pq}g^{rs}\left(H^{t}_{pr}\psi^{n}F_{qn}-H^{n}_{nq}\psi^{t}F_{pr}+H^{t}_{pn}\psi^{n}F_{qr}\right)+\frac{1}{16\eta}\times\\
&\left.\times h^{stpq}\psi^{r}\psi^{n}F_{pr}F_{qn}\right],
\end{aligned}
$$

$$
\frac{\partial}{\partial x^{m}}\left(\sqrt{-g}\,g^{pi}g^{qm}\psi_{n}\psi^{n}F_{pq}\right)=-4\eta\sqrt{-g}\,\lambda\psi^{i},
$$

$$
\frac{\partial L^{*}}{\partial \psi^{k}}=\lambda\varphi_{k}
$$

These equations in totality with (1.2.2.5) constitute a full system of differential equations relative to g_{ij}, φ_{i}, ψ^{k} and λ. The parameters φ_{i} in (4.1.2.9) contain F_{ij}, therefore they are determined with an accuracy of the addend $\frac{\partial u}{\partial x^{i}}$, where u is the arbitrary function. However, since φ_{i} is represented by (1.2.2.5), it is clear that u must satisfy the following first-order equation:

$$\psi^i \frac{\partial u}{\partial x^i} = 0 \qquad (4.1.2.10)$$

When solving the Cauchi problem relative to parameters g_{ij} and φ_i, for a certain three-dimensional variety

$$x^i = t^i(v^1, v^2, v^3) \qquad (4.1.2.11)$$

the function values g_{ij} and φ_i are specified. This means that $\frac{\partial u}{\partial x^i} = 0$ for (4.1.2.11), i.e.

$$\frac{\partial u}{\partial v^\alpha} = \frac{\partial u}{\partial x^i}\frac{\partial x^i}{\partial v^\alpha} = 0 \quad \text{at} \quad x^i = t^i(v^1, v^2, v^3),$$

or

$$u = const \quad \text{at} \quad x^i = t^i(v^1, v^2, v^3) \qquad (4.1.2.12)$$

(4.1.2.10) and (4.1.2.12) constitute the Cauchi problem for the linear uniform first-order differential equation with respect to the function u. As is known (see chapter II of the present work), it has the solution $u = const$ and this is the only possible solution.

Thus, the system of equations (4.1.2.9) and (1.2.2.5) and the Cauchi problem conditions for three-dimensional variety (4.1.2.11) with respect to g_{ij} and φ_i determine unambiguously (with an accuracy of a constant addend) the φ_i.

4.2. CHARACTERISTICS OF THE MATERIAL WORLD

4.2.1. ENERGY-MOMENTUM TENSOR OF THE MATTER

As has been mentioned in the Introduction of the present work, the unified GEH field, if its existence is possible, should form the basis of all physical phenomena known for us. In particular, using the concepts characterizing the GEH field, the essence of the material world should be explained. In other words, such classical concepts as mass, energy, momentum, electric field density, etc. by means of which in physics the material world is characterized should be expressed by the GEH field parameters. Today the establishment of this relation between the classical and the GEH field parameters is not limited by any fundamental requirements ensuring its unambiguity. The only requirement, according to which the asymptotic behavior of the introduced parameters must be of classical character, cannot provide unambiguity of relation and hence the solution of the problem under consideration has, to some extent, an intuitive character. The validity of the accepted solutions can be checked only by comparison of the obtained theoretical results with the relevant experimental data.

Basic parameters characterizing the state the of material world, which can be determined experimentally, are energy-momentum tensor T_i^k and current density vector J^k. The objective of the present paragraph is to define these parameters through the parameters of GEH field.

First let us discuss the energy-momentum tensor of the matter.

To define the dependence between T_i^k and g_{ik} and φ_i we shall use the known Einstein's gravitational field equation [3,4,5]:

$$G_i^k = -\chi T_i^k, \qquad (4.2.1.1)$$

where

$$G_i^k = R_i^k - \frac{1}{2}\delta_i^k R \; - \tag{4.2.1.2}$$

is Einstein's tensor, χ is a dimension coefficient $\left(\chi = \frac{8\pi G}{c^4}\right)$, and R_{ik} and R - Richci's tensor and scalar curvature. However, use of these equations exactly as they are is impossible since the constant χ contains such classic notion as the mass, the definition of which we are trying to make using the parameters g_{ik} and φ_i. To avoid tautology it is necessary to use only the law of proportional dependency between G_i^k and T_i^k from the equation system (4.2.1.1); as for the coefficient of proportionality it shall be formed only using GEH field parameters. It is natural since we do not have any other parameters.

It can be readily demonstrated, that the dimensionality of parameter χ^{-1} coincides with the dimensionality of quadrant of length vector φ_i i.e.

$$\left[\chi^{-1}\right] = \left[\varphi^p \varphi_p\right]$$

Basing only on this circumstance the proportionality coefficient between G_i^k and T_i^k is defined by following expression: $-\frac{\alpha}{2\pi} g^{pq}\varphi_p\varphi_q$, and for the tensors $\underset{e}{G}{}_i^k$ and $\underset{e}{T}{}_i^k$ - as generalization of the previous, is expressed by -

$$-\frac{\alpha}{2\pi} \underset{e}{g}{}^{pq} \underset{e}{\varphi}{}_p \underset{e}{\varphi}{}_q , \tag{4.2.1.3}$$

where α - is dimensionless constant, the value of which will be defined later while comparing the theoretical results and experimental data.

Thus, the energy-momentum tensor of the matter is defined by (equity):

$$\underset{e}{T}{}_i^k = -\frac{\alpha}{2\pi} \underset{e}{g}{}^{pq} \underset{e}{\varphi}{}_p \underset{e}{\varphi}{}_q \underset{e}{G}{}_i^k \tag{4.2.1.4}$$

In Einstein's relativistic gravitational field theory the equation (4.2.1.1) are used to define parameters g_{ij} assuming that the energy-momentum tensor of the matter T_i^k is the prescribed value. In comparison to this in the case under consideration the equations (4.2.1.4) are used to define the parameters of $\underset{e}{T}{}_i^k$ according to the prescribed values of $\underset{e}{g}{}_{ij}$ and $\underset{e}{\varphi}{}_i$ values which are solutions of the system (4.1.2.6), (4.1.2.7).

From the structure of the right part of (4.2.1.4) it is evident that $\frac{1}{\alpha}\underset{e}{T}{}_i^k$ is a mixed tensor of second order relative to the non-holonomic transformations belonging to the EH group.

Multiplying the identity (1.2.2.32) by the value (4.2.1.3) yields:

$$\frac{\partial \underset{e}{T}{}^{ki}}{\partial \underset{e}{x}{}^k} + H_{pk}^k \underset{e}{T}{}^{pi} + H_{pk}^i \underset{e}{T}{}^{kp} - \frac{\alpha}{2\pi} \underset{e}{G}{}^{ki} \frac{\partial}{\partial \underset{e}{x}{}^k}\left(\underset{e}{g}{}^{pq} \underset{e}{\varphi}{}_p \underset{e}{\varphi}{}_q\right) = 0$$

Hence, with allowance that

$$H_{pk}^k = \frac{1}{\sqrt{-\underset{e}{g}}} \frac{\partial \sqrt{-\underset{e}{g}}}{\partial \underset{e}{x}{}^p} + \frac{1}{2\eta} \underset{e}{\psi}{}^k \underset{e}{F}{}_{pk}$$

the following equation is received:

$$\frac{1}{\sqrt{-\underset{e}{g}}} \frac{\partial}{\partial \underset{e}{x}{}^k}\left(\sqrt{-\underset{e}{g}}\, \underset{e}{T}{}^{ki}\right) = -\underset{e}{F}{}^i , \tag{4.2.1.5}$$

where

$$-\underset{e}{F}^{i}=\left[\delta_{p}^{i}\frac{\partial}{\partial \underset{e}{x}^{k}}\left(\ln\left|\underset{e}{\varphi}_{n}\underset{e}{\varphi}^{n}\right|\right)-H_{pk}^{i}-\frac{1}{2\eta}\delta_{p}^{i}\underset{e}{\psi}^{q}\underset{e}{F}_{kq}\right]\underset{e}{T}^{kp}, \quad (4.2.1.6)$$

is a four-dimensional force density which characterizes the effect of GEH field on the matter [5]. From (4.2.1.6) it is evident that F^i does not represent the 4-vector. Irrespective of this the equations (4.2.1.5) are covariant relative to non-holonomic transformations belonging to EH group. This is evident since (4.2.1.5) is identical to (1.2.2.32) and this latter equity is covariant.

In $SR(x^k, \underset{k}{\vec{E}})$ we have

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}T^{ki}\right)=-F^i, \quad (4.2.1.7)$$

$$-F^i=\left[\delta_p^i\frac{\partial}{\partial x^k}\left(\ln\left|\varphi_n\varphi^n\right|\right)-H_{pk}^i-\frac{1}{2\eta}\delta_p^i\psi^q F_{kq}\right]T^{kp} \quad (4.2.1.8)$$

In modern literature of relativistic theory of gravitational field T^{00} is identified with the energy density, $T^{0\alpha}$ - with momentum density and $T^{\alpha\beta}$ - with the stress tensor of the matter. It should be mentioned that in a general case in the random reference system $SR(x^k, \underset{k}{\vec{E}})$ such differentiation of the energy-momentum tensor components by physical characteristics is not valid. It is only valid for $SR(x^k, \underset{k}{\vec{E}})$ with the metric divided into spatial and temporal parts (ref. Chapter II).

Similarly to this the components of tensor $\underset{e}{T}^{ik}$ defined by (4.2.1.4) in the random reference system $SR(x^k, \underset{k}{\vec{e}})$ cannot be differentiated in accord to the above rule; they are absolutely equipotent values. Using these components it is possible to define the energy and matter impulse density and stress tensor. Indeed, if $SR(x^k, \underset{k}{\vec{e}}')$ is the reference system with a divided metric, then energy $\underset{e}{T}'^{00}$ and impulse $\underset{e}{T}'^{0\alpha}$ densities, energy flow $\underset{e}{T}'^{\alpha 0}$ and stress tensor $\underset{e}{T}'^{\alpha\beta}$ are defined using evident equities:

$$\underset{e}{T}'^{00}=a_p^0 a_q^0\underset{e}{T}^{pq}, \quad \underset{e}{T}'^{0\alpha}=a_p^0 a_q^\alpha\underset{e}{T}^{pq},$$
$$\underset{e}{T}'^{\alpha 0}=a_p^\alpha a_q^0\underset{e}{T}^{pq}, \quad \underset{e}{T}'^{\alpha\beta}=a_p^\alpha a_q^\beta\underset{e}{T}^{pq}, \quad (4.2.1.9)$$

where a_i^k are transformation coefficients providing links between the systems $SR(x^k, \underset{k}{\vec{e}})$ and $SR(x^k, \underset{k}{\vec{e}}')$. In case when $SR(x^k, \underset{k}{\vec{E}})$ is a random reference system, and $SR(x^k, \underset{k}{\vec{E}}')$ is a reference system with divided metric, then

$$T'^{00}=\frac{\partial x'^0}{\partial x^p}\frac{\partial x'^0}{\partial x^q}T^{pq}, \quad T'^{0\alpha}=\frac{\partial x'^0}{\partial x^p}\frac{\partial x'^\alpha}{\partial x^q}T^{pq},$$
$$T'^{\alpha 0}=\frac{\partial x'^\alpha}{\partial x^p}\frac{\partial x'^0}{\partial x^q}T^{pq}, \quad T'^{\alpha\beta}=\frac{\partial x'^\alpha}{\partial x^p}\frac{\partial x'^\beta}{\partial x^q}T^{pq} \quad (4.2.1.10)$$

In $SR(x^k, \underset{k}{\vec{E}}')$ $g'^{0\alpha}=0$. Method of metric division, i.e. ensuring equities $g'^{0\alpha}=0$, is described in Chapter II of the present work as well as in [8].

4.2.2. ELECTRIC CURRENT DENSITY

In full analogy to the above for determination of electric current density, being one of main characteristics of material world. Let us use the equation of electromagnetic field in the curved space [5]

$$J^i = \frac{c}{4\pi}\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^m}\left(\sqrt{-g}\,g^{ip}g^{mq}F_{pq}\right), \qquad (4.2.2.1)$$

or

$$J^i = \frac{c}{4\pi}\left(g^{ip}g^{kq}F_{pq}\right)_{,k} \qquad (4.2.2.2)$$

Generalization of these identical equities in the considered case is either

$$\underset{e}{J}^i = \frac{c}{4\pi}\frac{1}{\sqrt{-\underset{e}{g}}}\frac{\partial}{\partial \underset{e}{x}^k}\left(\sqrt{-\underset{e}{g}}\,\underset{e}{g}^{ip}\,\underset{e}{g}^{kq}\,\underset{e}{F}_{pq}\right), \qquad (4.2.2.3)$$

or

$$\underset{e}{J}^i = \frac{c}{4\pi}\left(\underset{e}{g}^{ip}\,\underset{e}{g}^{kq}\,\underset{e}{F}_{pq}\right)_{,k} \qquad (4.2.2.4)$$

Below we shall demonstrate that tensor nature of both generalizations is identical however the electric current density defined by the equity (4.2.2.4) does not satisfy the charge conservation law even in $SR(x^k,\underset{k}{\vec{E}})$. For this reason the current density is subsequently defined according to the equity (4.2.2.3).

In the right part of this equity $\underset{e}{g}_{ik}$ and $\underset{e}{\varphi}_i$ are the solutions of the GEH field equations system (4.1.2.6) and (4.1.2.7). In the classic theory of electromagnetic field [5] the equations (4.2.2.1) were used to define the potential of the electromagnetic field φ_i in the curved space with the prescribed metric tensor g_{ik} for the known value of current density J^i. In contrast to this in the considered case the metric tensor $\underset{e}{g}_{ik}$ and potential $\underset{e}{\varphi}_i$ are the solutions of the system equations (4.1.2.6) and (4.1.2.7), which being substituted in (4.2.2.3) define the electric current density.

In classic theory of electromagnetic field [5] current density defined by (4.2.2.1) complies with the charge conservation law:

$$J^i_{,i} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(\sqrt{-g}\,J^i\right) = 0 \qquad (4.2.2.5)$$

This equity is the consequence of the identity:

$$\frac{\partial}{\partial x^i}\left(\sqrt{-g}\,J^i\right) = \frac{c}{4\pi}\frac{\partial^2}{\partial x^i \partial x^k}\left(\sqrt{-g}\,g^{ip}g^{kq}F_{pq}\right) \qquad (4.2.2.6)$$

and the anti-symmetry of F_{ik} tensor. In $SR(x^k,\underset{k}{\vec{e}})$ the charge conservation law is not valid. Besides, from the structure of the right part of equity (4.2.2.2) follows that J^i is contravariant vector.

To determine the vector character $\underset{e}{J}^i$ defined by (4.2.2.3) the following identity is used:

$$\frac{1}{\eta}\left(\underset{e}{g}^{ip}\underset{e}{g}^{kq}\underset{e}{F}_{pq}\right)_{,k}=\frac{4\pi}{\eta c}\underset{e}{J}^{i}+\frac{1}{2\eta^{2}}\underset{e}{\psi}^{k}\underset{e}{g}^{ip}\underset{e}{g}^{rq}\underset{e}{F}_{rk}\underset{e}{F}_{pq}+ \\ +\frac{1}{4\eta^{2}}\underset{e}{\psi}^{i}\underset{e}{g}^{rp}\underset{e}{g}^{kq}\underset{e}{F}_{rk}\underset{e}{F}_{pq} \qquad (4.2.2.7)$$

In accordance with the charge conservation law $\underset{e}{F}_{pq}$ (see (1.2.2.9)) the left part of this identity and the last two summands in the right part are contravariant vectors, hence $\frac{1}{\eta}\underset{e}{J}^{i}$ is a contravariant vector.

Similar to the energy-momentum tensor in general case, in the random $SR(x^k,\underset{k}{\vec{E}})$, one cannot claim that $\frac{1}{c}J^0$ is the charge density and J^α - the current density. Such a differentiation of the vactor components in accord to physical properties is valid only in $SR(x'^k,\underset{k}{\vec{E}})$ with divided into space and time parts of metric tensor, i.e. if $SR(x^k,\underset{k}{\vec{E}})$ is such, that $g_{0\alpha}=0$ and $g_{00}>0$, i.e. $\frac{1}{c}J'^0$ is a charge density and J'^α - is current density. These parameters can be expressed through the equipotent parameters J^i:

$$J'^0=\frac{\partial x'^0}{\partial x^p}J^p, \qquad J'^\alpha=\frac{\partial x'^\alpha}{\partial x^p}J^p \qquad (4.2.2.8)$$

The method of defining of $SR(x'^k,\underset{k}{\vec{E}})$ with divided metric, i.e. the method of defining the functional dependence $x'^k = =(x^0,x^1,x^2,x^3)$ is given in Chapter II.

4.2.3. MATERIAL BODY

In this section we shall attempt to define a material body basing on the basic notion of GEH field.

In [5] Einstein wrote: "Furthermore, there is an assertion that it is impossible to simultaneously maintain the field and particle concepts as elements of physical description. The concept of field requires absence of singularities while particles concept (being elementary concept) requires the singularities in the field. However, the field concept seems to be inevitable since otherwise it is impossible to formulate the general relativistic theory. The General Relativistic theory is the only means to avoid such unreal "thing as inertial system". Apart from this in [6] Einstein and Infield are developing the following idea: "We cannot build physics on the basis of the only notion – matter. But division into the matter and field on the background of recognition of mass and energy equivalency is somewhat artificial and vaguely defined. Could we rather abandon the notion of matter and build a pure field physics? What effects our senses in the form of matter actually is but a tremendous energy concentration in relatively small space. We could have considered the matter as the domains in space where the field is extremely strong. This would have allowed to develop the basics of a new philosophy. Its ultimate goal would have been to explain all events occurring in nature by structural laws, valid under all circumstances and everywhere. From this viewpoint a thrown stone is a changing field where the states of the highest field intensity move in the space with the speed developed by the stone".

The conservation law (4.2.1.5) obtained in 4.2.1 is similar to the identity (1.2.2.32), hence below we shall use (1.2.2.32) to solve the problem set in this section. With allowance of the results obtained in this very section we shall assume that in $SR(x^k, \underset{k}{\vec{E}})$ the metric tensor is divided into spatial and temporal parts to separate the energetic part from energy-momentum tensor. In such $SR(x^k, \underset{k}{\vec{E}})$ T^{00} - is matter energy density, $T^{0\alpha}$ – is the momentum density component, $T^{\alpha 0}$ – the energy flow component and $T^{\alpha\beta}$ – is a stress component.

In three-dimensional space let's isolate a certain volume V $x^0 = const$ and integrating the identity (1.2.2.32) in this volume, we shall get:

$$\int_S \sqrt{g_{00}}\, \underset{e}{G}^{\alpha i} d\sigma_\alpha + \frac{d}{dx^0}\int_v \underset{e}{G}^{0i} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x = = -\int_V H^{*i}_{\ pq}\, \underset{e}{G}^{qp} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x, \qquad (4.2.3.1)$$

where S – is a surface, limiting V, $d\sigma_\alpha$ - is a surface element,

$$H^{*i}_{pq} = H^{i}_{pq} + \frac{1}{2}\delta^i_p \psi^l F_{ql}, \quad \gamma = \det(g_{\alpha\beta}).$$

Multiplying the same identity by x^j and integrating it in V we get:

$$\int_S x^j \sqrt{g_{00}}\, \underset{e}{G}^{\alpha i} d\sigma_\alpha + \frac{d}{dx^0}\int_v x^j\, \underset{e}{G}^{0i} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x = = \int_V \underset{e}{G}^{ji} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - \int_V x^j H^{*i}_{pq}\, \underset{e}{G}^{qp} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x \qquad (4.2.3.2)$$

The average value of x^j coordinate is defined from the equity:

$$\bar{x}^j \int_V \underset{e}{G}^{00} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x = \int_V x^j\, \underset{e}{G}^{00} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x \qquad (4.2.3.3)$$

and let's introduce the following definition:

$$a^i = \int_V \sqrt{g_{00}}\, \underset{e}{G}^{0i} \sqrt{-\gamma}\, d^3x \qquad (4.2.3.4)$$

provided $\bar{x}^0 = x^0$. Using these definitions the (4.2.3.1) will receive the following form:

$$\frac{da^i}{dx^0} = \int_V H^{*i}_{pq}\, \underset{e}{G}^{qp} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - \int_S \ldots \qquad (4.2.3.5)$$

and (4.2.3.2) will get:

$$\frac{d\bar{x}^j a^i}{dx^0} = \int_V \underset{e}{G}^{ji} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - \int_V x^j H^{*i}_{pq}\, \underset{e}{G}^{qp} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - -\frac{d}{dx^0}\int_V (x^j - \bar{x}^j)\, \underset{e}{G}^{0i} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - \int_S \ldots \qquad (4.2.3.6)$$

With allowance of (4.2.3.5) the latter equity can be rewritten as:

$$\frac{a_i}{u_0}\frac{d\bar{x}^j}{ds} = \int_V \underset{e}{G}^{ji} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - O(x^j - \bar{x}^j) - \int_S \ldots, \qquad (4.2.3.7)$$

where $u^0 = \dfrac{dx^0}{ds}$,

$$O(x^j - \bar{x}^j) = \int_V (x^j - \bar{x}^j) H^{*i}_{pq}\, \underset{e}{G}^{qp} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x - -\frac{d}{dx^0}\int_V (x^j - \bar{x}^j)\, \underset{e}{G}^{0i} \sqrt{g_{00}}\sqrt{-\gamma}\, d^3x. \qquad (4.2.3.8)$$

The value $O(x^j - \bar{x}^j)$ tends to zero when $x^\alpha \to \bar{x}^\alpha$ and is equal to zero when $j = 0$.

From (4.2.3.3) it is evident, that $\bar{x}^j$ depend on x^0, if only the parameters of GEH field g_{ik} and φ_i depend on variable x^0, i.e. in four-dimensional spatial-and-temporal variety $\bar{x}^j(s)$ where the length of the arc s forms a certain trajectory. In case when g_{ik} and φ_i do not depend on x^0, the parameters $\bar{x}^i$ are constants, i.e. movement of the material body does not occur.

Let us consider the case when v is so small domain that the symbols $\overset{*}{H}{}^i_{pq}$ can be expanded in a Taylor series in the vicinity of point $\bar{x}^j$ and one can with sufficient degree of accuracy be confined to the linear terms relative to $x^j - \bar{x}^j$ i.e.

$$\overset{*}{H}{}^i_{pq} = \underset{0}{\overset{*}{H}}{}^i_{pq} + \underset{0}{\overset{*}{H}}{}^i_{pqr}(x^r - \bar{x}^r), \qquad (4.2.3.9)$$

where

$$\underset{0}{\overset{*}{H}}{}^i_{pq} = \overset{*}{H}{}^i_{pq}, \quad \underset{0}{\overset{*}{H}}{}^i_{pqr} = \frac{\partial \overset{*}{H}{}^i_{pq}}{\partial x^r} \text{ at } x^0 = \bar{x}^0,\ x^1 = \bar{x}^1,\ x^2 = \bar{x}^2, x^3 = \bar{x}^3$$

After substitution from (4.2.3.5) we get:

$$\frac{1}{u^0}\frac{da^i}{ds} + \overset{*}{H}{}^i_{pq}\int_V \underset{e}{G}{}^{qp}\sqrt{g_{00}}\sqrt{-\gamma}\,d^3x = O(x^j - \bar{x}^j) + \int_S \ldots \qquad (4.2.3.10)$$

With allowance of (4.2.3.7) this equity will acquire the following form:

$$\frac{da^i}{ds} + \underset{0}{\overset{*}{H}}{}^i_{pq} a^p \frac{d\bar{x}^q}{ds} = O(x^j - \bar{x}^j) + \int_S \ldots \qquad (4.2.3.11)$$

Assuming that the considered GEH field meets the following conditions:

1. Outside of V the GEH field falls very rapidly so that $\left|\int_S \ldots\right|$ is much more significantly lower compared to the volume integrals;
2. V is so small domain that $O(x^j - \bar{x}^j)$ is an arbitrary small value;
3. $\left|\int_V \underset{e}{G}{}^{00}\sqrt{g_{00}}\sqrt{-\gamma}\,d^3x\right|$ is much higher comparing volume integrals with other components of tensor $\underset{e}{G}{}^{ik}$ and compared to the changing of this very value by a unit of the trajectory length.

Such GEH field confined within the domain V will be indicated as a material body.

Basing on the first and second terms the equity (4.2.3.11) with a high degree of accuracy will have the following form:

$$\frac{da^i}{ds} + \overset{*}{H}{}^i_{pq} a^p \frac{dx^q}{ds} = 0 \qquad (4.2.3.12)$$

Considering that $\overset{*}{H}{}^i_{pq} = H^i_{pq} + \frac{1}{2}\psi^l F_{ql}\delta^i_p$, we get:

$$\frac{da^i}{ds} + H^i_{pq} a^p \frac{dx^q}{ds} = -\frac{1}{2}\psi^l F_{ql}\frac{dx^q}{ds} a^i \qquad (4.2.3.13)$$

When fulfilling the conditions of 1 and 2 from (4.2.3.7) with $i = 0$ we get:

$$\int_V \underset{e}{G}{}^{i0}\sqrt{g_{00}}\sqrt{-\gamma}\,d^3x = \frac{a^0}{u^0}\frac{dx^i}{ds},$$

which in totality with (4.3.2.4) yields

$$a^i = \frac{a^0}{u^0}\frac{dx^i}{ds} + \int_V \sqrt{g_{00}}\left(\underset{e}{G}{}^{0i} - \underset{e}{G}{}^{i0}\right)\sqrt{-\gamma}\,d^3x \qquad (4.2.3.14)$$

After substitution from (4.2.3.13) we get:

$$\frac{a^0}{u^0}\left(\frac{d^2x^i}{ds^2}+H^i_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}\right)+\frac{da^{*i}}{ds}+H^i_{pq}a^{*p}\frac{dx^q}{ds}+$$
$$+\frac{dx^i}{ds}\frac{d}{ds}\left(\frac{a^0}{u^0}\right)=-\frac{1}{2}\psi^l F_{ql}\frac{dx^q}{ds}a^i\,, \qquad (4.2.3.15)$$

where, $a^{*i}=\int_V\left(\underset{e}{G}^{0i}-\underset{e}{G}^{i0}\right)\sqrt{g_{00}}\sqrt{-\gamma}\,d^3x$.

Using the third condition the (4.2.3.15) with a high degree of accuracy is be reduced to the system of equations -

$$\frac{d^2x^i}{ds^2}+H^i_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}=-\frac{1}{2}\psi^l F_{ql}\frac{dx^q}{ds}a^i\,. \qquad (4.2.3.16)$$

These are the equations of motion of the charged material body.

These equations are more general than the equations of pseudo-geodesic line (3.1.2.3) and coincide with them only in cases when the following conditions are valid:

$$\frac{1}{2}F_{lq}\psi^l\frac{dx^q}{ds}=0 \qquad (4.2.3.17)$$

(4.2.3.17) is a necessary and sufficient condition for compatibility of the systems (3.1.2.3) and (4.2.3.16).

The pseudo-geodesic line equations (3.1.2.3) describe free motion of the charged material body in external GEH field, while equations (4.2.3.16), derived basing on using of energy-momentum tensor taking into account the force and energetic interactions between the individual parts of the matter – the compulsory motion accompanied by dissipation of a certain type of energy (as viewed by classic physics), namely there occurs the radiation of the GEH field. Hence it follows that not all charged material body can perform free motion in the given GEH field; for each given GEH field there is a relevant test body which is able to move freely in space without radiation of own GEH field.

This issue will be discussed more detail below when considering the motion of a charged test body in centrally symmetric GEH field.

Since F_{ij} is an anti-symmetric tensor of second order, the condition of (4.2.3.17) is automatically fulfilled if the following conditions are valid along entire pseudo-geodesic line:

$$\frac{1}{\psi^0}\frac{dx^0}{ds}=\frac{1}{\psi^1}\frac{dx^1}{ds}=\frac{1}{\psi^2}\frac{dx^2}{ds}=\frac{1}{\psi^3}\frac{dx^3}{ds}=\lambda\,, \qquad (4.2.3.18)$$

λ can be chosen so that the validity of condition (3.1.2.5) directly follows from the condition $g_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}=1$, which is valid for the case of free motion of the test body.

The mandatory nature of the condition (4.2.3.18) requires special investigation.

The system of equations of motion of uncharged material body is received from here with $F_{ij}=0$, or from the identity $G^{ik}{}_{,k}=0$ (where G^{ik} is Einstein's tensor) obtained from routine Bianchi's identity:

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}=0 \qquad (4.2.3.19)$$

From the viewpoint of the classic physics $mH^i_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}$ is a summed - gravitational + electromagnetic - force effecting the charged material body placed in the external gravitational and electromagnetic fields with potentials g_{ij} and φ_i respectively as if the material body and gravitational and electromagnetic fields are absolutely independent from each other unrelated realities. Such differentiation of the matter and field is a basis of investigation method in classic physics; here it is obtained as a consequence - in the form of integral law – laws of unified field.

From the standpoint of the unified field GEH field in V domain and outside its limits is a unified phenomenon; the field outside the V domain is an uninterrupted continuation of the internal field, which by definition is a material body; motion of the material body is a result of GEH field evolution in four-dimensional spatial-temporal variety.

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CHAPTER V

GEH FIELD OF THE CENTRAL SYMMETRY

In this chapter one of the specific cases of GEH field which has high applied importance is considered. It is a GEH field of the central symmetry.

Due to the extreme complexity of GEH field system of equations we shall attempt to investigate the raised problem directly while using the action function. It has rather simple form in the case in question which facilitates building up of system GEH field of equation.

General solution of this system in parametric form is provided and energy-momentum tensor and current density vector of the respective material world are defined. When using these latter and asymptotic characteristics of GEH field potentials the values of random constants resulting from the integrating of the system of equations are defined.

The issues of kinematics and dynamics in GEH field of central symmetry are also studied.

For the calculation simplicity we shall be limited by the reference system of $SR(x^k, \underset{k}{\vec{E}})$ type, with the system of coordinates x^k is chosen so that in the sites located at distances from the symmetry center it coincides with the spherical coordinates.

5.1. SYSTEM OF EQUATIONS OF GEH FIELD OF CENTRAL SYMMETRY AND ITS GENERAL SOLUTION

5.1.1. ACTION FUNCTION AND SYSTEM OF EQUATIONS OF GEH FIELD OF CENTRAL SYMMETRY

According to (4.1.1.11) the action function depends on parameters g_{ij} and φ_i and their partial derivatives of first order, therefore in the first turn we should define the form of these parameters. In the chosen reference system the components of the metric tensor g_{ij} have the form identical to the one encountered in the case of pure gravitational field of central symmetry:

$$(g_{ij}) = \begin{pmatrix} a(r) & 0 & 0 & 0 \\ 0 & -b(r) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}, \qquad (5.1.1.1)$$

where $a(r)$ and $b(r)$ are some sought functions, which in case in question differ from Schwarzschild's solution. As for φ_i and ψ^i they shall be selected in the following way:

$$\begin{aligned} &\varphi_0(r) = \varphi(r), \quad \varphi_1 = \varphi_2 = \varphi_3 = 0, \\ &\psi^0(r) = \psi(r), \quad \psi^1 = 0, \quad \psi^1 = \psi^2 = \psi^3 = 0, \end{aligned} \qquad (5.1.1.2)$$

where φ, and ψ are also sought functions satisfying the condition (1.2.2.5) $\varphi_i\psi^i = \beta$. From this condition $\psi(r)$ can be expressed through $\varphi(r)$, in particular

$$\psi(r) = \beta/\varphi(r) \qquad (5.1.1.3)$$

From (5.1.1.1) and (5.1.1.2) for covariant components ψ_i we get:

$$\psi_0 = a\psi, \quad \psi_1 = \psi_2 = \psi_3 = 0.$$

In further calculations we shall also need $\sqrt{-g}$ and g^{ik}, which according to (5.1.1.) are equal to:

$$\sqrt{-g} = r^2\sin\theta\sqrt{ab},$$

$$(g^{ij}) = \begin{pmatrix} 1/a(r) & 0 & 0 & 0 \\ 0 & -1/b(r) & 0 & 0 \\ 0 & 0 & -1/r^2 & 0 \\ 0 & 0 & 0 & -1/r^2\sin^2\theta \end{pmatrix}. \qquad (5.1.1.4)$$

The value of H_{ij}^k consists of two summands - Γ_{ij}^k and linear combination F_{ij}, while the form of Christoffel's symbol Γ_{ij}^k for gravitational field of central symmetry is known [1], and F_{ij} and $F_i^{\ j}$ according to (5.1.1.1) and (5.1.1.2) are defined using the following equities:

$$(F_{ij}) = \begin{pmatrix} 0 & \varphi' & 0 & 0 \\ -\varphi' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (F_i^{\ j}) = \begin{pmatrix} 0 & -\frac{1}{b}\varphi' & 0 & 0 \\ -\frac{1}{a}\varphi' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (5.1.1.5)$$

where $\varphi' = \frac{d\varphi}{dr}$, and in matrices (F_{ij}) and ($F_i^{\,j}$) index i is a number of the line and index j - is the number of the column $(i, j = 0,1,2,3)$.

Substituting these values of F_{ij} and $F_i^{\,j}$ and known values of Γ_{ij}^k from [1] in (1.2.2.12) we shall receive the following value for H_{ij}^k:

$$\left(H_{ij}^0\right) = \begin{pmatrix} 0 & a'/2a & 0 & 0 \\ \dfrac{a' - \frac{1}{\eta} a\psi\varphi'}{2a} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\left(H_{ij}^1\right) = \begin{pmatrix} \dfrac{\frac{1}{\eta} a\psi\varphi' - a'}{2b} & 0 & 0 & 0 \\ 0 & b'/2b & 0 & 0 \\ 0 & 0 & -r/b & 0 \\ 0 & 0 & 0 & -\dfrac{2\sin^2\theta}{b} \end{pmatrix}, \qquad (5.1.1.6)$$

$$\left(H_{ij}^2\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1/r & 0 \\ 0 & 1/r & 0 & 0 \\ 0 & 0 & 0 & -\sin\theta\cos\theta \end{pmatrix}, \quad \left(H_{ij}^3\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/r \\ 0 & 0 & 0 & ctg\theta \\ 0 & 1/r & ctg\theta & 0 \end{pmatrix}.$$

Here $a' = \frac{da}{dr}$, $b' = \frac{db}{dr}$. Substituting these values into (4.1.1.11) while taking into account, that the parameters do not depend on x^0 we'll get:

$$\delta\int_0^\infty \sqrt{\frac{a}{b}}\left[\frac{\beta^2 r^2}{8\eta^2}\left(\frac{\varphi'}{\varphi}\right)^2 - 2r\frac{a'}{a} - 2 - 2b\right]dr = 0 \qquad (5.1.1.7)$$

Let's introduce symbols

$$a = e^{\nu},\ b = e^{\lambda},\ \varphi = e^{\frac{2\eta f}{\beta}}, \qquad (5.1.1.8)$$

will yield the following form of (5.1.1.7):

$$\delta\int_0^\infty e^{\frac{\nu-\lambda}{2}}\left[\frac{r^2}{2}\left(\frac{df}{dr}\right)^2 - 2r\frac{d\nu}{dr} - 2 - 2e^{\lambda}\right]dr = 0 \qquad (5.1.1.9)$$

Here ν, λ and f are variable parameters. Hence for these unknown functions we have:

$$\begin{aligned} &\frac{d\lambda}{dr} - \frac{1}{r} + \frac{e^{\lambda}}{r} - \frac{r}{4}\left(\frac{df}{dr}\right)^2 = 0, \\ &\frac{d\nu}{dr} + \frac{1}{r} - \frac{e^{\lambda}}{r} - \frac{r}{4}\left(\frac{df}{dr}\right)^2 = 0, \\ &\frac{d}{dr}\left(r^2 e^{\frac{\nu-\lambda}{2}}\frac{df}{dr}\right) = 0. \end{aligned} \qquad (5.1.1.10)$$

General solution of this system is defined in [5] which after its substitution into (5.1.1.8) for a, b and φ will yield the following value:

$$\zeta = \frac{r}{r_e} = \frac{1}{\tilde{c}}\left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}} \left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}},$$

$$a = \tilde{c}^{\,2}\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^{n}, b = \frac{1}{p^2}\left(p - \sqrt{\frac{1-n}{1+n}}\right)\left(p + \sqrt{\frac{1+n}{1-n}}\right), \qquad (5.1.1.11)$$

$$\varphi = \varphi^0\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta\sqrt{1-n^2}}{\beta}},$$

where p is some parameter from the domain $\sqrt{\frac{1-n}{1+n}} \le p < \infty$, $\varphi_{0,}, n, r_e$ and $\tilde{c}$ - are random constants of integration, when $0 \le n \le 1,\ r_0 > 0,\ \tilde{c} > 0$. From (5.1.1.11) it is evident, that $0 \le r < +\infty$ when $\sqrt{\frac{1-n}{1+n}} \le p < +\infty$.

5.1.2. ASYMPTOTIC BEHAVIOR OF a, b AND φ INDICES WHEN $r \to \infty$

When $p \to \infty$ from the first equity of (5.1.1.11) system we get:

$$\zeta = O(p), \quad p \to \infty \qquad (5.1.2.1)$$

Besides,

$$\lim_{p\to\infty} a = \lim_{r\to\infty} a = \tilde{c}^{\,2} \lim_{p\to\infty}\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^{n} = \tilde{c}^{\,2}$$

On the other hand, in infinity the four-dimensional spatial-and-temporal variety is flat and if x^0 has dimension of length then the following equity is valid:

$$\lim_{r\to\infty} a = 1$$

Comparing the last equities we are getting $\tilde{c}^{\,2} = 1$ and since $\tilde{c} > 0$, then $\tilde{c} = 1$.

After substitution the system (5.1.1.11) will acquire the following form:

$$\zeta = \left(p - \sqrt{\frac{1-n}{1+n}} \right)^{\frac{1-n}{2}} \left(p + \sqrt{\frac{1+n}{1-n}} \right)^{\frac{1+n}{2}},$$

$$a = \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{n}, b = \frac{1}{p^2} \left(p - \sqrt{\frac{1-n}{1+n}} \right) \left(p + \sqrt{\frac{1+n}{1-n}} \right), \quad (5.1.2.2)$$

$$\varphi = \varphi^0 \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{\frac{2\eta\sqrt{1-n^2}}{\beta}}$$

One can use both asymptotic values of these indices representing known values in classic physics, and basic characteristics of material world indicated above to define the remaining integration constants.

Let us at first discuss the method of asymptotic values of the parameters in question. In case of large values of p decomposition of parameter a relative to $1/p$ yields:

$$a = 1 - n \left(\sqrt{\frac{1-n}{1+n}} + \sqrt{\frac{1+n}{1-n}} \right) \frac{1}{p} + \cdots, \quad p \to \infty$$

Hence according to (5.1.2.1) we'll have:

$$a = 1 - \frac{2n}{\sqrt{1-n^2}} \frac{1}{\zeta} + \cdots, \qquad \zeta \to \infty \qquad (5.1.2.3)$$

Comparing this expression with Schwarzschild's solution [1] we shall obtain:

$$2r_g = \frac{2MG}{C^2} = \frac{2nr_e}{\sqrt{1-n^2}}, \qquad (5.1.2.4)$$

where: $c = 3 \cdot 10^{10} \frac{cm}{s}$ is the velocity of light, $G = 6{,}67 \cdot 10^{-8} \frac{cm^3}{g \cdot s^2}$ - is gravitational constant, m - is total mass of the material body in question representing GEH field of central symmetry, r_g - gravitational radius.

From this equity it is obvious that:

$$r_e > r_g \quad \text{at} \quad n < \frac{1}{\sqrt{2}}, \quad r_e = r_g \quad \text{at} \quad n = \frac{1}{\sqrt{2}}$$

$$\text{and} \quad r_e < r_g \quad \text{at} \quad n > \frac{1}{\sqrt{2}} \qquad (5.1.2.5)$$

Equity (5.1.2.4) connects new parameters n and r_e with classic parameters M and G.

Asymptotic behavior of parameter b when $r \to \infty$ is defined from the third equity of system (5.1.2.2), in particular:

$$b = 1 + \frac{2n}{\sqrt{1-n^2}} \frac{1}{\zeta} + \cdots, \qquad \zeta \to \infty$$

Hence according to (5.1.24) we have:

$$b \approx \frac{1}{1 - \frac{2MG}{C^2} \cdot \frac{1}{r}}, \qquad r \to \infty \qquad (5.1.2.6)$$

This expression coincides with Schwarzschild's solution.

Let us calculate asymptotic value of parameter φ when $r \to \infty$. From the last equity of the system (5.1.2.2.) we have:

$$\varphi \approx \overset{0}{\varphi}\left(1 - \frac{4\eta}{\beta}\frac{1}{\zeta}\right), \qquad \zeta \to \infty. \tag{5.1.2.7}$$

The right hand part of this equity with the accuracy of constant summand $\overset{0}{\varphi}$ coincides with electric field potential of point source with a charge:

$$Q = -\frac{4\overset{0}{\varphi}\eta r_0}{\beta} \tag{5.1.2.8}$$

Here Q shall be assumed to be a full electric charge of the considered material world (ref. below). From (5.1.2.8) which establishes link between the new $\overset{0}{\varphi}$, r_0 and classic Q parameters it is possible to define the value of $\overset{0}{\varphi}$

$$\overset{0}{\varphi} = -\frac{Q\beta}{4\eta r_e} \tag{5.1.2.9}$$

It should be noted that asymptotic behavior of φ potential defined by the equity (5.1.2.7) in contrast to classic potential of point source is characterized by that, that at infinity it has a finite (nonzero) constant value. According to (5.1.1.3) the same requirement is met by asymptotic value of ψ, and this in its turn ensures fulfillment of the requirements of (4.1.1.6).

It should be mentioned that when x^0 has the dimension of time, the value of *a* defined in accordance with the second equity of the system (5.1.2.2) is replaced by the value:

$$c^2\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^n.$$

At the same time instead of (5.1.2.2) one should use the system:

$$\zeta = \left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}}\left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}},$$

$$b = \frac{1}{p^2}\left(p - \sqrt{\frac{1-n}{1+n}}\right)\left(p + \sqrt{\frac{1+n}{1-n}}\right), \tag{5.1.2.10}$$

$$a = c^2\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^n, \quad \varphi = \overset{0}{\varphi}\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}}.$$

Comparing expressions for a and φ makes it evident that c^2 for gravitational field plays the same role as $\overset{0}{\varphi}$ for electric field, in particular c^2 is asymptotic value of gravitational field potential (in classic understanding) at infinity. Further when discussing the specific problems we shall demonstrate that the value of $\overset{0}{\varphi}$ depends on the nature of considered material world of central symmetry; from the classic standpoint it means that to various values of classic characteristics $Q, M, \ldots$ correspond the various $\overset{0}{\varphi}$. In this connection the following hypothesis can be put fourth: c^2 also depends on the nature of discussed material world of central symmetry; for the Sun (for a certain GEH field of central symmetry) it has one value, for other heavenly objects – another.

5.1.3. STRUCTURE OF PARAMETERS a, b AND φ IN THE VICINITIES OF THE CENTRAL POINT

Of particular interest is behavior of the parameters *a, b* and φ in the vicinities of the central point $r = 0$, since in accordance with Einstein (ref. sector 4.2.3 of the present work) the field in this point shall be free from singularity [2,3].

From the first equity the system (5.1.2.2) due to the condition $1 - n \geq 0$, we obtain:

$$p \to \sqrt{\frac{1-n}{1+n}} \quad \text{at} \quad \zeta \to 0 \qquad (r \to 0),$$

with

$$p \to \sqrt{\frac{1-n}{1+n}} = O\left(\zeta^{\frac{2}{1-n}}\right), \quad \zeta \to 0 \quad (n \geq 0). \qquad (5.1.3.1)$$

From the second equity of the same system the following condition is valid:

$$a(\zeta) = O\left[\left(p - \sqrt{\frac{1-n}{1+n}}\right)^n\right] = O\left(\zeta^{\frac{2n}{1-n}}\right), \quad \zeta \to 0 \quad (n \geq 0), \qquad (5.1.3.2)$$

i.e., $a \to 0$ at $r \to 0$ as $r^{\frac{2n}{1-n}}$.

Quite similarly from the third and fourth equities of the systems (5.1.2.2) we obtain:

$$b(\zeta) = O\left(\zeta^{\frac{2}{1-n}}\right), \qquad (5.1.3.3)$$

$$\varphi(\zeta) = O\left(\zeta^{\frac{4\eta}{\beta}\sqrt{\frac{1+n}{1-n}}}\right), \quad \zeta \to 0, \quad (n \geq 0). \qquad (5.1.3.4)$$

Below we shall assume that the parameters η and β satisfy the term

$$\eta/\beta \geq 0 \qquad (5.1.3.5)$$

This term will ensure the limitation of the value $\varphi(r)$ in the vicinity of point $r = 0$ with $\varphi(r) \to 0$ at $r \to 0, \eta/\beta > 0$ and $\varphi(2) = const$ in the vicinity of point $r = 0$ at $\eta/\beta = 0$.

From (5.1.3.2) – (5.1.3.4) it is evident that the unified (GEH) field of central symmetry is free from singularity in the point $r = 0$.

5.2. KINEMATICS IN GEH FIELD OF CENTRAL SYMMETRY

5.2.1. DISTANCE BETWEEN TWO POINTS [4]

In accord to (5.1.1.2) the metric is divided into the spatial and temporal parts. Unlike Schwazschild's metric in the considered case $a \geq 0,\ b \geq 0$ for any r from the interval $0 \leq r < \infty$, i.e. x^0 is temporal-like and r – spatial-like coordinate lines in entire four-dimensional spatial-temporal variety. Hence the metric of three-dimensional space can be defined by the following equity:

$$\left(g_{\alpha\beta}\right)=\begin{pmatrix}-b & 0 & 0\\ 0 & -r^2 & 0\\ 0 & 0 & -r^2\sin^2\theta\end{pmatrix} \qquad (5.2.1.1)$$

In accordance with the results in Chapter II the distance between the two points $r_1,\theta_1,\ \phi_1$ and $r_2,\theta_2,\ \phi_2$ of three-dimensional space is defined as the arc length of geodesic line between these points. With this aim it should be solved the system of equations of geodesic lines of considered three-dimensional space with the metric (5.2.1.1):

$$\frac{d^2r}{ds^2}+\frac{b'}{2b}\left(\frac{dr}{2b}\right)^2-\frac{r}{b}\left(\frac{d\theta}{ds}\right)^2-\frac{r\sin^2\theta}{b}\left(\frac{d\phi}{ds}\right)^2=0,$$

$$\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}-\sin\theta\cos\theta\left(\frac{d\phi}{ds}\right)^2=0, \qquad (5.2.1.2)$$

$$\frac{d^2\phi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds}+2ctg\theta\frac{d\theta}{ds}\frac{d\phi}{ds}=0$$

With allowance of boundary conditions

$$\begin{matrix} r=r_1,\ \ \theta=\theta_1 & \text{and} & \phi=\phi_1 & \text{at} & s=0,\\ r=r_2,\ \ \theta=\theta_2 & \text{and} & \phi=\phi_2 & \text{at} & s=J_{1,2}\,,\end{matrix} \qquad (5.2.1.3)$$

where s is a current length of arc of geodesic line and $s_{1,2}$ is the length of the arc between the first and second points. It is the solution of the following equation -

$$b\left(\frac{dr}{ds}\right)^2+r^2\left(\frac{d\theta}{ds}\right)^2+r^2\sin^2\theta\left(\frac{d\phi}{ds}\right)^2=1 \quad \text{at} \quad s=s_{1,2}\,, \qquad (5.2.1.4)$$

where $r(s),\theta(s)$ and $\phi(s)$ are the solution of the problems (5.2.1.2) and (5.2.1.3).

In each special case the procedure indicated here can be realized with due regard to the nuances corresponding to a specific task. We shall consider here one special case when the considered points are located on one coordinate line $\theta=const$ and $\phi=const$, i.e. when $\theta_1=\theta_2$ and $\phi_1=\phi_2$. The solution (5.2.1.2) and (5.2.1.3) shall be sought in the following form: $r=r(s),\ \theta=const$ and $\phi=const$, where $r(s)$ is the Sought function. For this solution from (5.2.1.2) and (5.2.1.3) we shall obtain

$$\frac{d^2r}{ds^2}+\frac{b'}{2b}\left(\frac{dr}{ds}\right)^2=0, \qquad (5.2.1.5)$$

$$r=r_1 \quad \text{at} \quad s=0 \quad \text{and} \quad r=r_2 \quad \text{at} \quad s=s_{12}\,.$$

The first integral of this equation has the following form

$$\frac{dr}{ds}=\frac{\tilde{c}}{\sqrt{b(r)}}\,, \qquad (5.2.1.6)$$

where $\tilde{c}$ is a random constant equal to one since at $r\to\infty$ $\dfrac{dr}{ds}=1$ and $b(r)=1$. Then from (5.2.1.6) we get

$$s_{12}=\int_{r_1}^{r_2}\sqrt{b}dr \qquad (5.2.1.7)$$

From the above indicated consideration it is obvious that coordinate line $\theta=const$ and $\phi=const$ is a geodesic line and arc length of this line between two points are defined in accord to

(5.2.1.7). If $r_1 = 0$ (i.e. the first point of symmetry center) and r_2 is a random point ($r_2 = r$), then (5.2.1.7) can be reduced to the following equity:

$$R = \int_0^r \sqrt{b}dr \,, \qquad (5.2.1.8)$$

which defines the distance from the symmetry center $r = 0$ to a random point of three-dimensional space with a coordinate r in the presence of GEH field of central symmetry.

By substitution of values of b and r from (5.1.2.2) into (5.2.1.8) we get:

$$R = r_e \int_{\sqrt{\frac{1-n}{1+n}}}^{p} \left(\frac{t + \sqrt{\frac{1+n}{1-n}}}{t - \sqrt{\frac{1-n}{1+n}}} \right)^{n/2} dt \qquad (5.2.1.9)$$

The integrand in the point $p = \sqrt{\frac{1-n}{1+n}}$ has a singularity however since $n/2 < 1$, the integral in the right part of the last equity converges. Integrating by parts we shall get:

$$R = \frac{r_e}{1 - n/2} \left\{ \left(p - \sqrt{\frac{1-n}{1+n}} \right) \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{-n/2} - \frac{n}{2} R_{1-n/2}(p) \right\}, \qquad (5.2.1.10)$$

where

$$R_{1-n/2}(p) = \int_{\sqrt{\frac{1-n}{1+n}}}^{p} \left(\frac{t - \sqrt{\frac{1-n}{1+n}}}{t + \sqrt{\frac{1+n}{1-n}}} \right)^{1-n/2} dt$$

It is obvious that

$$R_{1-n/2}(p) = \frac{1}{2 - n/2} \left(p - \sqrt{\frac{1-n}{1+n}} \right) \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{1-n/2} + \frac{1 - n/2}{2 - n/2} R_{2-n/2}(p),$$

$$R_{2-n/2}(p) = \frac{1}{3 - n/2} \left(p - \sqrt{\frac{1-n}{1+n}} \right) \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{2-n/2} + \frac{2 - n/2}{3 - n/2} R_{3-n/2}(p),$$

....

$$R_{k-n/2}(p) = \frac{1}{k + 1 - n/2} \left(p - \sqrt{\frac{1-n}{1+n}} \right) \left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}} \right)^{k-n/2} + \frac{k - n/2}{k + 1 - n/2} R_{k+1-n/2}(p), \qquad (5.2.1.11)$$

where

$$R_{k-n/2}(p)=\int\limits_{\sqrt{\frac{1-n}{1+n}}}^{p}\left(\frac{t-\sqrt{\frac{1-n}{1+n}}}{t+\sqrt{\frac{1+n}{1-n}}}\right)^{k-n/2}dt,\quad k=1,2,... \qquad (5.2.1.12)$$

It can be readily demonstrated that

$$R_{k-n/2}(p)\to 0 \quad \text{at} \quad k\to\infty. \qquad (5.2.1.13)$$

Indeed the sequence

$$\left(\frac{t-\sqrt{\frac{1-n}{1+n}}}{t+\sqrt{\frac{1+n}{1-n}}}\right)^{k-n/2} \qquad k=1,2,...$$

uniformly converges to zero within the interval $\sqrt{\frac{1-n}{1+n}}\le t<p$, hence

$$\int\limits_{\sqrt{\frac{1-n}{1+n}}}^{p}\left(\frac{t-\sqrt{\frac{1-n}{1+n}}}{t+\sqrt{\frac{1+n}{1-n}}}\right)^{k-n/2}dt\to 0 \quad \text{at} \quad k\to\infty,$$

i.e. the condition (5.2.1.13) is valid for every fixed value of parameter p. If the respective values from (5.2.1.11) will be substituted into (5.2.1.10) then the following expression will be obtained for R:

$$R=-r_e\frac{n}{2}\left(p-\sqrt{\frac{1-n}{1+n}}\right)\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{-n/2}\left\{\frac{1}{\left(0-n/2\right)\left(1-n/2\right)}+\right.$$

$$\frac{1}{\left(1-n/2\right)\left(2-n/2\right)}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)+\frac{1}{\left(2-n/2\right)\left(3-n/2\right)}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{2}+...$$

$$\left....+\frac{1}{k-n/2}R_{k-n/2}(p)\right\}.$$

Continuing this process and considering the validity of condition (5.2.1.13) for $R(p)$ we finally get:

$$R(p)=-\frac{r_e n}{2}\left(p-\sqrt{\frac{1-n}{1+n}}\right)\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{-n/2}\sum_{k=0}^{\infty}\frac{1}{\left(k-n/2\right)\left(k+n/2\right)}\times$$
$$\times\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{k}. \qquad (5.2.1.14)$$

This equity in totality with the first equity of the system (5.1.2.2) defines (in parametric form) the functional dependence between R and r, i.e. it defines the distance from the symmetry center to the point with coordinate r.

The series in the right part of this equity uniformly converges in entire infinite region $\sqrt{\frac{1-n}{1+n}} \le p < \infty$ since its majorizing series

$$\sum_{k=0}^{\infty} \frac{1}{\left(k - n/2\right)\left(k+1-n/2\right)}$$

is a converging series. Easy enough to calculate the value of majorizing series; indeed

$$\sum_{k=0}^{\infty} \frac{1}{\left(k - n/2\right)\left(k+1-n/2\right)} = \frac{1}{\left(0-n/2\right)\left(1-n/2\right)} + \frac{1}{\left(1-n/2\right)\left(2-n/2\right)} +$$

$$+\frac{1}{\left(2-n/2\right)\left(3-n/2\right)} + \cdots = -\left(\frac{1}{n/2} + \frac{1}{1-n/2}\right) + \frac{1}{1-n/2} - \frac{1}{2-n/2} +$$

$$+\frac{1}{2-n/2} - \frac{1}{3-n/2} + \cdots = -\frac{2}{n}. \qquad (5.2.1.15)$$

By this value of the majorizing series it is possible to define the asymptotic value of parameter R at $p \to \infty$,$(r \to \infty)$ indeed

$$R(p) = -\frac{r_e n}{2}\left(p - \sqrt{\frac{1-n}{1+n}}\right)\sum_{k=0}^{\infty}\frac{1}{(k-n)\left(k+1-n/2\right)}, \quad p \to \infty,$$

i.e.

$$R(p) = r_e p \qquad \text{at} \qquad p \to \infty .$$

On the other hand from the first equity of (5.1.2.2) system we have:

$$\zeta = p \qquad \text{at} \qquad p \to \infty$$

Then it is evident that

$$R = r \qquad \text{at} \qquad r \to \infty . \qquad (5.2.1.16)$$

Hence, the dimensionless number r (a coordinate) in the infinity coincides with the distance from the symmetry center to a random point.

At $r \to 0$, i.e. at $p \to \sqrt{\frac{1-n}{1+n}}$ according to (5.2.1.14), $R \to 0$, really; in the right part the multiplier $p - \sqrt{\frac{1-n}{1+n}}$ is raised to $1 - n/2 > 0$ power.

Let's re-write the equity (5.2.1.14) in the following way:

$$R(p) = r_e\left(p - \sqrt{\frac{1-n}{1+n}}\right)\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right)^{-n/2} G_n\left(\frac{p - \sqrt{\frac{1-n}{1+n}}}{p + \sqrt{\frac{1+n}{1-n}}}\right), \qquad (5.2.1.17)$$

where

$$G_n(x) = -\frac{n}{2}\sum_{k=0}^{\infty}\frac{x^k}{\left(k-n/2\right)\left(k+1-n/2\right)}, \quad 0 \le x \le 1, \; 0 \le n \le 1. \qquad (5.2.1.18)$$

It is evident that $G_n(x)$ is a monotonously increasing function within the interval $0 \le x \le 1$, hence considering that the value

$$\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}} \quad \text{and} \quad \zeta = \left(p-\sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}} \left(p+\sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}}$$

are similarly monotonously increasing functions of argument p, than easy enough that $R(r)$ is a monotonously increasing function of its argument r in interval $0 \le r \le \infty$.

The values of $G_n(x)$ function for various values of argument x and parameter n are given in the Annex.

5.2.2 TIME [4]

The readings of the watch located in the point with a coordinate r according to the results of Chapter II can be defined by the following equity:

$$t = \frac{1}{c}\int_0^{x^0} \sqrt{a(r)}dx^0 \qquad (5.2.2.1)$$

Since the parameter $a(r)$ does not depend on x^0 we have:

$$t = \frac{x^0}{c}\sqrt{a(r)} \qquad (5.2.2.2)$$

If here instead of $a(r)$ we substitute its value from the second equity of (5.1.2.2) system we shall get:

$$t = \frac{x^0}{c}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2}. \qquad (5.2.2.3)$$

This equity in totality with the first equity of (5.1.2.2) system defines the time in the point with coordinate r; in various points the course of time is different. This effect is defined by multiplier:

$$\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2}$$

and by functional dependence on ζ from p. Thus, for example, in infinitely remote point

$$t_\infty = \frac{x^0}{c}$$

This result is natural since in the infinity the space is flat.

In accord to (5.2.2.3) the time interval Δt in the given point of the space is equal to

$$\Delta t = \frac{1}{c}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2}\Delta x^0. \qquad (5.2.2.4)$$

Let us consider two points with coordinates r_1 and r_2 (or p_1 and p_2) with $r_2 < r_1$, i.e. the point with coordinate r_2 lies closer to the symmetry center compared to the point with coordinate r_1 with $p_2 < p_1$ and hence

$$\left(\frac{p_2 - \sqrt{\frac{1-n}{1+n}}}{p_2 + \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} < \left(\frac{p_1 - \sqrt{\frac{1-n}{1+n}}}{p_1 + \sqrt{\frac{1+n}{1-n}}}\right)^{n/2}.$$

as well

$$\Delta t(r_2) < \Delta t(r_1). \tag{5.2.2.5}$$

Hence it follows that the course of time is faster in the points located at long distances from the symmetry center while as the distance between the point and symmetry center decreases the course of time slows down. The opposite is valid for the oscillation frequency:

$$v(r_2) > v(r_1) \tag{5.2.2.6}$$

This is the effect of red shift.

Basing on using the results of Chapter II and equity (3.1.5.13) the relation between the readings of the watches located in various points of three-dimensional space and coordinates r_1 and r_2 (p_1 and p_2), in particular,

$$t(r_2) = \left(\frac{p_2 - \sqrt{\frac{1-n}{1+n}}}{p_2 + \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} \left(\frac{p_1 + \sqrt{\frac{1-n}{1+n}}}{p_1 - \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} t(r_1). \tag{5.2.2.7}$$

Hence

$$\Delta t(r_2) = \left(\frac{p_2 - \sqrt{\frac{1-n}{1+n}}}{p_2 + \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} \left(\frac{p_1 + \sqrt{\frac{1-n}{1+n}}}{p_1 - \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} \Delta t(r_1), \tag{5.2.2.8}$$

from which it follows, that

$$v(r_2) = \left(\frac{p_2 + \sqrt{\frac{1+n}{1-n}}}{p_2 - \sqrt{\frac{1-n}{1+n}}}\right)^{n/2} \left(\frac{p_1 - \sqrt{\frac{1-n}{1+n}}}{p_1 + \sqrt{\frac{1+n}{1-n}}}\right)^{n/2} v(r_1) \tag{5.2.2.9}$$

If case, when the point with coordinate r_1 lies on the infinity $(r_1 \to \infty)$, and $r_2 = r$ is a random point, then from (5.2.2.7), (5.2.2.8) and (5.2.2.9) we get:

$$t(r)=\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2} t_\infty,$$

$$\Delta t(r)=\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2} \Delta t_\infty, \qquad (5.2.2.10)$$

$$v(r)=\left(\frac{p+\sqrt{\frac{1+n}{1-n}}}{p-\sqrt{\frac{1-n}{1+n}}}\right)^{n/2} v_\infty,$$

where $t_\infty, \Delta t_\infty$ and v_∞ are the values of parameters $t, \Delta t$ and v on the infinity, i.e. in the regions where three-dimensional space is flat.

These equities totality with the first equity of (5.1.2.2) system define the effect of red shift depending on the location of observation point with coordinate r.

5.3. ENERGY AND CHARGE DENSITIES OF GEH FIELD OF CENTRAL SYMMETRY

5.3.1.ENERGY DENSITY [5]

Since the considered metric is divided, according to paragraph 4.2.1. T_0^0 is the energy density

$$T_0^0=-\frac{\alpha}{2\pi}\varphi^p\varphi_p G_0^0. \qquad (5.3.1.1)$$

In the case under consideration (g_{ik} is a diagonal matrix)

$$\begin{aligned} G_0^0 &= g^{00}R_{00}-\frac{1}{2}\left(g^{00}R_{00}+g^{11}R_{11}+g^{22}R_{22}+g^{33}R_{33}\right)= \\ &=\frac{1}{2}\left(\frac{1}{a}R_{00}+\frac{1}{b}R_{11}+\frac{1}{r^2}R_{22}+\frac{1}{r^2\sin^2\theta}R_{33}\right) \end{aligned} \qquad (5.3.1.2)$$

Easy enough to show that

$$\begin{aligned} R_{00} &= -\frac{a''}{2b}+\frac{a'b'}{4b^2}+\frac{a'^2}{4ab}-\frac{a'}{rb}, \\ R_{11} &= -\frac{a''}{2a}-\frac{a'b'}{4ab}-\frac{a'^2}{4a^2}-\frac{b'}{rb}+\frac{\beta}{2\eta}\frac{2\varphi'}{r\varphi}, \\ R_{22} &= -\frac{rb'}{2b^2}+\frac{ra'}{2ab}+\frac{1}{b}-1-\frac{\beta}{2\eta}\frac{r\varphi'}{b\varphi}, \\ R_{33} &= R_{22}\sin^2\theta \end{aligned} \qquad (5.3.1.3)$$

In accordance with the above from (5.3.1.4) we get:

$$G_0^0 = -\frac{1}{4b}\left(\frac{\varphi'}{\varphi}\right)^2 = -\frac{r_e^{\,2}}{r^4 a}. \qquad (5.3.1.4)$$

After substitution for T_0^0 we get

$$T_0^0 = \frac{\alpha}{8\pi}\frac{\varphi'^2}{ab} = \frac{\alpha r_e^{\,2}}{2\pi}\frac{\varphi^2}{a^2 r^4} \qquad (5.3.1.5)$$

This equity with allowance of (5.1.2.2) will acquire the following form:

$$T_0^0 = \frac{\alpha \overset{0}{\varphi}{}^2}{2\pi r_e^{\,2}}\left(\frac{2\eta}{\beta}\right)^2 \frac{\left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{4\eta}{\beta}\sqrt{1-n^2}-2}}{\left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{4\eta}{\beta}\sqrt{1-n^2}+2}}. \qquad (5.3.1.6)$$

Hence it is evident, that at $\zeta \to \infty$, i.e. at $p \to \infty$, the energy density of GEH field of central symmetry is tending to zero and according to (5.1.2.2),

$$T_0^0(r) = O(r^{-4}) \qquad \text{at} \qquad r \to \infty.$$

With the accuracy of infinitely small high-order terms the equity (5.3.1.6) with $r \to \infty$ will assume the following form:

$$T_0^0(r) = \frac{\alpha \overset{0}{\varphi}{}^2 r_e^{\,2}}{2\pi}\left(\frac{2\eta}{\beta}\right)^2 \frac{1}{r^4} + \cdots, \qquad r \to \infty$$

Taking into account (5.1.2.8) this equity is reduced to the following equity:

$$T_0^0(r) = \alpha \frac{Q^2}{8\pi r^4} + \cdots, \quad r \to \infty \qquad (5.3.1.7)$$

The structure $T_0^0(r)$ in the vicinity of the central point $r = 0$, according to (5.1.2.2) is defined by the equity:

$$T_0^0(r) = O\left(r^{\frac{2\left(\frac{4\eta}{\beta}\sqrt{1-n^2}-2\right)}{1-n}} \right). \qquad (5.3.1.8)$$

Hence it is evident. that at

$$n \le \sqrt{1 - \left(\frac{\beta}{2\eta}\right)^2} \qquad (5.3.1.9)$$

$T_0^0(r)$ is free from singularity in central point $r = 0$.

With

$$n > \sqrt{1 - \left(\frac{\beta}{2\eta}\right)^2} \qquad (5.3.1.10)$$

the central point $r = 0$ for $T_0^0(r)$ is an isolated pole of $2(2 - \frac{2\eta}{\beta}\sqrt{1-n^2}) \big/ 1-n$ order.

When using (5.3.1.6) it is possible to define the quantity of energy of GEH field of central symmetry enclosed within the sphere with radius r and center located in the point $r = 0$:

$$E(r) = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{r} T_0^0(r)\sqrt{-g}\,dr$$

After substituting the value of T_0^0, $\sqrt{-g}$ and dr we shall get:

$$E(r) = \frac{\alpha r_e \overset{0}{\varphi}{}^2 \sqrt{1-n}}{\frac{4\eta}{\beta}\sqrt{1-n^2}-n} \left(\frac{2\eta}{\beta}\right)^2 \left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{\frac{4\eta}{\beta}\sqrt{1-n^2}-n} \qquad (5.3.1.11)$$

Full energy of GEH field of central symmetry is equal to:

$$E = \lim_{r\to\infty} E(r) = \frac{\alpha r_e \overset{0}{\varphi}{}^2 \sqrt{1-n^2}}{\frac{4\eta}{\beta}\sqrt{1-n^2}-n} \left(\frac{2\eta}{\beta}\right)^2 . \qquad (5.3.1.12)$$

One asymptotic character of T_0^0 parameter, defined according to (5.3.1.1) should be also indicated. With high values of r it should coincide with T_0^0 from (4.2.1.1). If high values of r the coefficient $\frac{\alpha\varphi_p\varphi^p}{2\pi}$ is tending to $\frac{\alpha \overset{0}{\varphi}{}^2}{2\pi}$, which according to (5.3.1.1) and (4.2.1.1) shall be equal to $1/\chi$, i.e.

$$\frac{\alpha \overset{0}{\varphi}{}^2}{2\pi} = \frac{c^4}{8\pi G}.$$

Hence,

$$\alpha \overset{0}{\varphi}{}^2 = \frac{c^4}{4G}. \qquad (5.3.1.13)$$

5.3.2. ELECTRIC CHARGE DENSITY [5]

Since in the case in question metric of four-dimensional spatial-temporal variety is divided into spatial and temporal parts, it is evident that the density of electric charge is defined by the equity:

$$\rho = \frac{1}{c} J^0 ,$$

where J^0 is a zero component of four-dimensional current density vector. From (4.2.2.3) it is obvious that in the case in question

$$J^0 = -\frac{c}{4\pi r_e^2} \frac{1}{\zeta^2\sqrt{ab}} \frac{d}{d\zeta}\left(\frac{\zeta^2}{\sqrt{ab}} \frac{d\varphi}{d\zeta}\right) \qquad (5.3.2.1)$$

Considering the values of parameters ζ, a, b and φ from (5.1.2.2) the latter equity assumes the following form:

$$\rho = -\frac{\overset{0}{\varphi}\left(\frac{2\eta}{\beta}\sqrt{1-n^2}-n\right)}{\pi r_e^2\sqrt{1-n^2}} \left(\frac{2\eta}{\beta}\right) \frac{\left(p-\sqrt{\frac{1-n}{1+n}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}-2}}{\left(p+\sqrt{\frac{1+n}{1-n}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}+2}} . \qquad (5.3.2.2)$$

Hence it is evident that in asymptote with $r \to \infty$ ρ is estimated by the equity:

$$\rho = O(r^{-4}), \quad r \to \infty . \qquad (5.3.2.3)$$

And in the vicinity of central point $r=0$ structure of electric charge density is defined by the equity:

$$\rho = O\left(r^{\frac{2\left(\frac{2\eta}{\beta}\sqrt{1-n^2}-2\right)}{1-n}}\right), \quad r \to 0 . \qquad (5.3.2.4)$$

If it is required to satisfy the condition

$$n \le \sqrt{1-\left(\frac{\beta}{\eta}\right)^2}\,, \qquad (5.3.2.5)$$

then ρ in the point $r=0$ is free from singularity. If (5.3.2.5) occurs then (5.3.1.9) is more valid. In case when

$$n > \sqrt{1-\left(\frac{\beta}{\eta}\right)^2} \qquad (5.3.2.6)$$

the central point $r=0$ is an isolated pole of electric charge density ρ of $2\left(\frac{2\eta}{\beta}\sqrt{1-\eta^2}-2\right)\Big/1-n$ order.

Quantity of electric charge enclosed in the sphere with radius r and center located in point $r=0$ is defined by the equity

$$Q(r) = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{r} \rho(r)\sqrt{-g}\,dr.$$

Hence from the according to (5.3.2.1) we have:

$$Q(r) = -\int_0^r \frac{d}{dr}\left(\frac{r^2}{\sqrt{ab}}\frac{d\varphi}{dr}\right)dr = -\frac{r^2}{\sqrt{ab}}\frac{d\varphi}{dr}\Big/\begin{matrix} r \\ 0 \end{matrix}.$$

After substitution of the respective values of parameters a, b and φ we get:

$$Q(r) = -2r_e \overset{0}{\varphi} \frac{2\eta}{\eta}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}-n}$$

Let Q is a full electric charge of the considered GEH field of central symmetry, then the following condition takes place:

$$Q = \lim_{r\to\infty} Q(r)$$

From these two latter equities we get:

$$Q = -\frac{4r_e \overset{0}{\varphi}\eta}{\beta} \qquad (5.3.2.7)$$

(5.3.2.7) fully coincides with (5.1.2.8), which early was obtained through absolutely different way. Here it was obtained through fulfilling the condition

$$n \le \frac{1}{\sqrt{1+\left(\frac{\beta}{2\eta}\right)^2}}. \qquad (5.3.2.8)$$

Necessity to satisfy this condition is determined by the fact that with $n > 1\Big/\sqrt{1+\left(\frac{\beta}{2\eta}\right)^2}$ the parameter $Q(r)$ is meaningless. On the other hand the inequity

$$\frac{1}{\sqrt{1+\left(\frac{\beta}{2\eta}\right)^2}} \geq \sqrt{1-\left(\frac{\beta}{\eta}\right)^2}$$

is valid and if n satisfies the condition (5.3.2.5) then condition (5.3.2.8) is automatically satisfied. In connection with this hereinafter we shall require that n satisfies condition (5.3.2.5), which makes sense if $\beta \leq \eta$.

5.4. MOTION OF MATERIAL BODY IN GEH FIELD OF CENTRAL SYMMETRY

5.4.1. MOTION OF UNCHARGED BODY WITH NON-ZERO MASS IN GEH FIELD OF CENTRAL SYMMETRY

In paragraph 4.2.3 material body was defined through using the notions of GEH field and from this definition as a consequence it follows that GEH field located outside of some region V containing material body in the form of GEH field which is characterized by certain features, is an external field independent of the material body. However actually GEH field external in relation to volume V is a continuation of internal field (material body).

Material body in GEH field of central symmetry from the standpoint of GEH field is a unified non-centrally symmetric field developing in time following a certain law and having some small domain V satisfying certain requirements of material body definition provided in 4.2.3. Besides, deviation of GEH field from central symmetry is so insignificant that it can be ignored. Thus a complex problem of development of some complex GEH field in time shall be reduced to a purely classic problem on motion of the point material body in the external static GEH field of central symmetry. Motion of this body is described by the system of equations of geodesic line (or by the system (4.2.3.19)) of four-dimensional spatial-temporal variety with a metric (5.1.1.1). This system has the following form:

$$\frac{d^2r}{ds^2}+\frac{1}{2b}\frac{db}{dr}\left(\frac{dr}{ds}\right)^2-\frac{r}{b}\left(\frac{d\theta}{ds}\right)-\frac{r\sin^2\theta}{b}\left(\frac{d\phi}{ds}\right)^2+$$
$$+\frac{1}{2b}\frac{da}{dr}\left(\frac{dx^0}{dr}\right)^2=0,$$

$$\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}-\sin\theta\cos\theta\left(\frac{d\phi}{ds}\right)^2=0,$$

$$\frac{d^2\phi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds}+2ctg\theta\frac{d\theta}{ds}\frac{d\phi}{ds}=0, \qquad (5.4.1.1)$$

$$\frac{d^2x^0}{ds^2}+\frac{1}{a}\frac{da}{dr}\frac{dr}{ds}\frac{dx^0}{ds}=0.$$

These equations contain exclusively characteristics $\left(\Gamma_{ij}^{k}\right)$ of gravitational field, in which the test body is moving. Solution of the system (5.4.1.1) under certain initial conditions allows to define the trajectory and motion law along this trajectory. This system does not contain gravitational charge (mass) of test body; various bodies with various gravitational charges but equal starting conditions are moving along one trajectory with a similar law. The system of equations considers only the gravitational impact of the gravitational field on the test body .

The first integral of the fourth equity of this system assumes the following form (ref. (3.2.1.16)):

$$a\frac{dx^0}{ds}=c_0=const \qquad (5.4.1.2)$$

In (5.4.1.1) ds is a length of elementary arc of geodesic line

$$ds=\sqrt{a(dx^0)^2-b(dr)^2-r^2(d\theta)^2-r^2\sin^2\theta(d\phi)^2}$$

According to (5.2.2.1)

$$a(dx^0)^2=c^2(dt)^2, \qquad (5.4.1.2')$$

where dt is a time interval measured by the watch located in the given point of the trajectory. Hence,

$$ds=c\sqrt{1-\frac{v^2}{c^2}}dt \qquad (5.4.1.3)$$

where

$$v^2=b\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\theta}{dt}\right)^2+r^2\sin^2\theta\left(\frac{d\phi}{dt}\right)^2$$

Considering the above from (5.4.1.2) we obtain:

$$\frac{\sqrt{a}}{\sqrt{1-\frac{v^2}{c^2}}}=c_0. \qquad (5.4.1.4)$$

If for potential of gravitational field the values of a defined by (5.1.2.10) are used, then in asymptote when the moving body is located far from the center of GEH field of central symmetry and its velocity of movement is low, i.e. when

$$\frac{MG}{c^2r}<<1, \qquad \frac{v}{c}<<1$$

according to (5.1.2.3) and (5.1.2.4)

$$\sqrt{a}=c\left(1-\frac{MG}{c^2r}\right),$$

$$\sqrt{1-\frac{v^2}{c^2}}=1-\frac{1}{2}\frac{v^2}{c^2}.$$

Hence

$$c\cdot c_0\approx c^2\left(1-\frac{MG}{c^2r}\right)\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\approx c^2+\frac{v^2}{2}-\frac{GM}{r}$$

If this equity is multiplied by rest mass of moving body we shall receive full mechanical energy of a moving body E_m

$$E_m = mc^2 + \frac{mv^2}{2} - \frac{GmM}{r} \qquad (5.4.1.5)$$

Hence the equity

$$E_m = \frac{mc\sqrt{a}}{\sqrt{1 - v^2/c^2}}, \quad \left(c_0 = \frac{E_m}{mc}\right) \qquad (5.4.16)$$

defines the total mechanical energy of a moving body in the field of central symmetry if a is defined in accordance with (5.1.2.10) and the equity (5.4.1.4) is a conservation law of mechanical energy of a moving body.

From (5.4.1.4) it is evident that dissipation of mechanical energy of the test body does not occur, in particular, no gravitational waves are radiated by the test body. Obviously in the system consisting only of such sub-systems the dynamic phenomena cannot develop – it is dead and hence unreasonable.

Since asymptotic value of the potential of gravitational field of central symmetry is equal to c^2, mc^2 is an energy of a material body with mass m in gravitational field which has potential c^2. Such definition of energy mc^2 is more natural than currently known name of "energy at rest".

When a is defined according to (5.1.2.2) total mechanical energy of the moving body is defined by the equity

$$E_m = \frac{mc^2\sqrt{a}}{\sqrt{1 - v^2/c^2}}, \quad \left(c_0 = \frac{E_m}{mc^2}\right). \qquad (5.4.1.7)$$

The potential of gravitational field in this case is $c^2 a$, not a.

Let us define other integrals of the system (5.4.1.1). Let us divide the third equation by $\frac{d\phi}{ds}$ and rewrite it in the following form:

$$\frac{d}{ds}\left(\ln r^2 \frac{d\phi}{ds}\right) + 2\frac{\cos\theta}{\sin\theta}\frac{d\theta}{ds} = 0$$

Hence

$$r^2 \sin^2\theta \frac{d\phi}{ds} = c_1 = const. \qquad (5.4.1.8)$$

In accord to this the first and second equities of the system (5.4.1.1) will get the following form:

$$\begin{aligned} &\frac{d^2 r}{ds^2} + \frac{1}{2b}\frac{db}{dr}\left(\frac{dr}{ds}\right)^2 - \frac{r}{b}\left(\frac{d\theta}{ds}\right)^2 - \frac{c_1}{br^3\sin^2\theta} + \frac{c_0^2}{2a^2 b}\frac{da}{dr} = 0, \\ &\frac{d^2\theta}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds} - \frac{c_1^2\cos\theta}{r^4\sin^3\theta} = 0. \end{aligned} \qquad (5.4.1.9)$$

If for θ the initial conditions are applied

$$\theta = \frac{\pi}{2}, \quad \frac{d\theta}{ds} = 0 \quad \text{at} \quad s = 0, \qquad (5.4.1.10)$$

then from the second equation of the system (5.4.1.9) θ we shall get for

$$\theta = \frac{\pi}{2}$$

In such case the first equation for r from system (5.4.1.9) can be rewritten in the following form:

$$\frac{d^2r}{ds^2}+\frac{1}{2b}\frac{db}{dr}\left(\frac{dr}{ds}\right)^2-\frac{c_1^2}{br^3}+c_0^2\frac{1}{2a^2b}\frac{da}{dr}=0\,. \qquad (5.4.1.11)$$

By multiplying this equity by $\frac{dr}{ds}$ we shall easily get the first integral, which coincides with the above defined $g_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds}=1$ integral of the system (5.4.1.1), in particular,

$$\frac{c_0^2}{a}-b\left(\frac{dr}{ds}\right)^2-\frac{c_1^2}{r^2}=1 \qquad (5.4.1.12)$$

This differential equation allows separation of the variables and its integral is defined in the following form:

$$s=\pm\int_{r_0}^{r}\frac{\sqrt{ab}dr}{\sqrt{-a-c_1^2\frac{a}{r^2}+c_0^2}}\,, \qquad (5.4.1.13)$$

where r_0 is an initial value of r, i.e. $r=r_0$ with $s=0$. This latter equity defines the functional dependence of $r(s)$. By substitution of $r(s)$ into (5.4.1.8) and (5.4.1.8) and by integration we get:

$$x^0(s)=x_0^0+c_0\int_0^s\frac{ds}{a},$$
$$\phi(s)=\phi_0+c_1\int_0^s\frac{ds}{r^2}, \qquad (5.4.1.14)$$

where x_0^0 and ϕ_0 are initial values of parameters x^0 and ϕ.

The equities (5.4.1.13) and (5.4.1.14) in aggregate with solution $\theta=\frac{\pi}{2}$ and initial conditions

$$r=r_0,\quad \frac{dr}{ds}=\dot{r}_0$$
$$\phi=\phi_0,\quad \frac{d\phi}{ds}=\tilde{\dot{\phi}}_0 \quad \text{at} \quad s=0 \qquad (5.4.1.15)$$

define the motion law and trajectory of moving material body in three-dimensional space. Arbitrary constants c_0 and c_1 are defined by these initial conditions, namely from (5.4.1.8) we get:

$$c_1=r_0^2\tilde{\dot{\phi}}_0 \qquad (5.4.1.16)$$

and from (5.4.1.12) we get

$$c_0=\pm\sqrt{a(r_0)}\sqrt{1+b(r_0)\dot{r}_0^2+r_0^2\dot{\phi}_0^2} \qquad (5.4.1.17)$$

From (5.4.1.13) and (5.4.1.12) the parameter *s* can be excluded and to receive the functional dependence between r and ϕ

$$\phi=\phi_0\pm c_1\int_{r_0}^{r}\frac{\sqrt{ab}dr}{r^2\sqrt{-a-c_1\frac{2a}{r^2}+c_0^2}}\,, \qquad (5.4.1.18)$$

which determines the shape of the motion trajectory of the material body.

The indicated here equities define the dependence of coordinates of moving point on s. To define the dependence of coordinates r, θ and ϕ on time let us apply equities (5.4.1.2) and (5.4.1.2’) as a result of which we shall obtain:

$$dt=\frac{1}{c}\sqrt{a}dx^0=\frac{c_0}{c}\frac{ds}{\sqrt{a}} \qquad (5.4.1.19)$$

Standard time will be defined by the readings of watches located infinitely far from the origin of coordinates, i.e. $t_c = t_\infty$. Then according to (5.2.2.10)

$$dt = \sqrt{a}\,dt_c$$

i.e.

$$dt_c = \frac{c_0}{c}\frac{ds}{a}.$$

Taking into account the above from (5.4.1.13) we shall obtain:

$$t_c = \pm\frac{c_0}{c}\int_{r_0}^{r}\frac{\sqrt{\frac{b}{a}}dr}{r^2\sqrt{-a - c_1^2\frac{a}{r^2} + c_0^2}}, \qquad (5.4.1.20)$$

which, in totality to (5.4.1.18) defines functional dependencies of $r(t)$ and $\phi(t)$, i.e. the motion law of material body according to the standard time.

From the practical standpoint the initial conditions (5.4.1.15) are not convenient since they are written for coordinates r, ϕ and for the parameter s. It is more convenient to formulate the initial conditions in parameters R, ϕ and t. Distance R from the central point to the point with r coordinate is defined by the equity (ref. (5.2.1.8))

$$R = \int_0^r \sqrt{b(r)}dr. \qquad (5.4.1.21)$$

From (5.4.1.19) and (5.4.1.21) we have:

$$\frac{dR}{dt} = \frac{c}{c_0}\sqrt{ab}\frac{dr}{ds},$$

$$\frac{d\phi}{dt} = \frac{c}{c_0}\sqrt{a}\frac{d\phi}{ds}.$$

Thus, for R and ϕ relative to t we have the following initial conditions:

$$R = R_0 = \int_0^{r_0}\sqrt{b(r)}dr, \quad \frac{dR}{dt} = \dot{R}_0 = \frac{c}{c_0}\sqrt{a(r_0)b(r_0)}\dot{r}_0,$$

$$\phi = \phi_0, \ \frac{d\phi}{dt} = \dot{\phi}_0 = \frac{c}{c_0}\sqrt{a(r)}\tilde{\dot{\phi}}_0, \qquad \text{at} \qquad t = 0.$$

From this if $a(r)$ is defined by the equity (5.1.2.2) we get:

$$r = r_0 = r(R_0), \quad \frac{dr}{ds} = \frac{E_m \dot{R}_0}{mc^3}\frac{1}{\sqrt{a(R_0)b(R_0)}},$$
$$\phi = \phi_0, \ \frac{d\phi}{ds} = \frac{E_m\dot{\phi}_0}{mc^3}\frac{1}{\sqrt{a(R_0)}}, \quad \text{at} \quad t = 0, \qquad (5.4.1.22)$$

where $r(R)$ is an inverse function of function (5.4.1.21).

Constants c_0, c_1 (and also E_m) can be expressed through R_0, $\phi_0, \dot{R}_0, \dot{\phi}_0$ in particular, from (5.4.1.16), (5.4.1.17) and (5.4.1.22) we have:

$$c_0^2 = \frac{a(R_0)}{1 - \frac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right]},$$

$$E_m^2 = \begin{cases} \dfrac{m^2 c^2 a(R_0)}{1 - \dfrac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right]} & \text{if } a \text{ is defined by (5.1.2.10),} \\ \dfrac{m^2 c^4 a(R_0)}{1 - \dfrac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right]} & \text{if } a \text{ is defined by (5.1.2.2),} \end{cases}$$

$$c_1 = \frac{E_m \dot{\phi}_0 r^2(R_0)}{mc^3\sqrt{a(R_0)}} = \frac{c_0}{c}\frac{\dot{\phi}_0 r^2(R_0)}{\sqrt{a(R_0)}}. \qquad (5.4.1.23)$$

These equities limit the initial values of the parameters R and ϕ and their derivatives in time, in particular motion of the material body is possible if the initial values meet the requirement:

$$1 - \frac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right] > 0$$

When using the system (5.1.2.2) the law of material body motion represented by the equities (5.4.1.2) and (5.4.1.18) will assume the following form:

$$t_c = \pm\frac{c_0 r_e{}^2}{c}\int_{p_0}^{p}\frac{d\zeta'}{\sqrt{A(\zeta')}},$$

$$A(\zeta') = c_0^2 r_e{}^2 \left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{2n} - r_e{}^2\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{3n} -$$

$$- \frac{c_1^2}{\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{4n},$$

$$\phi = \phi_0 \pm c_1\int_{p_0}^{p}\frac{d\zeta'}{\sqrt{B(\zeta')}},$$

$$B(\zeta') = c_0^2 r_e^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)^{2-2n}\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)^{2+2n} -$$

$$- r_e^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)^{2-n}\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)^{2+n} - \qquad (5.4.1.24)$$

$$- c_1^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right),$$

$$r = r_e\left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}}\left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}}$$

5.4.2. LIGHT BEAM PROPAGATION IN GEH FIELD OF CENTRAL SYMMETRY

According to classic physics propagation of the light beam is described by the system of equations (5.4.1.1), in which unlike the material body with rest mass m, s is some invariant parameter rather than length of geodesic line arc of four-dimensional spatial-temporal variety. In the case considered the light beam is propagated along the isotropic geodesic line $g_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds}=0$. In this connection integral (5.4.1.12) of the system (5.4.1.1) is replaced by integral:

$$\frac{c_0^2}{a}-b\left(\frac{dr}{ds}\right)^2-\frac{c_1^2}{r^2}=0\,, \qquad (5.4.2.1)$$

and equity (5.4.1.13) – by the equity:

$$s=\pm\int_{r_0}^{r}\frac{\sqrt{ab}d^2}{\sqrt{c_0^2-c_1^2\frac{a}{r^2}}} \qquad (5.4.2.2)$$

In addition, instead of (5.4.1.17) we shall get:

$$c_0=\pm\sqrt{a(r_0)}\sqrt{b(r_0)\dot{r}_0^2+r_0^2\tilde{\dot{\phi}}_0^2}\,, \qquad (5.4.2.3)$$

and instead of (5.4.1.20) and (5.4.1.18) – the equities:

$$\begin{aligned} t_c&=\pm\frac{c_0}{c}\int_{r_0}^{r}\frac{\sqrt{\frac{b}{a}}dr}{\sqrt{c_0^2-c_1^2\,a/r^2}}\,,\\ \phi&=\phi_0\pm c_1\int_{r_0}^{r}\frac{\sqrt{ab}dr}{r^2\sqrt{c_0^2-c_1^2\,a/r^2}}\,, \end{aligned} \qquad (5.4.2.4)$$

which define the functional dependences $r(t)$ and $\phi(t)$ i.e. trajectory and law of beam propagation in space.

For initial values the system (5.4.1.22) in considered case will assume the following form:

$$\begin{aligned} &r=r_0=r(R_0), \qquad \frac{dr}{ds}=\frac{c_0\dot{R}_0}{c}\frac{1}{\sqrt{a(R_0)b(R_0)}}\,,\\ &\phi=\phi_0\,, \quad \frac{d\phi}{ds}=\frac{c_0\dot{\phi}_0}{c}\frac{1}{\sqrt{a(R_0)}} \quad \text{at} \quad s=0. \end{aligned} \qquad (5.4.2.5)$$

Constant c_1 can be defined from (5.4.1.16) with allowance of (5.4.2.5), namely

$$c_1=\frac{c_0r^2(R_0)\dot{\phi}_0}{c}\frac{1}{\sqrt{a(R_0)}}\,. \qquad (5.4.2.6)$$

As for c_0 it cannot be defined from (5.4.2.3) and (5.4.2.5), as it was done in previous segment; indeed, if the values of $\dot{r}_0$ and $\tilde{\dot{\phi}}_0$ from (5.4.2.5) are substituted into (5.4.2.3), then c_0 is reduced and the following condition is obtained:

$$\dot{R}_0^2+r_0^2\dot{\phi}_0^2=c^2\,. \qquad (5.4.2.7)$$

Thus the real initial values of parameters R_0, ϕ_0 and their derivatives in time shall satisfy bond condition (5.4.2.7) which limits the degree of freedom of choice of initial conditions.

As it has been indicated s is a random invariant parameter provided taking into account, that the system (5.4.1.1) is invariant relative to transformations $s=\alpha s'$ where α is a random constant value and s' - a new invariant parameter then according to (5.4.1.2) it will become evident that c_0 can be equated to one.

Thus for the random constants c_0 and c_1 we have:

$$c_0 = 1, \quad c_1 = \frac{r^2(R_0)\dot{\phi}_0}{c\sqrt{a(R_0)}}. \qquad (5.4.2.8)$$

By using the system (5.1.2.2) the system (5.4.2.4) can be rewritten in the following form:

$$t_c = \pm\frac{r_e^{\,2}}{c}\int\limits_{p_0}^{p}\frac{d\zeta'}{\sqrt{A(\zeta')}},$$

$$A(\zeta') = r_e^{\,2}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{2n} - \frac{c_1^2}{\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)}\times$$

$$\times\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{4n},$$

$$\phi = \phi_0 \pm c_1\int\limits_{p_0}^{p}\frac{d\zeta'}{\sqrt{B(\zeta')}},$$

$$B(\zeta') = r_e^{\,2}\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)^{2-2n}\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)^{2+2n} - \qquad (5.4.2.9)$$

$$-c_1^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right),$$

$$r = \zeta\, r_e = r_e\left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}}\left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}}$$

5.4.3. MOTION OF A CHARGED MATERIAL BODY IN GEH FIELD OF CENTRAL SYMMETRY

According to (4.2.3.17) equations of free motion of a charged material body have the following form:

$$\frac{d^2x^k}{ds^2}+\Gamma^k_{pq}\frac{dx^k}{ds}\frac{dx^q}{ds}+\frac{1}{2\eta}F^k_p\frac{dx^p}{ds}\psi_q\frac{dx^q}{ds}=0\,. \qquad (5.4.3.1)$$

They contain solely characteristics of GEH field (they contain parameters H^k_{ij}) and do not contain characteristic parameters of the test body. It is indicative of the fact that not all kinds of test body can perform free motion in GEH field of central symmetry. Characteristic parameters of the test body shall meet certain conditions to enable free motion to take place.

In general case these conditions are difficult to identify, that is why we shall consider the cases when the distance between the symmetry center and test body is much larger than r_e.

For GEH field of central symmetry the third summand in the left part of the system (5.4.3.1) assumes the following form:

$$\frac{1}{2\eta}\left(\psi_0\frac{dx^0}{ds}\right)F^k_0\frac{dx^0}{ds}=\begin{cases}\dfrac{\beta}{2\eta c^2\varphi}\dfrac{d\varphi}{dr} & \text{at } k=1\\ 0 & \text{at } k\neq 1\end{cases}$$

From this it is evident, that equations of free motion of any test body was a small m and electric charge q, which meet the condition of selectivity

$$\frac{1}{2\eta}\left(\psi_0\frac{dx^0}{ds}\right)=\frac{\beta c}{2\eta\overset{0}{\varphi}}sign\left(\frac{dx^0}{ds}\right)=-\frac{q}{mc}\ - \qquad (5.4.3.2)$$

take the following form:

$$\frac{d^2x^k}{ds^2}+\Gamma^k_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}-\frac{q}{mc^2}F^k_p\frac{dx^p}{ds}=0 \qquad (5.4.3.3)$$

In the case in question the (5.4.3.2) has the following form:

$$\frac{\beta}{2\eta}\frac{a}{\varphi}\frac{dx^0}{ds}=-\frac{q}{mc^2}\,, \qquad (5.4.3.4)$$

and (5.4.3.3) is reduced to the system:

$$\begin{aligned}&\frac{d^2r}{ds^2}+\frac{1}{2b}\frac{db}{dr}\left(\frac{dr}{ds}\right)^2-\frac{r}{b}\left(\frac{d\theta}{ds}\right)^2-\frac{r\sin^2\theta}{b}\left(\frac{d\phi}{ds}\right)^2+\\&+\frac{1}{2b}\frac{da}{dr}\left(\frac{dx^0}{ds}\right)^2-\frac{q}{mc^2}F^1_0\frac{dx^0}{ds}=0,\\&\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}-\sin\theta\cos\theta\left(\frac{d\phi}{ds}\right)^2=0,\\&\frac{d^2\phi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds}+2ctg\theta\frac{d\theta}{ds}\frac{d\phi}{ds}=0,\\&\frac{d^2x^0}{ds^2}+\frac{1}{a}\frac{da}{dr}\frac{dx^0}{ds}\frac{dr}{ds}-\frac{q}{mc^2}F^0_1\frac{dr}{ds}=0.\end{aligned} \qquad (5.4.3.5)$$

Parameters of electron or positron satisfy condition (5.4.3.2) in cases when GEH field of central symmetry is proton. In case of positron the trajectory is infinite and in case of electron it is finite. Hence electron with proton form a stable system (hydrogen atom). It is stable since electron in GEH field of central symmetry being a proton performs free motion, it does not radiate its own GEH field.

Similarly to previous one, these equations allow flat motion and if this plane coincides with the plane $\theta=\pi/2$ the system (5.4.3.5), if here we substitute the values F^1_0 and F^0_1 from (5.1.1.5) will get such a form:

$$\frac{d^2r}{ds^2}+\frac{1}{2b}\frac{db}{dr}\left(\frac{dr}{ds}\right)^2-\frac{r}{b}\left(\frac{d\phi}{ds}\right)^2+\frac{1}{2b}\frac{da}{dr}\left(\frac{dx^0}{ds}\right)^2+$$
$$+\frac{q}{mc^2}\frac{1}{b}\frac{d\varphi}{dr}\frac{dx^0}{ds}=0,$$
$$\frac{d^2\phi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds}=0, \qquad (5.4.3.6)$$
$$\frac{d^2x^0}{ds^2}+\frac{1}{a}\frac{da}{dr}\frac{dx^0}{ds}+\frac{q}{mc^2}\frac{1}{a}\frac{d\varphi}{dr}\frac{dr}{ds}=0.$$

Similarly to previous one (or see (3.2.2.7)) the last equity of this system has the following first integral:

$$a\frac{dx^0}{ds}+\frac{q}{mc^2}\varphi=c_0=\frac{E_{me}}{mc^2}=const\,, \qquad (5.4.3.7)$$

where E_{me} is a total (mechanical and electric) energy of moving material GEH in body field of central symmetry. The (5.4.3.7) is a law of energy conservation (see the paragr. 3.2.2).

Considering the fact that

$$\sqrt{a}dx^0=cdt,\quad b\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2=v^2\,, \qquad (5.4.3.8)$$

where v is the velocity of motion of the body in question, the following is obtained from (5.4.3.7):

$$\frac{\sqrt{a}mc^2}{\sqrt{1-v^2/c^2}}+q\varphi=E_{me}\,. \qquad (5.4.3.9)$$

Let us consider asymptote of this equity at $r\to\infty$, i.e. at $p\to\infty$. Considering that:

$$\sqrt{a}=\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{n/2}\approx 1-\frac{nr_e}{\sqrt{1-n^2}}\cdot\frac{1}{r},\quad p>>1 \qquad (5.4.3.10)$$

which according to (5.1.2.4) can be re-written in the following way:

$$\sqrt{a}=1-{r_g}/{r}\,, \qquad (5.4.3.11)$$

when $r_g={MG}/{c^2}$. Quite similarly:

$$\varphi=\overset{0}{\varphi}\left(\frac{p-\sqrt{\frac{1-n}{1+n}}}{p+\sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta\sqrt{1-n^2}}{\beta}}\approx\overset{0}{\varphi}\left(1-\frac{4\eta r_e}{\beta}\cdot\frac{1}{r}\right),\quad p>>1 \qquad (5.4.3.12)$$

This equity according to (5.1.2.8) can be rewritten in the following way

$$\varphi=\overset{0}{\varphi}+\frac{Q}{r}\,. \qquad (5.4.3.13)$$

After substitution of values $\sqrt{a}$ and φ from (5.4.3.9) we shall have:

$$\frac{mc^2\left(1-{r_g}/{r}\right)}{\sqrt{1-v^2/c^2}}+q\overset{0}{\varphi}+\frac{qQ}{r}=E_{me}\,. \qquad (5.4.3.14)$$

Hence for lower (relative to c) velocities we get:

$$E_{me} = mc^2 + q\overset{0}{\varphi} + \frac{mv^2}{2} - \frac{GmM}{r} + \frac{qQ}{r}. \qquad (5.4.3.15)$$

The sum of three subsequent terms in the right part is a full energy of a moving body with mass m and charge q in the external gravitational and electric fields of point source with mass M and charge Q.

Thus when performing free motion of a charged test body while satisfying the condition (5.4.3.2) the full energy of the test body is preserved, no dissipation of the full energy occurs or which is the same, the test body does not radiate the respective GEH field. This circumstance is a basic reason of stability, for example of hydrogen atom. With manifestation of a excitation factor in the form of external GEH field a GEH field of complex structure is formed in the space resulting in violation of the basic condition (5.4.3.2) and in the hydrogen atom start to form dynamic phenomena accompanied either by atom ionization or irradiation of GEH field, etc. These dynamic phenomena are described by equations (4.2.3.16), in which g_{ik}, φ_i and ψ^k are potentials of the resulting dynamic field with a complex configuration.

At very large distances from the field source the space is flat and if the initial velocity of the moving material body is equal to zero, then the body will be in a state of rest at any moment of time and (5.4.3.15) is reduced to the equity:

$$E_{me} = mc^2 + q\overset{0}{\varphi}. \qquad (5.4.3.16)$$

This is the very energy of a motionless material body with a small m and charge q in gravitational field with potential c^2 and electrostatic field with potential $\overset{0}{\varphi}$, i.e. rest energy. It differs from so far known rest energy by the value $q\overset{0}{\varphi}$.

Similarly to previous case in question the motion integral is an equity (5.4.1.8). Taking this into account the motion integral $g_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds} = 1$ will acquire the following form:

$$a\left(\frac{dx^0}{ds}\right)^2 - b\left(\frac{dr}{ds}\right)^2 - \frac{c_1^2}{r^2} = 1.$$

Hence according to (5.4.3.7) we have:

$$\frac{1}{a}\left(c_0 - \frac{q\varphi}{mc^2}\right)^2 - b\left(\frac{dr}{ds}\right)^2 - \frac{c_1^2}{r^2} = 1. \qquad (5.4.3.17)$$

This differential equation allows separation of variables and its integral has the following form:

$$s = \pm\int_{r_0}^{r} \frac{\sqrt{ab}\,dr}{\sqrt{\left(c_0 - {q\varphi}/{mc^2}\right)^2 - a - a\,{c_1^2}/{r^2}}}, \qquad (5.4.3.18)$$

where r_0 is an initial value r, i.e. $r_0 = r$ with $s = 0$. (5.4.3.18) defines functional dependence $r(s)$. By substitution of $r(s)$ into (5.4.3.7) and (5.4.1.8) let's define $x^0(s)$ and $\phi(s)$:

$$x^0(s) = x_0^0 + \int_0^s \left(c_0 - {q\varphi}/{mc^2}\right)\frac{ds}{a},$$
$$\phi(s) = \phi_0 + c_1\int_0^s \frac{ds}{r^2}, \qquad (5.4.3.19)$$

where x_0^0 and ϕ_0 are initial values (with $s = 0$) of parameters x^0 and ϕ. The last three equities in the aggregate with initial conditions

$$r = r_0, \quad \frac{dr}{ds} = \dot{r}_0, \qquad \phi = \phi_0, \quad \frac{d\phi}{ds} = \tilde{\dot{\phi}}_0 \quad \text{at} \quad s = 0 \tag{5.4.3.20}$$

define the law of the charged material body motion in the GEH field of central symmetry. Random constants of c_0 and c_1 integration are defined from the initial conditions in particular, from (5.4.1.8) and (5.4.3.17) we get:

$$c_1 = r_0^2 \tilde{\dot{\phi}}_0, \qquad c_0 = \frac{q\varphi(r_0)}{mc^2} \pm \sqrt{a(r_0)}\sqrt{1 + b(r_0)r_0^2 + \frac{c_1^2}{r_0^2}}. \tag{5.4.3.21}$$

By excluding s from (5.4.3.18) and (5.4.3.19) we shall define the equation of the motion trajectory

$$\phi(r) = \phi_0 \pm c_1 \int_{r_0}^{r} \frac{\sqrt{ab}\,dr}{r^2\sqrt{\left(c_0 - \frac{q\varphi}{mc^2}\right)^2 - a - \frac{ac_1^2}{r^2}}}. \tag{5.4.3.22}$$

Dependence of the coordinates r, θ and ϕ on the real time t can be also defined. From (5.4.3.7) we get:

$$dt = \frac{1}{c}\sqrt{a}\,dx^0 = \frac{1}{c\sqrt{a}}\left(c_0 - \frac{q\varphi}{mc^2}\right)ds. \tag{5.4.3.23}$$

Taking this into account and similarly to previous one introducing standard time, we shall have:

$$t_c = \pm\frac{1}{c}\int_{r_0}^{r} \frac{\left(c_0 - \frac{q\varphi}{mc^2}\right)\sqrt{\frac{b}{a}}\,dr}{\sqrt{\left(c_0 - \frac{q\varphi}{mc^2}\right)^2 - a - \frac{ac_1^2}{r^2}}}, \tag{5.4.3.24}$$

which in aggregate with (5.4.3.22) defines the functional dependencies $r(t)$ and $\phi(t)$, i.e. the law of the charged material body motion in GEH field of central symmetry.

Similarly to previous one the initial values of the coordinates can be expressed through R_0, $\dot{R}_0$, ϕ_0 and $\dot{\phi}_0$, in particular:

$$r = r_0 = r(R_0), \quad \frac{dr}{ds} = \dot{r}_0 = \frac{\left(c_0 - \frac{q\varphi(R_0)}{mc^2}\right)}{c\sqrt{a(R_0)b(R_0)}}\dot{R}_0, \qquad \phi = \phi_0, \quad \frac{d\phi}{ds} = \tilde{\dot{\phi}}_0 = \frac{\left(c_0 - \frac{q\varphi(R_0)}{mc^2}\right)}{c\sqrt{a(R_0)}}\dot{\phi}_0 \quad \text{at} \quad s = o \tag{5.4.3.25}$$

Besides, for the constants c_0 and c_1 we have:

$$\left(c_0 - \frac{q\varphi(R_0)}{mc^2}\right)^2 = \frac{a(R_0)}{1 - \frac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right]}, \qquad c_1 = \frac{\left(c_0 - \frac{q\varphi(R_0)}{mc^2}\right)r^2(R_0)\dot{\phi}_0}{c\sqrt{a(R_0)}}. \tag{5.4.3.26}$$

According to (5.4.3.7)

$$E_{me} = q\varphi(R_0) + \frac{mc^2\sqrt{a(R_0)}}{\sqrt{1-\frac{1}{c^2}\left[\dot{R}_0^2 + r^2(R_0)\dot{\phi}_0^2\right]}} \qquad (5.4.3.27)$$

In a given paragraph we always took into account the fact that a is defined from equity (5.1.2.2). In case when a is defined from (5.1.2.10) in all above expressions mc^2 shall be replaced by mc.

When using (5.1.2.2) the law of motion (5.4.3.22), (5.4.3.24) will acquire the following form:

$$t_c = \pm\frac{r_e^{\ 2}}{c}\int_{p_0}^{p}\frac{[A(\zeta')]d\zeta'}{\sqrt{B(\zeta')}},\quad A(\zeta') = c_0 - \frac{q\overset{0}{\varphi}}{mc^2}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}},$$

$$B(\zeta') = r_e^{\ 2}\left[c_0 - \frac{q\overset{0}{\varphi}}{mc^2}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}}\right]^2 \times$$

$$\times\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{2n} - r_e^2\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{3n} -$$

$$-\frac{c_1^2}{\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{4n},$$

$$\phi = \phi_0 \pm c_1\int_{p_0}^{p}\frac{d\zeta'}{\sqrt{C(\zeta')}},$$

$$C(\zeta') = r_e^{\ 2}\left[c_0 - \frac{q\overset{0}{\varphi}}{mc^2}\left(\frac{\zeta' - \sqrt{\frac{1-n}{1+n}}}{\zeta' + \sqrt{\frac{1+n}{1-n}}}\right)^{\frac{2\eta}{\beta}\sqrt{1-n^2}}\right]^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)^{2-2n} \times$$

$$\times\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)^{2+2n} - r_e^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)^{2-n}\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right)^{2+n} -$$

$$-c_1^2\left(\zeta' - \sqrt{\frac{1-n}{1+n}}\right)\left(\zeta' + \sqrt{\frac{1+n}{1-n}}\right),$$

$$r = r_e\left(p - \sqrt{\frac{1-n}{1+n}}\right)^{\frac{1-n}{2}}\left(p + \sqrt{\frac{1+n}{1-n}}\right)^{\frac{1+n}{2}} \qquad (5.4.3.28)$$

In general case the motion of the charged material body is described by the system of equations (4.2.3.16). To use these equations in case of two charged material bodies (in the sense of classic physics) of which one is a central body and the other – a test body, it is necessary to make use of the GEH field, which is a model of two bodies. This is the GEH field having a dynamic nature with a complex spatial configuration acquires the nature of static GEH field of central symmetry in the vicinity of the center of the central body, while being placed near the center of the test body – the nature of dynamic GEH field.

Since the equations (4.2.3.16) contain all characteristic parameters required for both central and test bodies some conditions of selectivity, similar to condition (5.4.3.2) become unnecessary.

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CHAPTER VI

SOME APPLICATIONS OF THEORY OF GEH FIELD OF CENTRAL SYMMETRY

Here we shall try to use the results obtained in previous chapter for approximate description of internal structure of material world possessing the central symmetry. If material object is at large distances from the external world and if it consists of a large number of grains (in classic sense) than for accuracy can be considered high decree of centrally-symmetric material world. Centrally-symmetric material world are for example heavenly bodies, heavy atomic nuclei, elementary particles, etc.

Approximate coincidence of the model (GEH field) with respective material world apart from the above mentioned reasons is also conditioned by the fact that the model is statistic and characteristic parameters of GEH field are regular values within entire space and do not allow existence of individual sub-regions with strong and weak concentrations of GEH field clearly demarcated by closed surfaces.

6.1. APPLICATION OF THE THEORY OF GEH FIELD OF CENTRAL SYMMETRY IN ASTROPHYSICS

6.1.1. STATIC MODEL OF THE SUN

The world is a unified GEH field in the process of its evolution, while the Sun as the part of the entire world as well is GEH field changing in time, However it is changing so slowly that generations of people do not notice this change and the Sun seems to be a static object (for several generations). In this connection for creation model of the Sun we can use a static GEH field. Besides, in some region of the space representing a sphere with radius L of the order of about half of the distance from the Sun to the nearest stars GEH field can be assumed (approximately) centrally-symmetrical.

Investigation of evolution of the entire GEH field, filling entire space is practically an inaccessible problem for the human being and not only because that the the respective Cauchi problem is extremely complex but also because baseline information about the state of the world (GEH field) in the initial moment of time is required to solve this problem; it is practically impossible to collect such information.

In this connection to develop a static model of the Sun we shall use a semi-classic method according to which the Sun is considered as a material body placed in some external gravitational and electromagnetic fields, which actually are the continuation of the Sun considering it as a GEH field.

According to the above mentioned in some cases we consider the Sun as a static GEH field of central symmetry and in other – as a classic material body; this is the very essence of semi-classic consideration of the problem under study.